Bargaining and Reputation
Abreu and Gul – 2000

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Sargent Reading Group
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• Two players bargain over a pie of size 1.
  • Take turns to make offers
  • Consider the continuous-time limit

• Complete information, alternating offers (Rubinstein, 1982)

\[ 1 - v^1 = \delta_2 v^2 \quad 1 - v^2 = \delta_1 v^1 \]

• This paper: uncertainty about strategic posture
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  • Take turns to make offers
  • Consider the continuous-time limit

• Complete information, alternating offers (Rubinstein, 1982)

\[ v^1(\delta_1, \delta_2) = \frac{1 - \delta_2}{1 - \delta_1 \delta_2} \]

• This paper: uncertainty about strategic posture
Types

Perturb the Rubinstein game

- Player $i$ is *behavioral* with probability $z^i$
- With cond. probability $\pi^i(\alpha)$, $i$ is type $\alpha$
  - Type $\alpha$ characterized by its strategy
  - Demands $\alpha$, accepts offers $\geq \alpha$

- With probability $1 - z^i$, $i$ is rational and *strategic*.

Two questions

- Does the *rational* type *pretend* to be crazy?
- Does it matter (in the limit as $z \to 0$)?
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Two questions

- Does the *rational* type *pretend* to be crazy? ✓
- Does it matter (in the limit as $z \to 0$)? ✓
• Time 0
  • P1 chooses a demand $\alpha_1$
    $\implies$ P1 is either rational or behavioral of type $\alpha_1$
  • P2 can accept or demand $\alpha_2 > 1 - \alpha_1$
    Maybe P2 is type $\alpha_2$, maybe strategically
  • P1 can concede or reject

• Time $t > 0$
  • Protocol calls for $i$ to make demands at certain (frequent) times
  • Changing the demand = revealing rationality
  • State variable: posterior of crazy
  • Revealing rationality $\preceq$ conceding
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  • Changing the demand = revealing rationality
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• Player $i$ chooses
  • A (distribution over) type to mimic $\alpha^i$
  • A probability of concession before time $t$, $F^i_{\alpha^1, \alpha^2}(t)$
    • $F^i_{\alpha^1, \alpha^2}(0) > 0$ means concession after initial demands are made

• Subgame after initial demands: war of attrition / chicken
• Indifference conditions $\sim$ mixed strategies:
  1. At most one player concedes at time 0
  2. Player $i$ concedes with constant hazard rate

$$\lambda^i = r^i \frac{1 - \alpha^i}{\alpha^i - (1 - \alpha^i)}$$

3. At some $T^0 < \infty$ the posterior of crazy reaches 1 for both players
• Player $i$ chooses
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Concessions

Concession rate

\[ \lambda^i = r^i \frac{1 - \alpha^i}{\alpha^i - (1 - \alpha^i)} \]

- Concede faster against more **impatient** opponents
- Concede more slowly against more **greedy** opponents
- Concede slowly when demand is high

- ‘Time to exhaustion’ \( T^i \):
  
  \[ 1 - z^i = F^i_{\alpha^i, \alpha^2}(T^i) = 1 - e^{-z^i} \]

- If \( T^i > T^j \), \( i \) has to make an initial concession
  
  \- Low \( T^i \) is **strength** in the war of attrition
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Concessions

Graph showing two curves labeled $F_1$ and $F_2$. The $x$-axis ranges from 0 to 300, and the $y$-axis ranges from 0.0 to 0.8. The curves ascend as the $x$-values increase, with $F_2$ generally higher than $F_1$. The graph includes grid lines and a dotted vertical line at 200 on the $x$-axis.
Suppose P1 demands $\alpha^1$ and P2 does not concede

$$T^2 = -\frac{1}{\lambda^2} \log \frac{z^2 \pi^2(\alpha^2)}{z^2 \pi^2(\alpha^2) + (1 - z^2) \mu_{\alpha^1}(\alpha^2)}$$

- Expected payoff: $\alpha^2 \times q^1 + (1 - \alpha^1) \times (1 - q^1)$
  - $q^1$ is the probability of immediate concession by P1
  - $q^1$ decreasing in $T^2$, so decreasing in $\alpha^2$

- Indifference across mimicked types pins down $\mu^2$
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- Indifference **across** mimicked types pins down $\mu^2$
Let $z \to 0$ with $\lim \frac{z_1}{z_2} \in (0, 1)$.

- $i$ can guarantee any payoff below

$$v^i = \frac{r^j}{r^1 + r^2}$$

- $\lambda^i > \lambda^j$ makes $j$ concede immediately wpa1

- $\lambda^1 > \lambda^2$ iff $r^1(1 - \alpha^2) < r^2(1 - \alpha^1)$
  - But for $\alpha^1 \leq v^1$, “$>$” means that $\alpha^2 < 1 - v^1$
Concluding Remarks

- Theory of strategic postures in bargaining
- Perturbation gives an incentive to mimic the crazy types
  - Indifference conditions pin down play
  - Indifference conditions pin down distribution of mimicked types
- In the limit
  - Outcome robust to bargaining protocol
  - Only mimics the Rubinstein outcome (if allowed)