

Bargaining and Reputation

Abreu and Gul – 2000

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Sargent Reading Group

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- Two players **bargain** over a pie of size 1.
 - Take **turns** to make offers
 - Consider the continuous-time limit
- **Complete** information, alternating offers (Rubinstein, 1982)

$$1 - v^1 = \delta_2 v^2 \qquad 1 - v^2 = \delta_1 v^1$$

- This paper: uncertainty about *strategic posture*

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$$v^1(\delta_1, \delta_2) = \frac{1 - \delta_2}{1 - \delta_1 \delta_2}$$

- This paper: uncertainty about *strategic posture*

Perturb the Rubinstein game

- Player i is *behavioral* with probability z^i
- With cond. probability $\pi^i(\alpha)$, i is type α
 - Type α characterized by its strategy
 - Demands α , accepts offers $\geq \alpha$
- With probability $1 - z^i$, i is rational and *strategic*.

Two questions

- Does the *rational* type *pretend* to be crazy?
- Does it matter (in the limit as $z \rightarrow 0$)?

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- Does it matter (in the limit as $z \rightarrow 0$)? ✓

- Time 0
 - P1 chooses a demand α^1
 - \implies P1 is either rational or behavioral of type α^1
 - P2 can accept or demand $\alpha^2 > 1 - \alpha^1$
 - Maybe P2 is type α^2 , maybe strategically
 - P1 can concede or reject
- Time $t > 0$
 - Protocol calls for i to make demands at certain (frequent) times
 - Changing the demand = revealing rationality
 - State variable: posterior of crazy
 - Revealing rationality \lesssim conceding

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- Player i chooses
 - A (distribution over) type to mimic α^i
 - A probability of **concession** before time t , $F_{\alpha^1, \alpha^2}^i(t)$
 - $F_{\alpha^1, \alpha^2}^i(0) > 0$ means concession after initial demands are made
- Subgame after initial demands: **war of attrition** / chicken
- Indifference conditions \sim mixed strategies:
 1. At most one player concedes at time 0
 2. Player i concedes with constant hazard rate

$$\lambda^i = r^j \frac{1 - \alpha^j}{\alpha^j - (1 - \alpha^i)}$$

3. At some $T^0 < \infty$ the posterior of crazy reaches 1 for **both** players

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Concession rate

$$\lambda^i = r^j \frac{1 - \alpha^i}{\alpha^j - (1 - \alpha^i)}$$

- Concede faster against more **impatient** opponents
- Concede more slowly against more **greedy** opponents
- Concede slowly when demand is high

- 'Time to exhaustion' T^i :

$$1 - z^i = F_{\alpha^i, \alpha^i}^i(T^i) = \frac{1 - \alpha^i}{1 - \alpha^i \alpha^i T^i}$$

- If $T^i > T^j$, i has to make an initial concession
 - Low T^i is **strength** in the war of attrition

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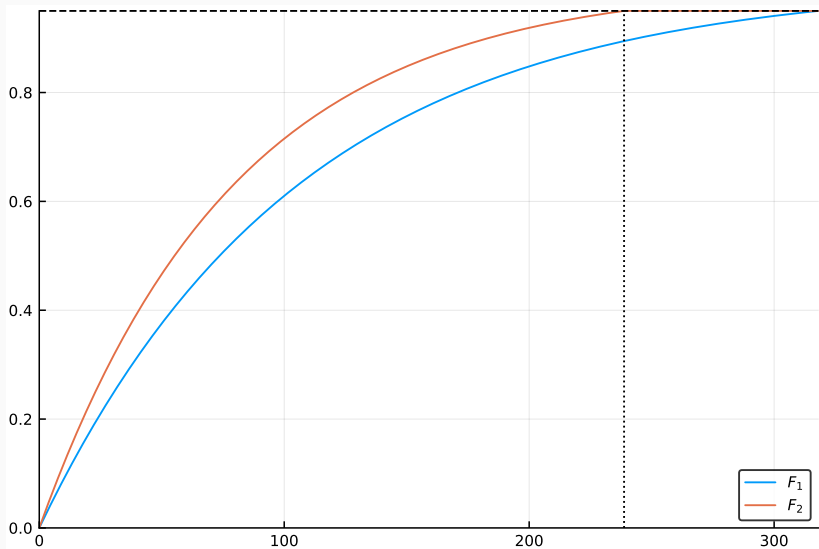
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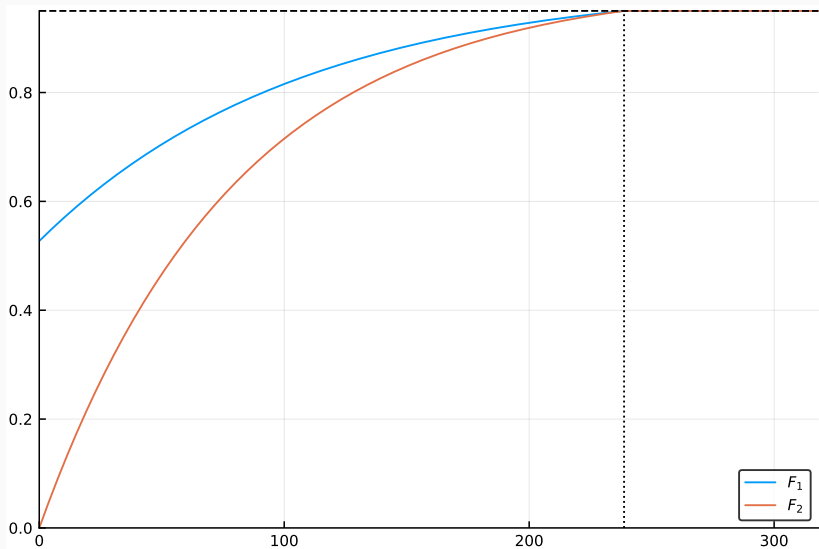
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WHO TO MIMIC?

Suppose P1 demands α^1 and P2 does **not** concede

$$T^2 = -\frac{1}{\lambda^2} \log \frac{z^2 \pi^2(\alpha^2)}{z^2 \pi^2(\alpha^2) + (1 - z^2) \mu_{\alpha^1}^2(\alpha^2)}$$

- Expected payoff: $\alpha^2 \times q^1 + (1 - \alpha^1) \times (1 - q^1)$
 - q^1 is the probability of **immediate** concession by P1
 - q^1 decreasing in T^2 , so decreasing in α^2
- Indifference **across** mimicked types pins down μ^2

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Let $z \rightarrow 0$ with $\lim \frac{z_1}{z_2} \in (0, 1)$.

- i can **guarantee** any payoff below

$$\underline{v}^i = \frac{r^j}{r^1 + r^2}$$

- $\lambda^i > \lambda^j$ makes j concede **immediately** wpa1
- $\lambda^1 > \lambda^2$ iff $r^1(1 - \alpha^2) < r^2(1 - \alpha^1)$
 - But for $\alpha^1 \leq \underline{v}^1$, “ \geq ” means that $\alpha^2 < 1 - \underline{v}^1$

CONCLUDING REMARKS

- Theory of strategic **postures** in bargaining
- Perturbation gives an incentive to mimic the **crazy** types
 - Indifference conditions pin down play
 - Indifference conditions pin down **distribution** of mimicked types
- In the limit
 - Outcome **robust** to bargaining protocol
 - Only mimics the **Rubinstein** outcome (if allowed)