Bargaining and Reputation

Abreu and Gul – 2000

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Sargent Reading Group
October 2017

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Outline

- Two players bargain over a pie of size 1.
    - Take **turns** to make offers
    - Consider the continuous-time limit
- Complete information, alternating offers (Rubinstein, 1982)

$$1 - v^1 = \delta_2 v^2 \qquad\qquad 1 - v^2 = \delta_1 v^1$$

- This paper: uncertainty about *strategic posture*

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$$v^1(\delta_1, \delta_2) = \frac{1 - \delta_2}{1 - \delta_1 \delta_2}$$

- This paper: uncertainty about *strategic posture*

Types

Perturb the Rubinstein game

- Player i is *behavioral* with probability z^i
- With cond. probability $\pi^i(\alpha)$, i is type α
    - Type α characterized by its strategy
    - Demands α, accepts offers $\geq \alpha$
- With probability $1 - z^i$, i is rational and ***strategic***.

Two questions

- Does the *rational* type pretend to be crazy?
- Does it matter (in the limit as $z \to 0$)?

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Play

- Time 0
  - P1 chooses a demand α^1

    $\Longrightarrow$ P1 is either rational or behavioral of type α^1
  - P2 can accept or demand $\alpha^2 > 1 - \alpha^1$

    Maybe P2 is type α^2, maybe strategically
  - P1 can concede or reject
- Time $t > 0$
  - Protocol calls for i to make demands at certain (frequent) times
  - Changing the demand = revealing rationality
  - State variable: posterior of crazy
  - Revealing rationality $\lesssim$ conceding

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- Player i chooses
  - A (distribution over) type to mimic α^i
  - A probability of concession before time t, $F^i_{\alpha^1,\alpha^2}(t)$
    - $F^i_{\alpha^1,\alpha^2}(0) > 0$ means concession after initial demands are made
- Subgame after initial demands: war of attrition / chicken
- Indifference conditions $\sim$ mixed strategies:
  1. At most one player concedes at time 0
  2. Player i concedes with constant hazard rate

$$\lambda^i = r^j \frac{1-\alpha^j}{\alpha^j - (1-\alpha^i)}$$

  3. At some $T^0 < \infty$ the posterior of crazy reaches 1 for *both* players

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Concession rate

$$\lambda^i = r^j \frac{1 - \alpha^i}{\alpha^j - (1 - \alpha^i)}$$

- Concede faster against more impatient opponents
- Concede more slowly against more greedy opponents
- Concede slowly when demand is high

- 'Time to exhaustion' T^i:

$$1 - z^i = F^i_{\alpha^1,\alpha^2}(T^i) = 1 - e^{-\lambda^i T^i}$$

- If $T^i > T^j$, i has to make an initial concession
  - Low T^i is strength in the war of attrition

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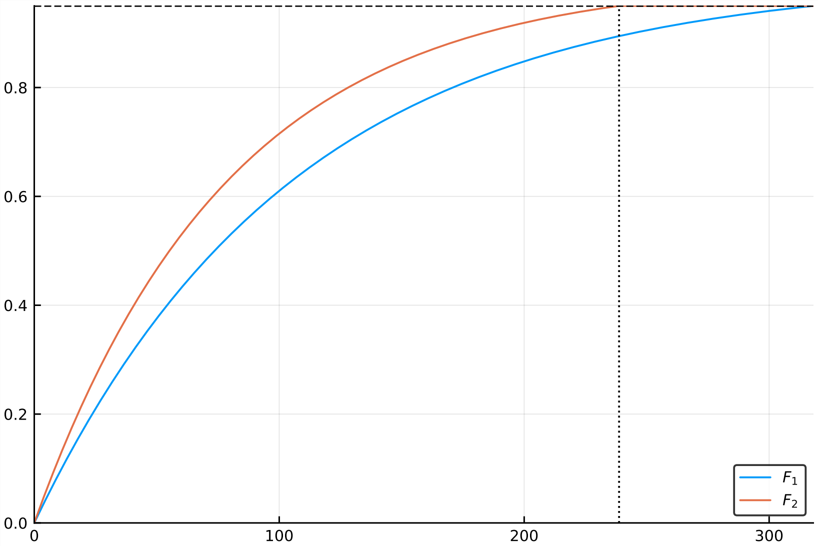
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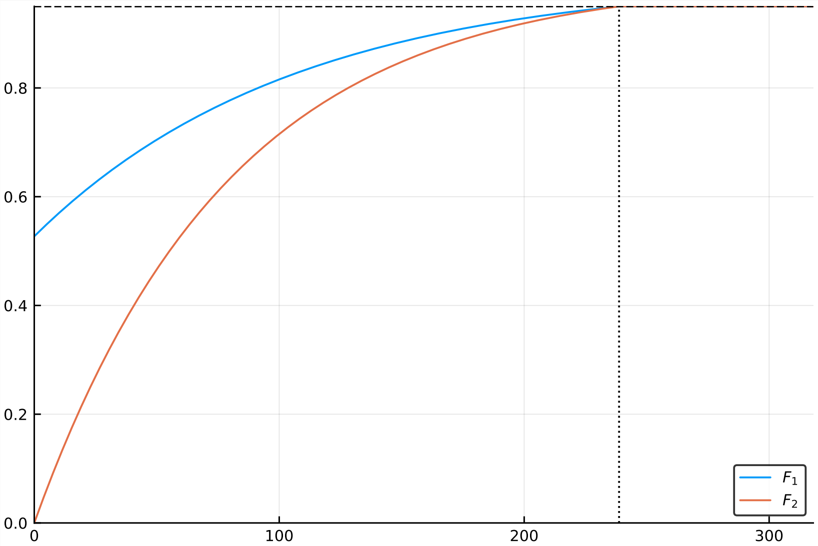
Concessions



Who to Mimic?

Suppose P1 demands α^1 and P2 does not concede

$$T^2 = -\frac{1}{\lambda^2} \log \frac{z^2 \pi^2(\alpha^2)}{z^2 \pi^2(\alpha^2) + (1 - z^2)\mu^2_{\alpha^1}(\alpha^2)}$$

- Expected payoff: $\alpha^2 \times q^1 + (1 - \alpha^1) \times (1 - q^1)$
  - q^1 is the probability of immediate concession by P1
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The Rational Limit

Let $z \to 0$ with $\lim \frac{z_1}{z_2} \in (0, 1)$.

- i can guarantee any payoff below

$$\underline{v}^i = \frac{r^j}{r^1 + r^2}$$

- $\lambda^i > \lambda^j$ makes j concede **immediately** wpa1
- $\lambda^1 > \lambda^2$ iff $r^1(1 - \alpha^2) < r^2(1 - \alpha^1)$
  - But for $\alpha^1 \leq \underline{v}^1$, "$\geq$" means that $\alpha^2 < 1 - \underline{v}^1$

Concluding Remarks

- Theory of strategic postures in bargaining
- Perturbation gives an incentive to mimic the crazy types
  - Indifference conditions pin down play
  - Indifference conditions pin down **distribution** of mimicked types
- In the limit
  - Outcome **robust** to bargaining protocol
  - Only mimics the Rubinstein outcome (if allowed)