SPECULATIVE INVESTOR BEHAVIOR AND LEARNING

Stephen Morris 1996 - Quarterly Journal of Economics

Chase Coleman

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Sargent Reading Group

INTRODUCTION

OVERVIEW

This paper is a follow up to the Harrison Kreps 1978 paper, ”Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations”

Explores speculative premia when agents are Bayesian learners

In a given period, an individual values an asset according to

$$(1+r)p_t = E_t[y_{t+1} + p_{t+1}]$$

If all agents are of one type, then this is just pinned down by:

$$p_t^* = E_t\left[\sum_{\tau=1}^{\infty}\left(\frac{1}{1+r}\right)^{\tau} y_{t+\tau}\right]$$

If agents differ in beliefs, you may think you can sell it to someone else at a higher price tomorrow

$$(1+r)p_t = E_{i,t}[y_{t+1}] + E_{i,t}[p_{t+1}] \overset{?}{>} (1+r)p_t^*$$

SPECULATIVE PREMIA

Refer to $p_t - p_t^*$ as the *speculative premia*

SPECULATIVE INVESTOR BEHAVIOR AND LEARNING

- Two assets:
  - Risk-free asset that can be traded at $\frac{1}{r}$ and pays 1 in following period.
  - Risky asset that can be bought and sold (no short selling) which pays 1 with probability θ. The price at which it trades clears the market.
- J types of risk-neutral traders with a continuum of each type
  - Agents do not know the true θ; they will instead learn about it as Bayesians.
  - Types differ only in the prior which they place over values of θ ($\pi_i(\theta)$).
  - Assume π_i is twice differentiable and uniformly bounded below.

BAYESIAN UPDATING

Given a prior distribution of $\pi_i(\theta)$, an agent who has seen s payments of the dividend in t periods has a posterior given by

$$\xi_i(\theta|x,t) = \frac{\theta^s(1-\theta)^{t-s}\pi_i(\theta)}{\int_{\zeta=0}^1 \zeta^s(1-\zeta)^{t-s}\pi_i(\zeta)d\zeta}$$

Which results in an expected dividend of

$$\mu_i(s,t) = \int_{\theta=0}^1 \theta\xi_i(\theta|x,t)d\theta = \frac{\int_{\theta=0}^1 \theta^{s+1}(1-\theta)^{t-s}\pi_i(\theta)d\theta}{\int_{\theta=0}^1 \theta^s(1-\theta)^{t-s}\pi_i(\theta)d\theta}$$

For all $\theta \in [0,1]$ and $i \in \mathcal{I}$, it can be shown that $\lim_{t\to\infty} \mu_i(s,t) = \theta$

FUNDAMENTAL EVALUATION OF ASSET

Assume there is only one type and that we have seen s payments in t periods.

At what price would the asset trade?

$$p_i^*(s,t) = \sum_{\tau=1}^{\infty} \left(\frac{1}{1+r}\right)^{\tau} \mu_i(s,t) = \frac{1}{r}\mu_i(s,t)$$

From this point forward, I will refer to $\mu_i(s,t)$ as type i's *fundamental value*.

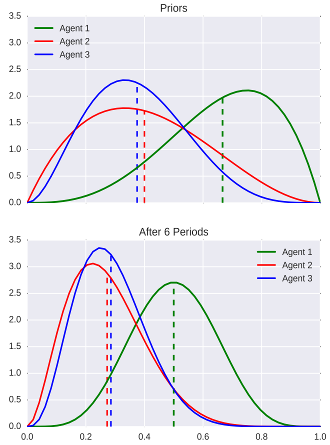
Two definitions of optimist

- **Definition 1**: Trader k is a *global optimist* if $\mu_k(s,t) \geq \mu_i(s,t)$ for all $i \in \mathcal{I}$ and all histories (s,t).
- **Definition 2**: Trader k is a *local optimist* if there exists a history (s,t) such that for all histories (s',t') following (s,t), $\mu_k(s',t') \geq \mu_i(s',t')$.

Why do we care about optimists?

OPTIMISTS

What does an optimist look like?



We would like to find conditions on the priors which ensure no optimists (local or global) exist

- **Definition 3**: Beliefs $\{\pi_i\}_{i\in\mathcal{I}}$ satisfy *perpetual switching* if, for every $i \in \mathcal{I}$ and history (s,t), there exists $j \neq i$ and a history (s',t') following (s,t) such that $\mu_j(s',t') > \mu_i(s',t')$.

RATE DOMINANCE

Different concept to think about what is required to get exclude optimists

- **Definition 4**: Trader k is *rate dominant* if $d/d\theta \ln(\pi_k(\theta)) \geq d/d\theta \ln(\pi_i(\theta))$ for all $i \in \mathcal{I}$ and $\theta \in [0, 1]$
- **Note**: Trader k is rate dominant if and only if his prior density satisfies the monotonic likelihood ratio property with all other priors

Can be shown that

- global optimist $\Leftrightarrow$ local optimist $\Leftrightarrow$ rate dominant

A simple class of priors that are clean in seeing the equivalence

Rate dominance is then:

$$d/d\theta \ln(\pi_i(\theta)) = \frac{\alpha_i - 1}{\theta} - \frac{\beta_i - 1}{1 - \theta}$$

The difference can be written

$$d/d\theta \ln(\pi_k(\theta)) - d/d\theta \ln(\pi_i(\theta)) = \frac{\alpha_k - \alpha_i}{\theta} + \frac{\beta_i - \beta_k}{1 - \theta}$$

Thus rate dominant iff $\alpha_k \geq \alpha_i$ and $\beta_k \leq \beta_i$

Global optimist is $\mu_k(s,t) > \mu_i(s,t)$

The beta distribution is a conjugate prior for a Bernoulli likelihood, so:

- If there are s realizations of the dividend in t periods then posterior is a beta distribution with parameters ($\alpha = \alpha_i + s$, $\beta = \beta_i + t - s$)
- Mean of a beta distribution is just $\frac{\alpha}{\alpha+\beta}$

Thus global optimist is just

$$\frac{\alpha_k + s}{\alpha_k + \beta_k + t} \geq \frac{\alpha_i + s}{\alpha_i + \beta_i + t} \Leftrightarrow \alpha_k \geq \alpha_i \& \beta_k \leq \beta_i$$

Recall the first slide. Prices will satisfy

$$p_t = \frac{1}{1+r} E_t[y_{t+1} + p_{t+1}]$$

What will the price be that they can get tomorrow? It will be the price at which the most optimistic agent tomorrow values it.

Define

$$\mu^*(s,t) = \max_{i \in \mathcal{I}} \mu_i(s,t)$$

Then prices satisfy

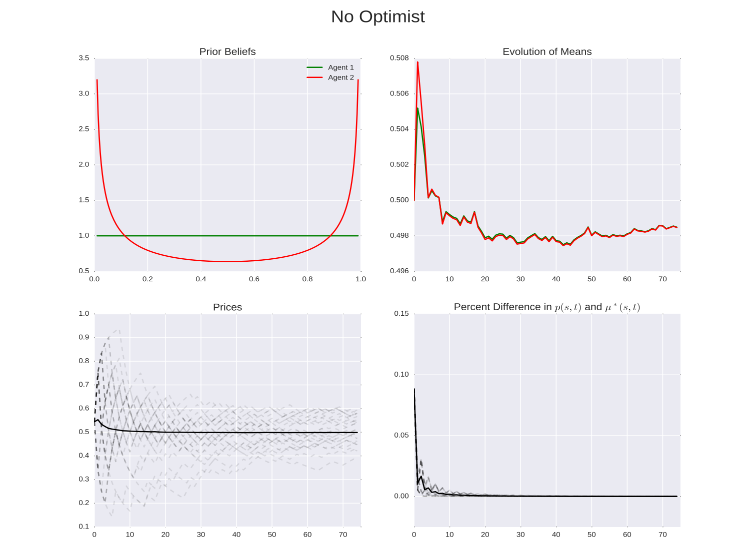
$$P(s,t) = \frac{1}{1+r}\left[\mu^*(s,t)(1 + P(s+1,t+1)) + (1-\mu^*(s,t))P(s,t+1)\right]$$

or (in terms of the risk-free asset)

$$p(s,t) = \frac{1}{1+r}\left[\mu^*(s,t)(r + p(s+1,t+1)) + (1-\mu^*(s,t))p(s,t+1)\right]$$

Theorem 2:

- If trader k is an optimist, then $p(s,t) = \mu_k(s,t)$ for all histories (s,t).
- If there is no optimist, then $p(s,t) > \mu_i(s,t)$ for all histories (s,t) and $i \in \mathcal{I}$
- As $t \to \infty$, $p(\theta t, t) \to \mu_i(\theta t, t) \to \theta$ for all $\theta \in [0,1]$ and traders $i \in \mathcal{I}$


No Optimist
Prior Beliefs
Agent 1
Agent 2
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.2
0.4
0.6
0.8
1.0
Evolution of Means
0.508
0.506
0.504
0.502
0.500
0.498
0.496
0
10
20
30
40
50
60
70
Prices
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10
20
30
40
50
60
70
Percent Difference in $p(s,t)$ and $\mu^*(s,t)$
0.15
0.10
0.05
0.00
0
10
20
30
40
50
60
70

ECONOMY WITH OPTIMIST