Optimal Government Debt Maturity under Limited Commitment (2014)

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Introduction

- Angeletos (2002), Buera and Nicolini (2004): Non-state contingent debt, with commitment, can implement optimal policy
  - Optimal structure "tilted long," with gov't purchasing short-term assets and selling long-term debt
  - Short and long positions large relative to GDP
- Lucas and Stokey (1983): State-contingent debt, limited commitment, can implement optimal policy
  - Optimal structure flat
- This paper: Non-state contingent debt, limited commitment, cannot implement optimal policy
  - Tradeoff between insurance (tilted long) and commitment (flat)
  - Quantitative exercise: Short-term bond is 3.1% of GDP, long-term bond is 77% of GDP with annual payouts of 3.2% of GDP
  - Maturity structure nearly flat: Lack of commitment more important than lack of state-contingency

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Outline

1. Model
2. Three period example
3. Quantitative exercise

Model

- Spender-saver model of Mankiw (2000)
- Non-contingent bond market, uniform lump sum taxes/subsidies
- Mass $\lambda \in (0, 1)$ of spenders with preferences

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \theta_t u^p(c_t^p), \quad c_t^p = y^p - \tau_t, \; u^{p\prime} > 0, \; u^{p\prime\prime} \leq 0$$

- Mass $1 - \lambda$ of savers with preferences

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t u^r(c_t^r), \quad u^{r\prime} > 0, \; u^{r\prime\prime} \leq 0$$

- Saver budget constraint

$$c_t^r = y^r - \tau_t + \sum_{j=1}^{\infty} q_t^{t+j} \left(b_{t-1}^{t+j} - b_t^{t+j} \right) + b_{t-1}^t$$

Model

- Government with preferences

$$\psi \mathbb{E} \sum_{t=0}^{\infty} \beta^t \theta_t u^p(c_t^p) + (1-\psi) \mathbb{E} \sum_{t=0}^{\infty} \beta^t u^r(c_t^r), \quad \psi \in [0,1]$$

- Government budget constraint

$$\tau_t = \sum_{j=1}^{\infty} q_t^{t+j} \left(B_{t-1}^{t+j} - B_t^{t+j} \right) + B_{t-1}^t$$

- Bond market clearing

$$B_t^{t+j} = (1-\lambda) b_t^{t+j}$$

- Resource constraint

$$\lambda c_t^p + (1-\lambda) c_t^r = \lambda y^p + (1-\lambda) y^r$$

Model

A Markov Perfect Competitive Equilibrium consists of stochastic consumption and debt sequences $\left\{c_t^p, c_t^r, \left\{b_t^{t+j}\right\}_{j=1}^{\infty}\right\}_{t=0}^{\infty}$, a stochastic policy sequence $\left\{\tau_t, \left\{B_t^{t+j}\right\}_{j=1}^{\infty}\right\}_{t=0}^{\infty}$, and a stochastic bond price sequence $\left\{\left\{q_t^{t+j}\right\}_{j=1}^{\infty}\right\}_{t=0}^{\infty}$ such that:

1. $c_t^p = y^p - \tau_t$
2. $\left\{c_t^r, \left\{b_t^{t+j}\right\}_{j=1}^{\infty}\right\}_{t=0}^{\infty}$ maximizes saver utility subject to b.c.
3. $\left\{\tau_t, \left\{B_t^{t+j}\right\}_{j=1}^{\infty}\right\}_{t=0}^{\infty}$ maximizes gov't utility subject to b.c.
4. Bond market clears

3 period example

- $t = 0, 1, 2$, with $\delta \in [0, 1)$ and

$$\begin{aligned}\theta_0 &\geq 1 + \delta \\ \theta_1 &\in \{1 - \delta, 1 + \delta\} \text{ with equal probability} \\ \theta_2 &= 1\end{aligned}$$

- Gov't chooses $\tau_t, B_0^1, B_0^2, B_1^2(\theta_1)$
- $\lambda = 1/2$ (equal number of spenders/savers) and $\psi = 1$ (gov't only values spenders)
- $u^p(c_t^p) = c_t^p$ and $u^r(c_t^r) = \log c_t^r$
- Consumption allocation

$$\alpha = \left\{c_0^i, \left\{c_1^i(\theta_1), c_2^i(\theta_1)\right\}_{\theta_1 = 1-\delta, 1+\delta}\right\}_{i=p,r}$$

3 period example (incomplete mkts, full commitment)

- Primal approach: gov't solves

$$\min_{\alpha, B_0^1, B_0^2} \{\theta_0 c_0^r + \beta \mathbb{E}[\theta_1 c_1^r + \beta c_2^r]\}, \quad \text{subject to}$$

$$0 = \left(\frac{y^r - c_0^r}{c_0^r}\right) + \beta \mathbb{E}\left(\frac{y^r - c_1^r(\theta_1)}{c_1^r(\theta_1)}\right) + \beta^2 \mathbb{E}\left(\frac{y^r - c_2^r(\theta_1)}{c_2^r(\theta_1)}\right),$$

$$0 = \left(\frac{y^r + B_0^1 - c_1^r(\theta_1)}{c_1^r(\theta_1)}\right) + \beta\left(\frac{y^r + B_0^2 - c_2^r(\theta_1)}{c_2^r(\theta_1)}\right) \ \forall \theta_1$$

- Solution (same allocation as complete mkts + full commitment):

$$c_t^r(\theta_t) = \frac{y^r}{\theta_t^{1/2}} \frac{\mathbb{E}\left[\sum_{t=0}^2 \beta^t \theta_t^{1/2}\right]}{1 + \beta + \beta^2} \ \forall t, \quad B_0^1 = -y^r,$$

$$B_0^2 = y^r \left(\frac{1+\beta}{\beta} \frac{\mathbb{E}\left[\sum_{t=0}^2 \beta^t \theta_t^{1/2}\right]}{1 + \beta + \beta^2} - 1\right)$$

3 period example (complete mkts, limited commitment)

- $\theta_0 \geq 1 + \delta, \theta_1 = \theta_2 = 1$
- Date 1 gov't solves

$$\min_{c_1^r, c_2^r, B_1^2} \{c_1^r + \beta c_2^r\} \tag{1}$$

$$\text{s.t.} \quad 0 = \left(\frac{y^r + B_0^1 - c_1^r}{c_1^r} \right) + \beta \left(\frac{y^r + B_0^2 - c_2^r}{c_2^r} \right)$$

- Date 0 gov't solves

$$\min_{\alpha, B_0^1, B_0^2} \{\theta_0 c_0^r + \beta c_1^r + \beta^2 c_2^r\} \quad \text{such that (1) and}$$

$$0 = \left(\frac{y^r - c_0^r}{c_0^r} \right) + \beta \left(\frac{y^r - c_1^r}{c_1^r} \right) + \beta^2 \left(\frac{y^r - c_2^r}{c_2^r} \right)$$

3 period example (complete mkts, limited commitment)

- Solution (same allocation as complete mkts + full commitment):

$$
\begin{aligned}
c_t^r(\theta_t) &= \frac{y^r}{\theta_t^{1/2}} \frac{\mathbb{E}\left[\sum_{t=0}^{2} \beta^t \theta_t^{1/2}\right]}{1+\beta+\beta^2} \; \forall t \\
B_0^1 = B_0^2 &= y^r \left(\frac{\mathbb{E}\left[\sum_{t=0}^{2} \beta^t \theta_t^{1/2}\right]}{1+\beta+\beta^2} - 1 \right)
\end{aligned}
$$

3 period example (incomplete mkts, limited commitment)

- Date 1 gov't solves

$$\min_{c_1^r(\theta_1), c_2^r(\theta_1), B_1^2} \{\theta_1 c_1^r + \beta c_2^r\}$$

$$\text{s.t.} \quad 0 = \left(\frac{y^r + B_0^1 - c_1^r}{c_1^r}\right) + \beta\left(\frac{y^r + B_0^2 - c_2^r}{c_2^r}\right)$$

- Obtain continuation value

$$V(B_0^1, B_0^2) = -\frac{1}{1+\beta}\mathbb{E}\left(\sum_{t=1}^{2} \beta^{t-1}\theta_t^{1/2}(y^r + B_0^t)^{1/2}\right)^2$$

- Date 0 gov't solves

$$\max_{B_0^1, B_0^2} \left\{\theta_0(q_0^1(B_0^1, B_0^2)B_0^1 + q_0^2(B_0^1, B_0^2)B_0^2) + \beta V(B_0^1, B_0^2)\right\}$$

3 period example (incomplete mkts, limited commitment)

- Define
$$\kappa = \frac{y^r + B_0^2}{y^r + B_0^1}$$
- Slope of yield curve $q_0^1(B_0^1, B_0^2)/q_0^2(B_0^1, B_0^2)$ depends on κ and not on total level of debt issuance, and increases in κ
- **Proposition**

incomplete markets, full commitment	$\kappa = \infty$
complete markets, limited commitment	$\kappa = 1$
incomplete markets, limited commitment	$\kappa \in (1, \infty)$

Quantitative exercise

- $t = 0, 1, 2, \ldots$
- CRRA utilities, coefficients σ^p, σ^r
- Two bonds:
  - One-period bond b^S
  - Perpetuity b^L, with payoff stream $1, \gamma, \gamma^2, \ldots, \gamma \in (0, 1]$
- Saver budget constraint

$$c_t^r = y^r - \tau_t - q_t^S b_t^S + q_t^L(\gamma b_{t-1}^L - b_t^L) + (b_{t-1}^S + b_{t-1}^L)$$

- Government budget constraint

$$\tau_t = -q_t^S B_t^S + q_t^L(\gamma B_{t-1}^L - B_t^L) + (B_{t-1}^S + B_{t-1}^L)$$

- $\theta_t = \{1 - \delta, 1 + \delta\}$, with $\Pr\{\theta_{t+1} = \theta_t\} = \rho$

Quantitative exercise

Parameter	Value	Explanation
β	0.96	Period = 1 year
σ^p	0	Robust
σ^r	1	Robust
ψ	1	Robust
γ	1	Allows flat maturity structure
λ	0.4	SCF: Fraction of hand-to-mouth households
y^p/y^r	0.5	SCF: Hand-to-mouth HHs' relative income
B^S_{-1}	0.1625	US maturity structure, 1988-2013
B^L_{-1}	0.0175	US maturity structure, 1988-2013
ρ	0.5	i.i.d. shocks
δ	0.1	Std. dev. in net borrowing, 1988-2013

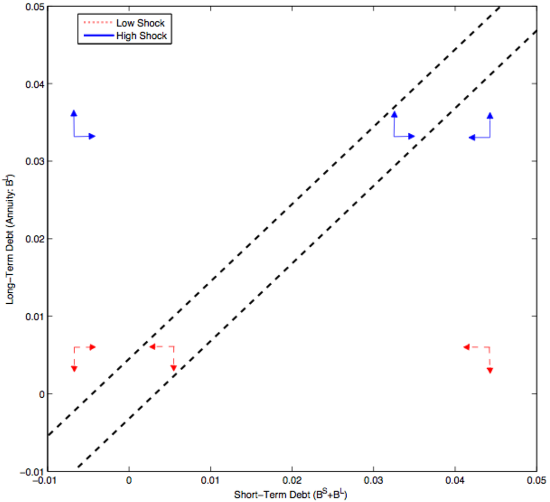
Table 1: Statistics under Baseline Parameter Values

Variables	Statistics	Commitment	No Commitment
Short-Term Debt	Mean	-1.860	0.031
	Std. Dev	0	0.025
	Autocorr.	–	0.982
Long-Term Debt (annuity)	Mean	0.107	0.032
	Std. Dev	0	0.025
	Autocorr.	–	0.982
Long-Term Debt (market value)	Mean	2.585	0.770
	Std. Dev	0.124	0.610
	Autocorr.	-0.002	0.944
Total Debt	Mean	0.725	0.801
	Std. Dev	0.124	0.634
	Autocorr.	-0.002	0.946

Notes: The table reports average statistics across 1000 simulations of 500 periods each, disregarding the first 200 observations.

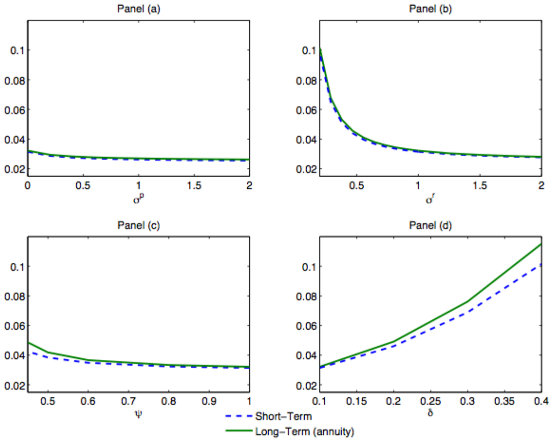
Conclusion

- Borrowing long provides a hedging benefit: Value of outstanding liabilities declines when short-term interest rates rise
- ...however this also lowers fiscal discipline for governments with limited commitment, increasing future short-term rates
- Quantitative exercise: Short-term bond is 3.1% of GDP, long-term bond is 77% of GDP with annual payouts of 3.2% of GDP
- Maturity structure nearly flat: Lack of commitment more important than lack of state-contingency



Notes: The figure shows the phase diagram. The horizontal axis displays short-term debt $(B^S + B^L)$. The vertical axis displays long-term debt annuity payouts. The blue solid line and the red dotted line refer to the high and low shocks, respectively. The dynamics of the phase diagram depend on whether the the debt levels are in between, above, or below the black dashed lines.

Figure 3: Debt Positions – No Commitment



Notes: The figure shows short-term debt, $B^S + B^L$ and long-term debt, B^L, as a function of parameters in the model with lack of commitment. Panel (a) varies the risk aversion of the spender households σ^p; panel (b) varies the risk aversion of the saver households σ^r; panel (c) varies the welfare weight on the spender households ψ; and panel (d) varies the variance of the shock δ.