Optimal Government Debt Maturity under Limited Commitment (2014)

Davide Debortoli, Ricardo Nunes, and Pierre Yared

Presented by Kevin Dick
September 23, 2014
Introduction

- Angeletos (2002), Buera and Nicolini (2004): Non-state contingent debt, with commitment, can implement optimal policy
  - Optimal structure “tilted long,” with gov’t purchasing short-term assets and selling long-term debt
  - Short and long positions large relative to GDP
- Lucas and Stokey (1983): State-contingent debt, limited commitment, can implement optimal policy
  - Optimal structure flat
- This paper: Non-state contingent debt, limited commitment, cannot implement optimal policy
  - Tradeoff between insurance (tilted long) and commitment (flat)
  - Quantitative exercise: Short-term bond is 3.1% of GDP, long-term bond is 77% of GDP with annual payouts of 3.2% of GDP
  - Maturity structure nearly flat: Lack of commitment more important than lack of state-contingency
Introduction

- Angeletos (2002), Buera and Nicolini (2004): Non-state contingent debt, with commitment, can implement optimal policy
  - Optimal structure “tilted long,” with gov’t purchasing short-term assets and selling long-term debt
  - Short and long positions large relative to GDP
- Lucas and Stokey (1983): State-contingent debt, limited commitment, can implement optimal policy
  - Optimal structure flat
- This paper: Non-state contingent debt, limited commitment, cannot implement optimal policy
  - Tradeoff between insurance (tilted long) and commitment (flat)
  - Quantitative exercise: Short-term bond is 3.1% of GDP, long-term bond is 77% of GDP with annual payouts of 3.2% of GDP
  - Maturity structure nearly flat: Lack of commitment more important than lack of state-contingency
Outline

1. Model
2. Three period example
3. Quantitative exercise
Model

- Spender-saver model of Mankiw (2000)
- Non-contingent bond market, uniform lump sum taxes/subsidies
- Mass $\lambda \in (0, 1)$ of spenders with preferences

$$
\mathbb{E} \sum_{t=0}^{\infty} \beta^t \theta_t u^p(c^p_t), \quad c^p_t = y^p - \tau_t, \quad u^p' > 0, \quad u^p'' \leq 0
$$

- Mass $1 - \lambda$ of savers with preferences

$$
\mathbb{E} \sum_{t=0}^{\infty} \beta^t u^r(c^r_t), \quad u^r' > 0, \quad u^r'' \leq 0
$$

- Saver budget constraint

$$
c^r_t = y^r - \tau_t + \sum_{j=1}^{\infty} q^{t+j}_t \left( b^{t+j}_{t-1} - b^{t+j}_t \right) + b^t_{t-1}
$$
Model

- **Government with preferences**

  \[ \psi \mathbb{E} \sum_{t=0}^{\infty} \beta^t \theta u^p(c^p_t) + (1 - \psi) \mathbb{E} \sum_{t=0}^{\infty} \beta^t u^r(c^r_t), \quad \psi \in [0, 1] \]

- **Government budget constraint**

  \[ \tau_t = \sum_{j=1}^{\infty} q^{t+j} \left( B^{t+j}_{t-1} - B^{t+j}_t \right) + B^t_{t-1} \]

- **Bond market clearing**

  \[ B^{t+j} = (1 - \lambda)b^{t+j}_t \]

- **Resource constraint**

  \[ \lambda c^p_t + (1 - \lambda)c^r_t = \lambda y^p + (1 - \lambda)y^r \]
A Markov Perfect Competitive Equilibrium consists of stochastic consumption and debt sequences \( \left\{ c^p_t, c^r_t, \{ b^{t+j}_t \}_{j=1}^\infty \right\}_{t=0}^\infty \), a stochastic policy sequence \( \left\{ \tau_t, \{ B^{t+j}_t \}_{j=1}^\infty \right\}_{t=0}^\infty \), and a stochastic bond price sequence \( \left\{ \{ q^{t+j}_t \}_{j=1}^\infty \right\}_{t=0}^\infty \) such that:

1. \( c^p_t = y^p - \tau_t \)

2. \( \left\{ c^r_t, \{ b^{t+j}_t \}_{j=1}^\infty \right\}_{t=0}^\infty \) maximizes saver utility subject to b.c.

3. \( \left\{ \tau_t, \{ B^{t+j}_t \}_{j=1}^\infty \right\}_{t=0}^\infty \) maximizes gov’t utility subject to b.c.

4. Bond market clears
3 period example

- $t = 0, 1, 2$, with $\delta \in [0, 1)$ and

  $\theta_0 \geq 1 + \delta$

  $\theta_1 \in \{1 - \delta, 1 + \delta\}$ with equal probability

  $\theta_2 = 1$

- Gov’t chooses $\tau_t, B^1_0, B^2_0, B^2_1(\theta_1)$

- $\lambda = 1/2$ (equal number of spenders/savers) and $\psi = 1$ (gov’t only values spenders)

- $u^p(c^p_t) = c^p_t$ and $u^r(c^r_t) = \log c^r_t$

- Consumption allocation

  $$\alpha = \left\{c^i_0, \left\{c^i_1(\theta_1), c^i_2(\theta_1)\right\}_{\theta_1 = 1 - \delta, 1 + \delta}\right\}_{i = p, r}$$
3 period example (incomplete mkts, full commitment)

- Primal approach: gov’t solves

$$\min_{\alpha, B_0^1, B_0^2} \{ \theta_0 c_r^0 + \beta \mathbb{E}[\theta_1 c_r^1 + \beta c_r^2] \}, \text{ subject to}$$

$$0 = \left( \frac{y^r - c_r^0}{c_r^0} \right) + \beta \mathbb{E} \left( \frac{y^r - c_r^1(\theta_1)}{c_r^1(\theta_1)} \right) + \beta^2 \mathbb{E} \left( \frac{y^r - c_r^2(\theta_1)}{c_r^2(\theta_1)} \right),$$

$$0 = \left( \frac{y^r + B_0^1 - c_r^1(\theta_1)}{c_r^1(\theta_1)} \right) + \beta \left( \frac{y^r + B_0^2 - c_r^2(\theta_1)}{c_r^2(\theta_1)} \right) \forall \theta_1$$

- Solution (same allocation as complete mkts + full commitment):

$$c_r^t(\theta_t) = \frac{y^r}{\theta_t^{1/2}} \frac{\mathbb{E} \left[ \sum_{t=0}^2 \beta^t \theta_t^{1/2} \right]}{1 + \beta + \beta^2} \forall t, \quad B_0^1 = -y^r,$$

$$B_0^2 = y^r \left( \frac{1 + \beta \mathbb{E} \left[ \sum_{t=0}^2 \beta^t \theta_t^{1/2} \right]}{\beta \left( \frac{1}{1 + \beta + \beta^2} - 1 \right)} - 1 \right)$$
3 period example (complete mkts, limited commitment)

- $\theta_0 \geq 1 + \delta, \theta_1 = \theta_2 = 1$
- Date 1 gov’t solves

$$\min_{c_1^r, c_2^r, B_1^2} \{ c_1^r + \beta c_2^r \}$$

s.t. $0 = \left( y^r + \frac{B_0^1 - c_1^r}{c_1^r} \right) + \beta \left( \frac{y^r + B_0^2 - c_2^r}{c_2^r} \right)$

- Date 0 gov’t solves

$$\min_{\alpha, B_0^1, B_0^2} \{ \theta_0 c_0^r + \beta c_1^r + \beta^2 c_2^r \} \text{ such that (1) and }$$

$$0 = \left( \frac{y^r - c_0^r}{c_0^r} \right) + \beta \left( \frac{y^r - c_1^r}{c_1^r} \right) + \beta^2 \left( \frac{y^r - c_2^r}{c_2^r} \right)$$
Solution (same allocation as complete mkts + full commitment):

\[ c_t^r(\theta_t) = \frac{y^r}{\theta_t^{1/2}} \frac{\mathbb{E} \left[ \sum_{t=0}^{2} \beta^t \theta_t^{1/2} \right]}{1 + \beta + \beta^2} \quad \forall t \]

\[ B_0^1 = B_0^2 = y^r \left( \frac{\mathbb{E} \left[ \sum_{t=0}^{2} \beta^t \theta_t^{1/2} \right]}{1 + \beta + \beta^2} - 1 \right) \]
3 period example (incomplete mkts, limited commitment)

- **Date 1 gov’t solves**

\[
\min_{c^r_1(\theta_1), c^r_2(\theta_1), B_1^2} \left\{ \theta_1 c^r_1 + \beta c^r_2 \right\}
\]

\[
s.t. \quad 0 = \left( \frac{y^r + B_0^1 - c^r_1}{c^r_1} \right) + \beta \left( \frac{y^r + B_0^2 - c^r_2}{c^r_2} \right)
\]

- **Obtain continuation value**

\[
V(B_0^1, B_0^2) = -\frac{1}{1 + \beta} \mathbb{E} \left( \sum_{t=1}^{2} \beta^{t-1} \theta_t^{1/2} (y^r + B_t^1)^{1/2} \right)^2
\]

- **Date 0 gov’t solves**

\[
\max_{B_0^1, B_0^2} \left\{ \theta_0 (q^1_0(B_0^1, B_0^2)B_0^1 + q^2_0(B_0^1, B_0^2)B_0^2) + \beta V(B_0^1, B_0^2) \right\}
\]
3 period example (incomplete mkts, limited commitment)

- Define

\[ \kappa = \frac{y^r + B^2_0}{y^r + B^1_0} \]

- Slope of yield curve \( q^1_0(B^1_0, B^2_0)/q^2_0(B^1_0, B^2_0) \) depends on \( \kappa \) and not on total level of debt issuance, and increases in \( \kappa \)

- Proposition

  - incomplete markets, full commitment \( \kappa = \infty \)
  - complete markets, limited commitment \( \kappa = 1 \)
  - incomplete markets, limited commitment \( \kappa \in (1, \infty) \)
Quantitative exercise

- $t = 0, 1, 2, \ldots$
- CRRA utilities, coefficients $\sigma^p, \sigma^r$
- Two bonds:
  - One-period bond $b^S$
  - Perpetuity $b^L$, with payoff stream $1, \gamma, \gamma^2, \ldots, \gamma \in (0, 1]$
- Saver budget constraint
  \[
  c^r_t = y^r_t - \tau_t - q^S_t b^S_t + q^L_t (\gamma b^L_{t-1} - b^L_t) + (b^S_{t-1} + b^L_{t-1})
  \]
- Government budget constraint
  \[
  \tau_t = -q^S_t B^S_t + q^L_t (\gamma B^L_{t-1} - B^L_t) + (B^S_{t-1} + B^L_{t-1})
  \]
- $\theta_t = \{1 - \delta, 1 + \delta\}$, with $\Pr\{\theta_{t+1} = \theta_t\} = \rho$
## Quantitative exercise

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.96</td>
<td>Period = 1 year</td>
</tr>
<tr>
<td>$\sigma^p$</td>
<td>0</td>
<td>Robust</td>
</tr>
<tr>
<td>$\sigma^r$</td>
<td>1</td>
<td>Robust</td>
</tr>
<tr>
<td>$\psi$</td>
<td>1</td>
<td>Robust</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1</td>
<td>Allows flat maturity structure</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.4</td>
<td>SCF: Fraction of hand-to-mouth households</td>
</tr>
<tr>
<td>$y^p/y^r$</td>
<td>0.5</td>
<td>SCF: Hand-to-mouth HHs’ relative income</td>
</tr>
<tr>
<td>$B^S_{-1}$</td>
<td>0.1625</td>
<td>US maturity structure, 1988-2013</td>
</tr>
<tr>
<td>$B^L_{-1}$</td>
<td>0.0175</td>
<td>US maturity structure, 1988-2013</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.5</td>
<td>i.i.d. shocks</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.1</td>
<td>Std. dev. in net borrowing, 1988-2013</td>
</tr>
</tbody>
</table>
Table 1: Statistics under Baseline Parameter Values

<table>
<thead>
<tr>
<th>Variables</th>
<th>Statistics</th>
<th>Commitment</th>
<th>No Commitment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-Term Debt</td>
<td>Mean</td>
<td>-1.860</td>
<td>0.031</td>
</tr>
<tr>
<td></td>
<td>Std. Dev</td>
<td>0</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
<td>–</td>
<td>0.982</td>
</tr>
<tr>
<td>Long-Term Debt (annuity)</td>
<td>Mean</td>
<td>0.107</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>Std. Dev</td>
<td>0</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
<td>–</td>
<td>0.982</td>
</tr>
<tr>
<td>Long-Term Debt (market value)</td>
<td>Mean</td>
<td>2.585</td>
<td>0.770</td>
</tr>
<tr>
<td></td>
<td>Std. Dev</td>
<td>0.124</td>
<td>0.610</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
<td>-0.002</td>
<td>0.944</td>
</tr>
<tr>
<td>Total Debt</td>
<td>Mean</td>
<td>0.725</td>
<td>0.801</td>
</tr>
<tr>
<td></td>
<td>Std. Dev</td>
<td>0.124</td>
<td>0.634</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
<td>-0.002</td>
<td>0.946</td>
</tr>
</tbody>
</table>

Notes: The table reports average statistics across 1000 simulations of 500 periods each, disregarding the first 200 observations.
Conclusion

- Borrowing long provides a hedging benefit: Value of outstanding liabilities declines when short-term interest rates rise
- ...however this also lowers fiscal discipline for governments with limited commitment, increasing future short-term rates
- Quantitative exercise: Short-term bond is 3.1% of GDP, long-term bond is 77% of GDP with annual payouts of 3.2% of GDP
- Maturity structure nearly flat: Lack of commitment more important than lack of state-contingency
Notes: The figure shows the phase diagram. The horizontal axis displays short-term debt \((B^S + B^L)\). The vertical axis displays long-term debt annuity payouts. The blue solid line and the red dotted line refer to the high and low shocks, respectively. The dynamics of the phase diagram depend on whether the debt levels are in between, above, or below the black dashed lines.
Figure 3: Debt Positions – No Commitment

Panel (a) and Panel (b) illustrate the relationship between debt and risk aversion parameters for short-term and long-term debts, respectively. Panel (c) and Panel (d) depict the effects of the welfare weight and variance of the shock on the debt positions.

Notes: The figure shows short-term debt, $B^S + B^L$ and long-term debt, $B^L$, as a function of parameters in the model with lack of commitment. Panel (a) varies the risk aversion of the spender households $\sigma^p$; panel (b) varies the risk aversion of the saver households $\sigma^s$; panel (c) varies the welfare weight on the spender households $\psi$; and panel (d) varies the variance of the shock $\delta$. 