Optimal Security Design and Dynamic Capital Structure in a Continuous-Time Agency Model

DeMarzo and Sannikov, Journal of Finance, 2006

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March 24, 2015

Overview

- A principal hires an agent to manage a risky project.
- The agent can misreport cashflows and divert some cash to private consumption.
- The principal decides a compensation scheme as a function of the agent's reports.
- What is the optimal contract?

Technology

- A **project** produces cash flows: $dY_t = \mu dt + \sigma dZ_t$
  - The **agent** observes Y_t.
  - The **principal** does not.
- The agent
  - reports $\hat{Y}_t$ to the principal,
  - diverts the remainder $(Y_t - \hat{Y}_t)$, where $\hat{Y}_t \leq Y_t$,
  - and gets consumption $\lambda(Y_t - \hat{Y}_t)$, where $\lambda < 1$.
- The **contract** specifies
  - payments dI_t to the agent
  - a termination time τ

  as functions of the reports $\hat{Y}_t$.

Preferences

- Both agents are **risk neutral**.
  - The agent discounts at rate γ.
  - The principal discounts at rate $r < \gamma$.
- When the project is terminated, the agent receives outside option R and the principal receives L.
- The **agent's expected payoff at date 0** is

$$W_0 = \mathbb{E}\left[\int_0^\tau e^{-\gamma s} dC_s + e^{-\gamma\tau} R\right]$$

where $dC_s = \lambda\left(dY_s - d\hat{Y}_s\right) + dI_s$.

- The **principal's expected payoff at date 0** is

$$b_0 = \mathbb{E}\left[\int_0^\tau e^{-rs}\left(d\hat{Y}_s - dI_s\right) + e^{-r\tau} L\right].$$

The optimal contract: preliminaries

- Deadweight loss from diverting cash flows.
  $\Rightarrow$ We can focus on contracts which induce truth-telling.
- Define the **agent's promised value** $W_t(\hat{Y}_t)$ after a history of reports $(\hat{Y}_s,\, 0 \leq s \leq t)$ as:

$$W_t\left(\hat{Y}_t\right) = \mathbb{E}_t\left[\int_t^\tau e^{-\gamma(s-t)} dI_s + e^{-\gamma(\tau-t)} R\right]$$

- **Lemma** There exists a process β_t such that

$$dW_t = \gamma W_t dt - dI_t + \beta_t \left(d\hat{Y}_t - \mu dt\right)$$

- **Lemma** Truth-telling is incentive compatible if and only if $\beta_t \geq \lambda$ for all $t \leq \tau$.
  - **Intuition**: Steal $dY_t - d\hat{Y}_t$: immediately gain $\lambda(dY_t - d\hat{Y}_t)$, but lose $\beta_t(dY_t - d\hat{Y}_t)$ in future expected payoff.

The principal's value function

- Let $b(W)$ be the principal's value function.
- The principal can always provide the agent with W by
  - paying him a lump-sum transfer of $dI > 0$,
  - and then moving to the optimal contract with payoff $W - dI$
- Therefore, it must be that, $\forall W$

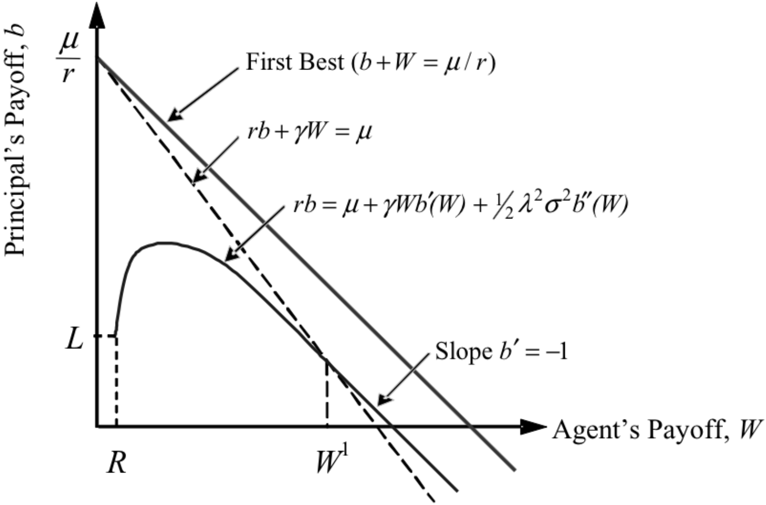
$$b(W) \geq b(W - dI) - dI$$

which implies that

$$b'(W) \geq -1, \forall W$$

- Define W^1, the lowest W such that $b'(W) \leq -1$.
It is optimal to pay the agent according to

$$dI = \max\left(W - W^1, 0\right)$$


Principal's Payoff, b
μ/r
First Best (b + W = μ / r)
rb + γW = μ
rb = μ + γWb′(W) + ½ λ²σ²b″(W)
L
Slope b′ = –1
Agent's Payoff, W
R
W¹

The principal's value function

- On the region to the left of W^1, $b(W)$ solves the HJB equation

$$rb(W) = \max_{\beta \geq \lambda} \mu + \gamma W b'(W) + \frac{1}{2}\beta^2 \sigma^2 b''(W)$$

- $\beta = \lambda$ is optimal.
  - **Intuition:** termination is inefficient ($\mu > rL + \gamma R$), so principal wants to reduce the risk that agent's promised value falls to R.
- Boundary conditions: $b(R) = L$ and 1st and 2nd derivatives at W^1.

The optimal contract in full

Agent's promised utility W_t evolves according to

$$dW_t = \gamma W_t dt - dI_t + \lambda \left(d\hat{Y}_t - \mu dt \right)$$

Compensation

When $W_t \in [R, W^1)$, $dI_t = 0$.
When $W_t = W^1$, payments dI_t cause W_t to reflect at W^1.

Principal's payoff $b(W_t)$ satisfies

$$rb(W) = \mu + \gamma W b'(W) + \frac{1}{2}\lambda^2\sigma^2 b''(W) \quad \text{on } [R, W^1]$$
$$b'(W) = -1 \text{ for } W \geq W^1$$

and boundary conditions $b(R) = L$ and $rb(W^1) = \mu - \gamma W^1$.

Termination The contract is terminated at time τ, when W_t reaches R.

Capital structure implementation

- Optimal contract can be implemented (not uniquely) with
  1. **Equity**, a fraction λ of which is held by the agent,
  2. **Long-term debt**, with face value D,
  3. **A credit line** with limit C^L and interest rate γ
- For the right choice of **long-term debt** D and **credit line limit** C^L, it is incentive compatible for the agent to
  - always pay down the credit line
  - roll over the long-term debt
  - pay any excess cash flow as dividends

  thereby **implementing the optimal contract**.

Capital structure implementation: comments

1. Agent's equity holding eliminates incentive to divert.
2. What stops the agent from overpaying dividends?
    - Assume the agent's promised value under the contract follows

$$W_t = R + \lambda(C^L - M_t) \qquad (*)$$

      where M_t is the **current draw on the credit line**.
    - Now suppose the agent considers (i) borrowing up to the limit on the credit line, (ii) paying a large dividend, and (iii) defaulting.

$$\textbf{payoff} = \underbrace{\lambda(C^L - M_t)}_{\text{dividend}} + \underbrace{R}_{\text{termination value}}$$

        - By $(*)$, the agent does just as well by behaving.
    - How to ensure $(*)$ holds?
        - Choose long-term debt D and credit limit C^L such that the firm's profit rate is such that $(*)$ holds.
        - In particular: $rD = \mu - \gamma R/\lambda - \gamma C^L$ and $C^L = \lambda^{-1}(W^1 - R)$.

Conclusion

- The optimal contract is consistent with a capital structure that is widely observed in the data.
- Prediction: firms
  1. issue long-term debt
  2. use credit lines to absorb transient cashflow shocks
  3. pay out dividends once some cashflow threshold has been reached
- Everything in this presentation is in DeMarzo and Fishman (2007), the same setup but in discrete time.
- The main benefit of the continuous time model is **analytical comparative statics** and **security values**.
- **Extensions**
  - Hidden saving.
  - Hidden effort instead of diverting cash flow.
  - Endogenized termination payoffs.
  - Renegotiation.
  - Private benefits of control.