Optimal Security Design and Dynamic Capital Structure in a Continuous-Time Agency Model

DeMarzo and Sannikov, Journal of Finance, 2006

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Overview

- A principal hires an agent to manage a risky project.
- The agent can misreport cashflows and divert some cash to private consumption.
- The principal decides a compensation scheme as a function of the agent’s reports.
- What is the optimal contract?
A project produces cash flows: \( dY_t = \mu dt + \sigma dZ_t \)
- The agent observes \( Y_t \).
- The principal does not.

The agent
- reports \( \hat{Y}_t \) to the principal,
- diverts the remainder \( (Y_t - \hat{Y}_t) \), where \( \hat{Y}_t \leq Y_t \),
- and gets consumption \( \lambda(Y_t - \hat{Y}_t) \), where \( \lambda < 1 \).

The contract specifies
- payments \( dl_t \) to the agent
- a termination time \( \tau \)

as functions of the reports \( \hat{Y}_t \).
Preferences

- Both agents are risk neutral.
  - The agent discounts at rate $\gamma$.
  - The principal discounts at rate $r < \gamma$.
- When the project is terminated, the agent receives outside option $R$ and the principal receives $L$.
- The agent’s expected payoff at date 0 is
  $\mathcal{W}_0 = \mathbb{E} \left[ \int_0^T e^{-\gamma s} dC_s + e^{-\gamma \tau} R \right]$\where $dC_s = \lambda \left( dY_s - d\hat{Y}_s \right) + dl_s$.
- The principal’s expected payoff at date 0 is
  $b_0 = \mathbb{E} \left[ \int_0^T e^{-rs} \left( d\hat{Y}_s - dl_s \right) + e^{-r \tau} L \right]$. 
The optimal contract: preliminaries

- Deadweight loss from diverting cash flows.
  \[ \Rightarrow \text{We can focus on contracts which induce truth-telling.} \]
- Define the **agent’s promised value** \( W_t(\hat{Y}_t) \) after a history of reports \((\hat{Y}_s, 0 \leq s \leq t)\) as:
  \[
  W_t(\hat{Y}_t) = \mathbb{E}_t \left[ \int_t^\tau e^{-\gamma(s-t)} dl_s + e^{-\gamma(\tau-t)} R \right]
  \]

- **Lemma** There exists a process \( \beta_t \) such that
  \[
  dW_t = \gamma W_t dt - dl_t + \beta_t \left( d\hat{Y}_t - \mu dt \right)
  \]

- **Lemma** Truth-telling is incentive compatible if and only if \( \beta_t \geq \lambda \) for all \( t \leq \tau \).
  - **Intuition:** Steal \( dY_t - d\hat{Y}_t \): immediately gain \( \lambda(dY_t - d\hat{Y}_t) \), but lose \( \beta_t(dY_t - d\hat{Y}_t) \) in future expected payoff.
The principal’s value function

- Let $b(W)$ be the principal’s value function.
- The principal can always provide the agent with $W$ by
  - paying him a lump-sum transfer of $dl > 0$,
  - and then moving to the optimal contract with payoff $W - dl$
- Therefore, it must be that, $\forall W$

\[
b(W) \geq b(W - dl) - dl
\]

which implies that

\[
b'(W) \geq -1, \ \forall W
\]

- Define $W^1$, the lowest $W$ such that $b'(W) \leq -1$.
  It is optimal to pay the agent according to

\[
dl = \max(W - W^1, 0)
\]
First Best \((b + W = \mu / r)\)

\[ rb + \gamma W = \mu \]

\[ rb = \mu + \gamma W b'(W) + \frac{1}{2} \lambda^2 \sigma^2 b''(W) \]

Slope \(b' = -1\)
The principal’s value function

- On the region to the left of $W^1$, $b(W)$ solves the HJB equation

$$rb(W) = \max_{\beta \geq \lambda} \mu + \gamma Wb'(W) + \frac{1}{2} \beta^2 \sigma^2 b''(W)$$

- $\beta = \lambda$ is optimal.
  - **Intuition:** termination is inefficient ($\mu > rL + \gamma R$), so principal wants to reduce the risk that agent’s promised value falls to $R$.
- Boundary conditions: $b(R) = L$ and 1st and 2nd derivatives at $W^1$. 
The optimal contract in full

Agent’s promised utility $W_t$ evolves according to

$$dW_t = \gamma W_t dt - dl_t + \lambda \left( d\hat{Y}_t - \mu dt \right)$$

Compensation
When $W_t \in [R, W^1)$, $dl_t = 0$.
When $W_t = W^1$, payments $dl_t$ cause $W_t$ to reflect at $W^1$.

Principal’s payoff $b(W_t)$ satisfies

$$rb(W) = \mu + \gamma Wb'(W) + \frac{1}{2} \lambda^2 \sigma^2 b''(W) \quad \text{on } [R, W^1]$$

$$b'(W) = -1 \quad \text{for } W \geq W^1$$

and boundary conditions $b(R) = L$ and $rb(W^1) = \mu - \gamma W^1$.

Termination The contract is terminated at time $\tau$, when $W_t$ reaches $R$. 
Optimal contract can be implemented (not uniquely) with

1. **Equity**, a fraction $\lambda$ of which is held by the agent,
2. **Long-term debt**, with face value $D$,
3. **A credit line** with limit $C^L$ and interest rate $\gamma$

For the right choice of long-term debt $D$ and credit line limit $C^L$, it is incentive compatible for the agent to

- always pay down the credit line
- roll over the long-term debt
- pay any excess cash flow as dividends

thereby implementing the optimal contract.
Capital structure implementation: comments

1. Agent’s equity holding eliminates incentive to divert.

2. What stops the agent from overpaying dividends?
   - Assume the agent’s promised value under the contract follows
     \[ W_t = R + \lambda(C_L - M_t) \]  
     where \( M_t \) is the current draw on the credit line.
   - Now suppose the agent considers (i) borrowing up to the limit on the credit line, (ii) paying a large dividend, and (iii) defaulting.
     \[ \text{payoff} = \lambda(C_L - M_t) + R \]
     \[ \text{dividend} \quad \text{termination value} \]
   - By (\( \ast \)), the agent does just as well by behaving.
   - How to ensure (\( \ast \)) holds?
     - Choose long-term debt \( D \) and credit limit \( C_L \) such that the firm’s profit rate is such that (\( \ast \)) holds.
     - In particular: \( rD = \mu - \gamma R / \lambda - \gamma C_L \) and \( C_L = \lambda^{-1}(W_1 - R) \).
Conclusion

• The optimal contract is consistent with a capital structure that is widely observed in the data.
• Prediction: firms
  1. issue long-term debt
  2. use credit lines to absorb transient cashflow shocks
  3. pay out dividends once some cashflow threshold has been reached
• Everything in this presentation is in DeMarzo and Fishman (2007), the same setup but in discrete time.
• The main benefit of the continuous time model is **analytical comparative statics** and **security values**.
• Extensions
  • Hidden saving.
  • Hidden effort instead of diverting cash flow.
  • Endogenized termination payoffs.
  • Renegotiation.
  • Private benefits of control.