Identifying Ambiguity Shocks

Bhandari, Borovicka and Ho (2016)

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Survey expectations

Households' vs. professional forecasters' survey expectations ...

... about future aggregate variables, y_t

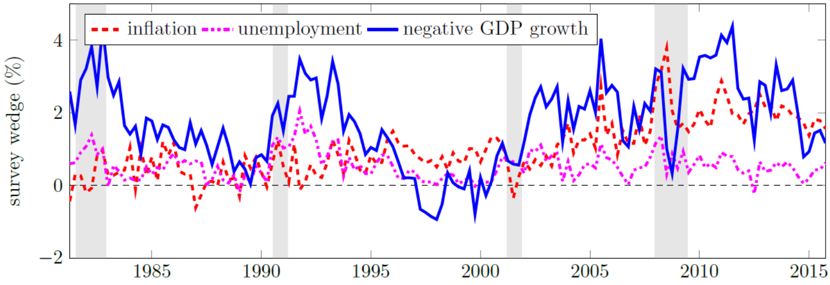
inflation, GDP growth, unemployment rate

... professional forecasts $\approx$ rational expectations

... household's deviation from this: bias, belief wedge

$$\Delta_t^{(\tau)}(y) \equiv \mathbb{E}_t^H \left[y_{t+\tau}\right] - \mathbb{E}_t^P \left[y_{t+\tau}\right]$$

Systematic, pessimistic bias with a common BC component



	mean	$\Delta_t^{(4)}(u)$	$\Delta_t^{(4)}(\pi)$	$-\Delta_t^{(4)}(d\mathcal{Y})$
$\Delta_t^{(4)}(u)$	0.55	1.00	0.04	0.63
$\Delta_t^{(4)}(\pi)$	1.01		1.00	0.32
$-\Delta_t^{(4)}(d\mathcal{Y})$	1.73			1.00

Factor model of distorted beliefs

Physical vs subjective beliefs

Characterize the joint dynamics of y_t and $\mathbb{E}_t^H[y_{t+1}]$

Allowing for differences in physical, P and subjective measure, $\tilde{P}$

- filtered prob space with P generated by the k-dim normal w_{t+1}
- likelihood ratio/change of measures to get $\tilde{P}$

$$m_{t+1} = \exp\left(\nu_t \cdot w_{t+1} - \frac{1}{2}|\nu_t|^2\right)$$

...”unrestricted” version of $\frac{\exp(-\theta V_{t+1})}{\mathbb{E}_t[\exp(-\theta V_{t+1})]}$

- essentially mean shifts in the shocks:

$$\widetilde{\mathbb{E}}_t[w_{t+1}] = \nu_t = \mathbb{E}_t[m_{t+1} w_{t+1}]$$

Factor-augmented VAR

Key restrictions:

- $\nu_{\mathbf{t}} = \mathbf{Hf_t}$, where H is $k \times 1$ and f_t is univariate
- f_t is a common (latent) factor of belief wedges $\Delta_t^{(\tau)}(y)$

  $\Rightarrow f_t$ is "time-varying measure of pessimism" of the household
- $k-1$ economic variables y_t and the factor f_t are related:

$$\begin{pmatrix} y_{t+1} \\ f_{t+1} \end{pmatrix} = \begin{pmatrix} \psi_{\mathbf{y}} & \psi_{\mathbf{f}}\rho_{\mathbf{f}} \\ 0 & \rho_{\mathbf{f}} \end{pmatrix} \begin{pmatrix} y_t \\ f_t \end{pmatrix} + \begin{pmatrix} \psi_{\mathbf{w}} & \psi_{\mathbf{f}}\sigma_{\mathbf{f}} \\ 0 & \sigma_{\mathbf{f}} \end{pmatrix} \begin{pmatrix} w_{t+1}^{y} \\ w_{t+1}^{f} \end{pmatrix}$$

with the observation equation:

$$\Delta_{t+1}^{(4)} = \psi_{\Delta} f_{t+1} + \sigma_{\mathbf{\Delta}} \varepsilon_{t+1}$$

where ψ_{Δ} is a known function of ψ_y, ψ_w and H

Bayesian estimation I

Persistent pessimism

$$f_{t+1} = \underset{(.73,\,.88)}{0.8}\ f_t \ + \ \underset{(.0028,\,.0063)}{0.0047}\ w^f_{t+1}$$

Variance decomposition at the posterior modes

	u_t	π	$d\mathcal{Y}_t$	R_t	$\Delta^{(4)}_t(u)$	$\Delta^{(4)}_t(\pi)$	$\Delta^{(4)}_t(d\mathcal{Y})$
w^f	21.0	4.0	13.1	5.3	23.2	9.7	67.5
w^{-f}	79.0	96.0	86.9	94.7			
meas err.					76.8	90.3	32.5

Bayesian estimation II – contractionary ambiguity shock

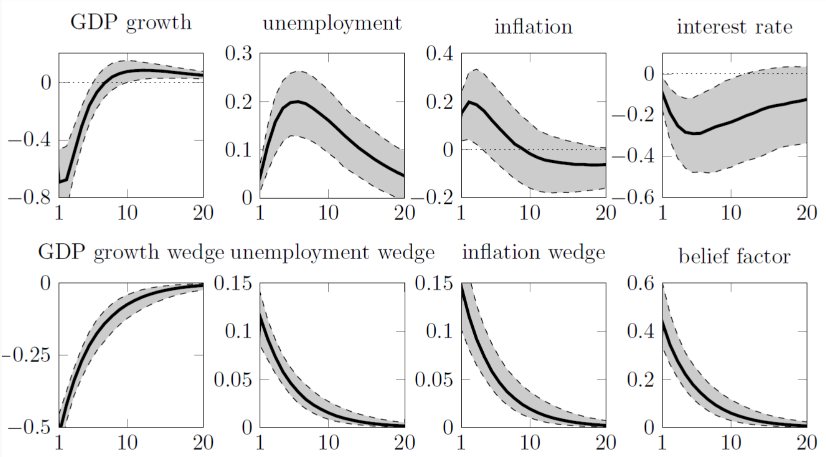


Figure 1: Bayesian IRFs to the pessimism shock w^f (10^{th}-90^{th} perc)

Structural model of distorted belief

Robust preferences

Aim: "rationalize" these results with a New-Keynesian DSGE model in which the representative household has robust preferences

"Endogenous" deviations from the physical measure P:

- ν_t is derived from continuation values V_{t+1} (equilibrium object)
- how we specify the dynamic model with shocks becomes essential
$$\Rightarrow V_{t+1} \Rightarrow \nu_t \Rightarrow m_{t+1} = \frac{\exp(-\theta_t V_{t+1})}{\mathbb{E}_t[\exp(-\theta_t V_{t+1})]}$$

Proposal: multiplier preferences with time-varying $\theta_t = \widetilde{\theta} x_t = f_t$

$$V_t = \min_{E_t[m_{t+1}]=1} \max_{c_t} \; u(x_t) + \frac{\beta}{\theta_t}\mathbb{E}_t\left[m_{t+1}\log m_{t+1}\right] + \beta\mathbb{E}_t\left[m_{t+1}V_{t+1}\right]$$
$$x_{t+1} = \psi(x_t, w_{t+1}) \qquad \text{(equilibrium law of motion)}$$

New-Keynesian model: key components

- **AR(1) robust concern:** $\theta_{t+1} = (1-\rho_\theta)\bar{\theta} + \rho_\theta \theta_t + \sigma_\theta w^\theta_{t+1}$
  - exogenous, but equilibrium endogenously determines the low V_{t+1} states that become more likely under the worst-case model
- **Production**:
  comp final good + monopolistic retailers (Calvo) + comp wholesaler
- **Frictional labor market**: hiring decisions are determined by surplus
  - evaluated under the worst-case $\widetilde{P}$
  - Hall-Milgrom bargaining $\Rightarrow$ more procyclical surplus (than Nash)
- **Shock structure**: independent normals under P
  - neutral technology shock: $\mathbf{w}^{\mathbf{A}}$ (iid growth, unit root)
  - investment specific shock: $\mathbf{w}^{\mathbf{\Psi}}$ (mean-reverting growth)
  - monetary policy shock: $\mathbf{w}^{\mathbf{R}}$
  - ambiguity shock: $\mathbf{w}^\theta$

Linear approximation – Borovicka and Hansen (2014)

- Approximate the dynamics around the deterministic SS, $\bar{x} = \psi(\bar{x}, 0)$

$$x_{1t+1} = \psi_{\mathbf{q}} + \psi_x x_{1t} + \psi_w w_{t+1}$$
$$V_{1t} = V_x x_{1t} + V_{\mathbf{q}}$$

where $\mathbf{q}$ scales the volatility of w_{t+1} and $x_{jt} = \left.\frac{d^j y_t}{d\mathbf{q}^j}\right|_{\mathbf{q}=0}$

- Critical step in expanding the continuation value recursion
  - ... $\mathbf{q}$ scales **jointly** the volatility of w_{t+1} and the magnitude of θ_t
  - $\Rightarrow$ worst-case and benchmark do not converge as $\mathbf{q} \to 0$
- Under $\widetilde{P}$ the perceived innovations are

$$w_{t+1} \sim \mathcal{N}\left(-(\bar{\theta} + \theta_{1t})(V_x \psi_w)', \quad I_{4\times 4}\right)$$

V_x: sensitivity of continuation value to the states

ψ_w: loadings of the states on the shocks

Approximation and estimation

- State dynamics under the worst-case model

$$x_{1t+1} = \left[\psi_{\mathbf{q}} - \psi_w \psi_w' V_x' \widetilde{\theta} \bar{x}\right] + \left[\psi_x - \psi_w \psi_w' V_x' \widetilde{\theta}\right] x_{1t} + \psi_w \widetilde{w}_{t+1}$$

- Model-implied belief wedge

$$\Delta_t^{(1)} = \psi_w \underbrace{\widetilde{\mathbb{E}}\left[w_{t+1}\right]}_{=\nu_t} = \underbrace{-(\bar{\theta} + \theta_{1t})}_{=f_t}(\psi_w \underbrace{\psi_w')V_x'}_{=H}$$

. . . **key model restriction**: H is a function of model parameters

- Assumption: surveyed households provide answers using $\widetilde{P}$

  worst-case model ≈ subjective beliefs

- Linear approximation facilitates **Bayesian estimation** methods

  model parameters and latent factor, $\{\theta_t\}$

Bayesian estimation

Aim: quantify the role of ambiguity shocks

"more" than FAVAR: impact is restricted by the structural model, $\mathbf{V_x}$

Persistent pessimism

$$\theta_{t+1} = \underset{(.85,\,.89)}{0.87}\ \theta_t + \underset{(.01,\,.02)}{0.02}\ w^{\theta}_{t+1}$$

Variance decomposition at the posterior modes

	u_t	π	$d\mathcal{Y}_t$	R_t	Hiring	$\Delta^{(4)}_t(u)$	$\Delta^{(4)}_t(\pi)$	$\Delta^{(4)}_t(d\mathcal{Y})$
Model								
w^{θ}	13.6	2.6	24.8	1.9	21.8	6.3	22.0	16.4
m.e.						93.7	78.0	83.6
FAVAR								
w^{f}	21.0	4.0	13.1	5.3		23.2	9.7	67.5

Contractionary ambiguity shock

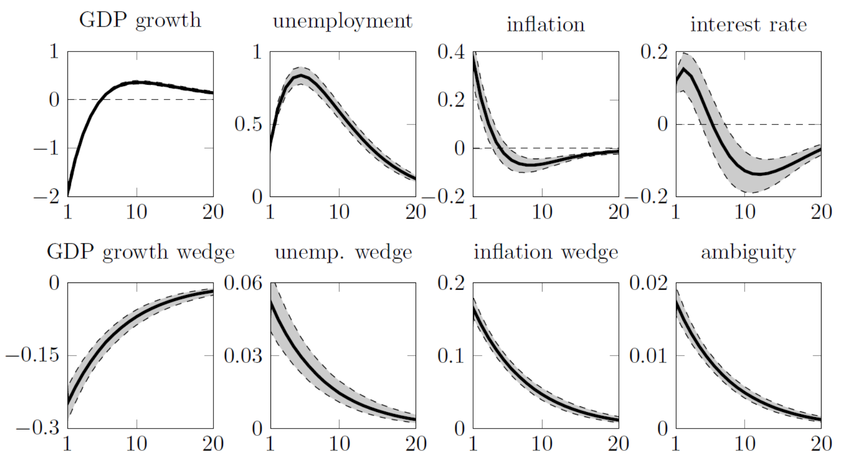
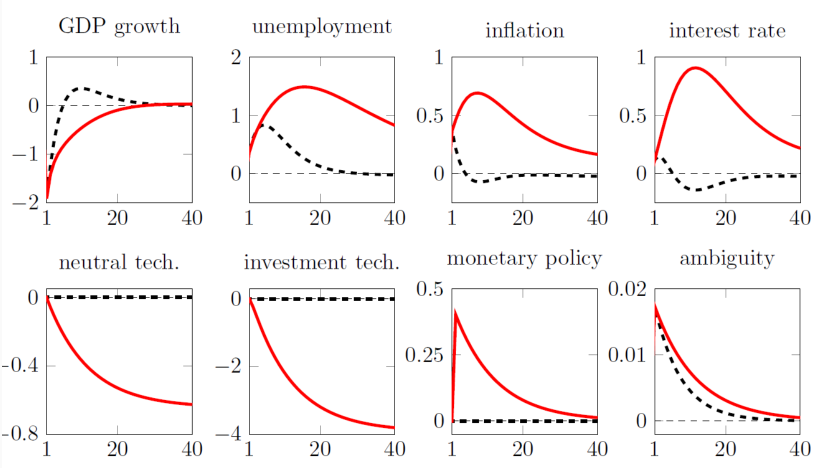


Figure 2: Bayesian IRFs to the pessimism shock w^f (10^{th}-90^{th} perc)

Ambiguity shocks under P (dashed) and $\widetilde{P}$ (red)

Main force: interaction of w^{θ} and (w^{A}, w^{Ψ}, w^{R})

independent under P vs. correlated (according to V_x) under $\widetilde{P}$



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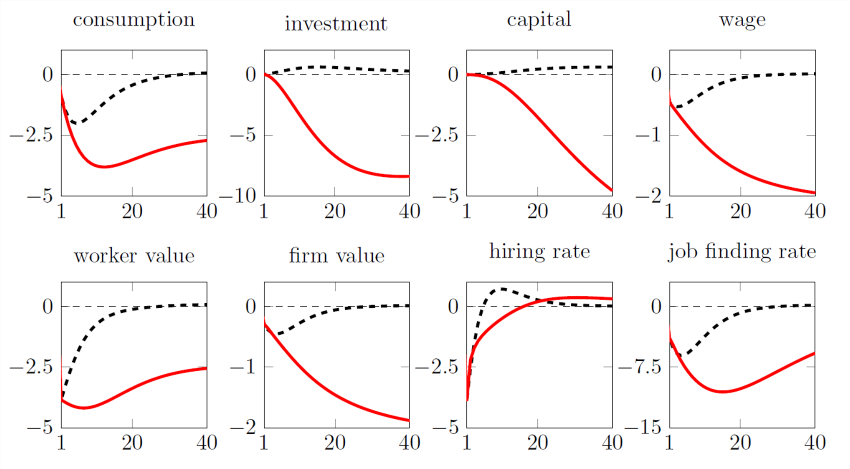

NOT a story of belief heterogeneity . . .

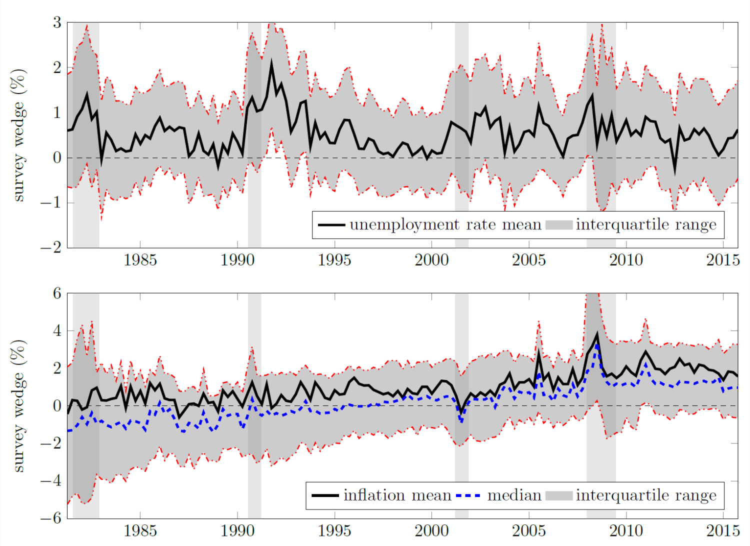

Figure 3: Dispersion in survey expectations in the Michigan Survey

Single factor is a measure of pessimism. . .

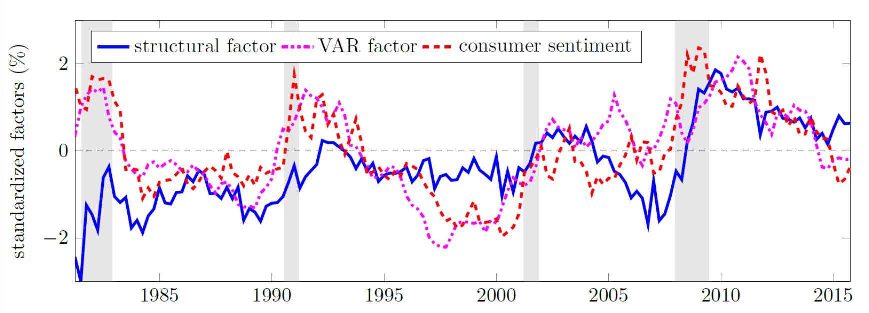

Figure 4: Extracted ambiguity factors with the negative of the Index of Consumer Sentiment (Michigan Survey)