The Time Structure of Self-Enforcing Agreements

Debraj Ray

Econometrica,Vol. 70 (2) (March 2002), , 547-582.

Presented by Tomek Piskorski

Objective of the Paper

Objective of the Paper

- Consider a repeated relationship between two individuals, a principal and an agent.

Objective of the Paper

- Consider a repeated relationship between two individuals, a principal and an agent.
- Suppose an agent is not committed to honoring the terms of the contract and the principal's discounted sum of profits must be nonnegative.

Objective of the Paper

- Consider a repeated relationship between two individuals, a principal and an agent.
- Suppose an agent is not committed to honoring the terms of the contract and the principal's discounted sum of profits must be nonnegative.
- At any time the agent may take an advantage of the going agreement and shirk his responsibilities or repudiate his obligations.

Objective of the Paper

- Consider a repeated relationship between two individuals, a principal and an agent.
- Suppose an agent is not committed to honoring the terms of the contract and the principal's discounted sum of profits must be nonnegative.
- At any time the agent may take an advantage of the going agreement and shirk his responsibilities or repudiate his obligations.
- What are the properties of Pareto optimal sequences of agreements that are proof against deviations by the agent and at the same time generate nonnegative profits for the principal?

Set-up

Set-up

Time: Discrete, Infinite, $t = 0, 1, \ldots\ldots$

Set-up

Time: Discrete, Infinite, $t = 0, 1, \ldots..$

There is one principal and one agent (worker).

Set-up

Time: Discrete, Infinite, $t = 0, 1, \ldots..$

There is one principal and one agent (worker).

At every t an agreement $\xi_t \in \Xi$ is specified.

Set-up

Time: Discrete, Infinite, $t = 0, 1, \ldots$..

There is one principal and one agent (worker).

At every t an agreement $\xi_t \in \Xi$ is specified.

Ξ is the set of physically feasible agreements.

Set-up

Time: Discrete, Infinite, $t = 0, 1, \ldots.$

There is one principal and one agent (worker).

At every t an agreement $\xi_t \in \Xi$ is specified.

Ξ is the set of physically feasible agreements.

For any t, $\xi_t = (c_t, m_t)$ where c_t takes value in compact metric space and $m_t \in R$.

Set-up

Time: Discrete, Infinite, $t = 0, 1, \ldots.$

There is one principal and one agent (worker).

At every t an agreement $\xi_t \in \Xi$ is specified.

Ξ is the set of physically feasible agreements.

For any t, $\xi_t = (c_t, m_t)$ where c_t takes value in compact metric space and $m_t \in R$.

Example: Employment relationship, where c_t hours worked at t, and m_t stands for the wage paid to the worker at t.

Self-Enforcing Agreements

Self-Enforcing Agreements

Consider the sequence of agreements $\{\xi_t\}$. Assume that it is honored. From any date t onwards define

Self-Enforcing Agreements

Consider the sequence of agreements $\{\xi_t\}$. Assume that it is honored. From any date t onwards define

$\alpha_t = \sum_{s=t}^{\infty} \beta^{s-t} A(\xi_s)$ is the agent's continuation payoff ($A_m > 0$),

Self-Enforcing Agreements

Consider the sequence of agreements $\{\xi_t\}$. Assume that it is honored. From any date t onwards define

$\alpha_t = \sum_{s=t}^{\infty} \beta^{s-t} A(\xi_s)$ is the agent's continuation payoff ($A_m > 0$),

$\pi_t = \sum_{s=t}^{\infty} \beta^{s-t} P(\xi_s)$ is the principal's continuation payoff ($P_m < 0$).

Self-Enforcing Agreements

Consider the sequence of agreements $\{\xi_t\}$. Assume that it is honored. From any date t onwards define

$\alpha_t = \sum_{s=t}^{\infty} \beta^{s-t} A(\xi_s)$ is the agent's continuation payoff ($A_m > 0$),

$\pi_t = \sum_{s=t}^{\infty} \beta^{s-t} P(\xi_s)$ is the principal's continuation payoff ($P_m < 0$).

Note that a sequence of agreements $\{\xi_t\}$ in Ξ generates a sequence of continuation payoffs $\{\alpha_t, \pi_t\}$.

Self-Enforcing Agreements

Consider the sequence of agreements $\{\xi_t\}$. Assume that it is honored. From any date t onwards define

$\alpha_t = \sum_{s=t}^{\infty} \beta^{s-t} A(\xi_s)$ is the agent's continuation payoff ($A_m > 0$),

$\pi_t = \sum_{s=t}^{\infty} \beta^{s-t} P(\xi_s)$ is the principal's continuation payoff ($P_m < 0$).

Note that a sequence of agreements $\{\xi_t\}$ in Ξ generates a sequence of continuation payoffs $\{\alpha_t, \pi_t\}$.

Let $V(\xi_t)$ be the agent's value of outside option at agreement ξ_t.

Self-Enforcing Agreements

Consider the sequence of agreements $\{\xi_t\}$. Assume that it is honored. From any date t onwards define

$\alpha_t = \sum_{s=t}^{\infty} \beta^{s-t} A(\xi_s)$ is the agent's continuation payoff ($A_m > 0$),

$\pi_t = \sum_{s=t}^{\infty} \beta^{s-t} P(\xi_s)$ is the principal's continuation payoff ($P_m < 0$).

Note that a sequence of agreements $\{\xi_t\}$ in Ξ generates a sequence of continuation payoffs $\{\alpha_t, \pi_t\}$.

Let $V(\xi_t)$ be the agent's value of outside option at agreement ξ_t.

A sequence of agreements $\{\xi_t\}$ in Ξ is **self-enforcing** (SE) if

Self-Enforcing Agreements

Consider the sequence of agreements $\{\xi_t\}$. Assume that it is honored. From any date t onwards define

$\alpha_t = \sum_{s=t}^{\infty} \beta^{s-t} A(\xi_s)$ is the agent's continuation payoff ($A_m > 0$),

$\pi_t = \sum_{s=t}^{\infty} \beta^{s-t} P(\xi_s)$ is the principal's continuation payoff ($P_m < 0$).

Note that a sequence of agreements $\{\xi_t\}$ in Ξ generates a sequence of continuation payoffs $\{\alpha_t, \pi_t\}$.

Let $V(\xi_t)$ be the agent's value of outside option at agreement ξ_t.

A sequence of agreements $\{\xi_t\}$ in Ξ is **self-enforcing** (SE) if

(i) $\alpha_t \geq V(\xi_t)$ for all $t \geq 0$,

Self-Enforcing Agreements

Consider the sequence of agreements $\{\xi_t\}$. Assume that it is honored. From any date t onwards define

$$\alpha_t = \sum_{s=t}^{\infty} \beta^{s-t} A(\xi_s)$$ is the agent's continuation payoff ($A_m > 0$),

$$\pi_t = \sum_{s=t}^{\infty} \beta^{s-t} P(\xi_s)$$ is the principal's continuation payoff ($P_m < 0$).

Note that a sequence of agreements $\{\xi_t\}$ in Ξ generates a sequence of continuation payoffs $\{\alpha_t, \pi_t\}$.

Let $V(\xi_t)$ be the agent's value of outside option at agreement ξ_t.

A sequence of agreements $\{\xi_t\}$ in Ξ is **self-enforcing** (SE) if

(i) $\alpha_t \geq V(\xi_t)$ for all $t \geq 0$, (ii) $\pi_t \geq 0$ for all $t \geq 0$.

Efficient Self-Enforcing (ESE)
Sequence of Agreements

Efficient Self-Enforcing (ESE) Sequence of Agreements

A SE agreement sequence $\{\xi_t\}$ in Ξ is **efficient** if the payoffs it generates (α_0, π_0) are such that given π_0, α_0 is the maximum payoff to the agent among all SE agreements sequences that yield at least π_0 to the principal.

Efficient Self-Enforcing (ESE) Sequence of Agreements

A SE agreement sequence $\{\xi_t\}$ in Ξ is **efficient** if the payoffs it generates (α_0, π_0) are such that given π_0, α_0 is the maximum payoff to the agent among all SE agreements sequences that yield at least π_0 to the principal.

An **agent's best self-enforcing sequence** of agreements is the one that maximizes the agent payoff among the class of all ESE sequences of agreements.

The Structure of ESE Sequences

The Structure of ESE Sequences

Theorem 1: Let $\{\xi_t\}$ be an ESE agreement sequence. Then there exists a finite date such that an agent's best self-enforcing sequence is followed thereafter.

The Structure of ESE Sequences

Theorem 1: Let $\{\xi_t\}$ be an ESE agreement sequence. Then there exists a finite date such that an agent's best self-enforcing sequence is followed thereafter.

A sequence of agreements $\{\xi_t^*\}$ is **periodic** if there exists a date N and a finite collection of agreements $\{\xi_0, .., \xi_N\}$ s.t. for all t, $\xi_t^* = \xi_i$ where $i =$Reminder$(t/(N+1))$.

The Structure of ESE Sequences

Theorem 1: Let $\{\xi_t\}$ be an ESE agreement sequence. Then there exists a finite date such that an agent's best self-enforcing sequence is followed thereafter.

A sequence of agreements $\{\xi_t^*\}$ is **periodic** if there exists a date N and a finite collection of agreements $\{\xi_0, .., \xi_N\}$ s.t. for all t, $\xi_t^* = \xi_i$ where $i =$Reminder$(t/(N+1))$.

Theorem 2:

(i) There exists an agent's best SE sequence that is periodic.

The Structure of ESE Sequences

Theorem 1: Let $\{\xi_t\}$ be an ESE agreement sequence. Then there exists a finite date such that an agent's best self-enforcing sequence is followed thereafter.

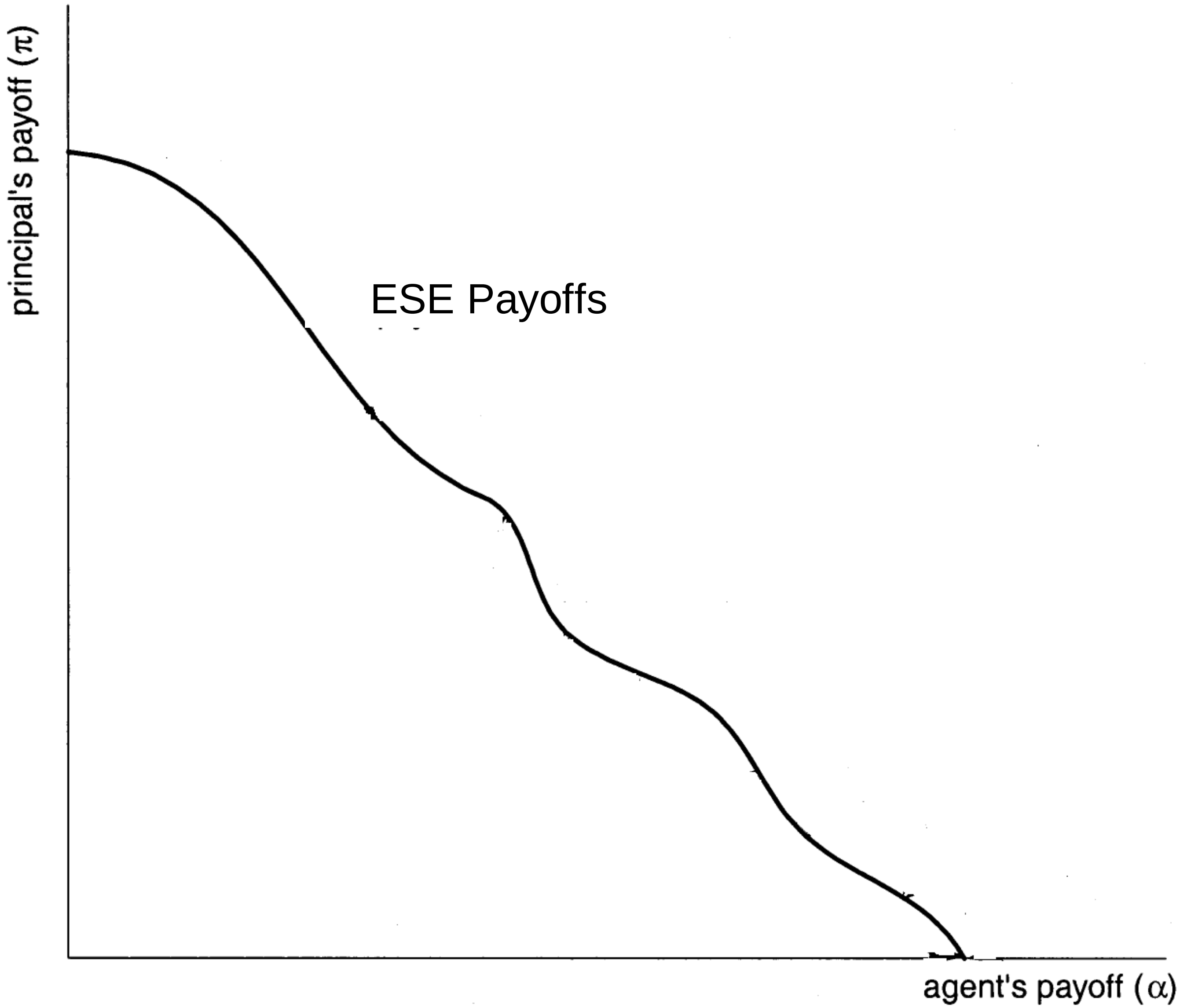
A sequence of agreements $\{\xi_t^*\}$ is **periodic** if there exists a date N and a finite collection of agreements $\{\xi_0, .., \xi_N\}$ s.t. for all t, $\xi_t^* = \xi_i$ where $i =$Reminder($t/(N+1)$).

Theorem 2:

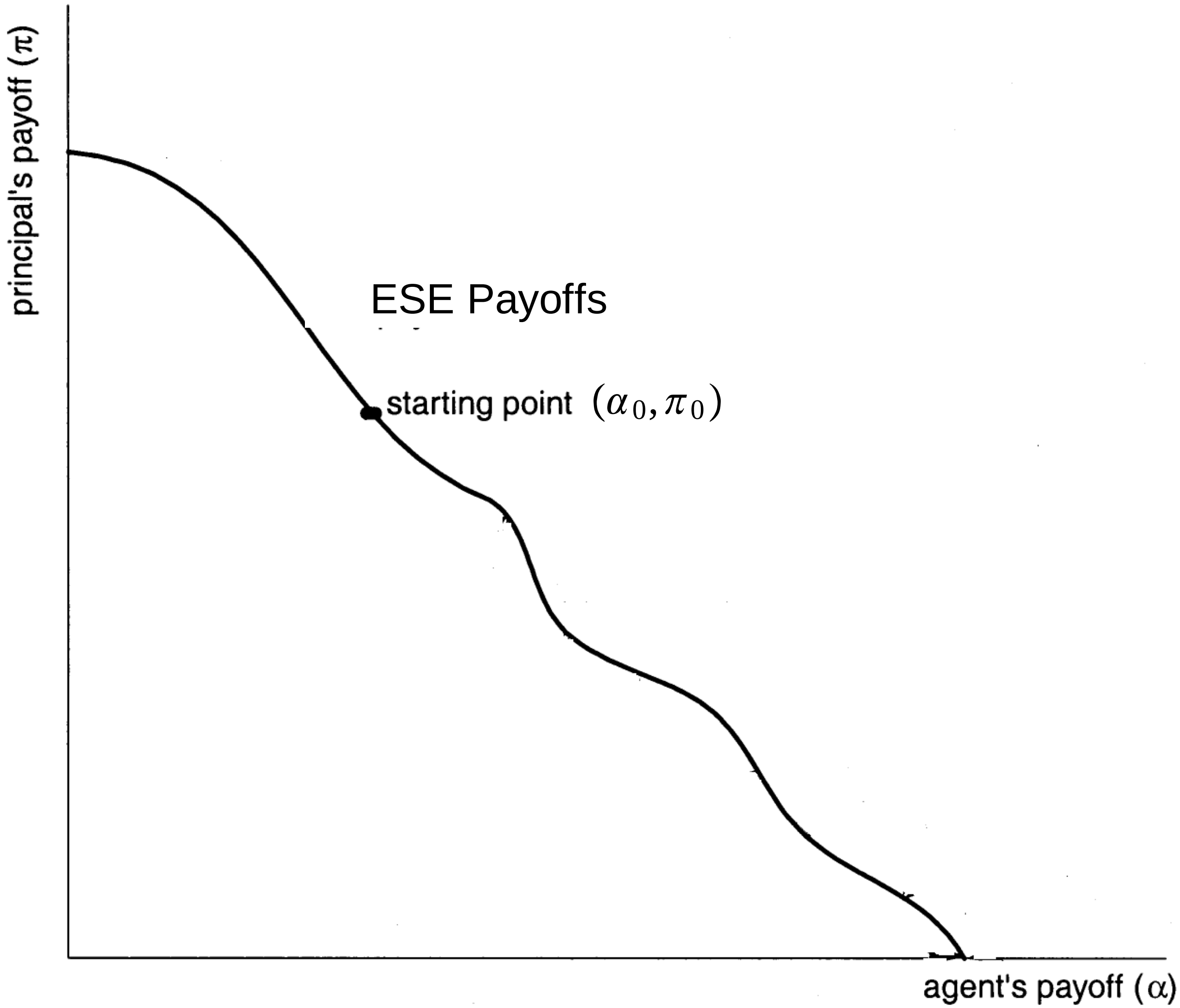
(i) There exists an agent's best SE sequence that is periodic.

(ii) Let $\{\xi_t\}$ be an ESE sequence. Then there exists a finite date s.t. an agent's best SE periodic sequence is followed thereafter.

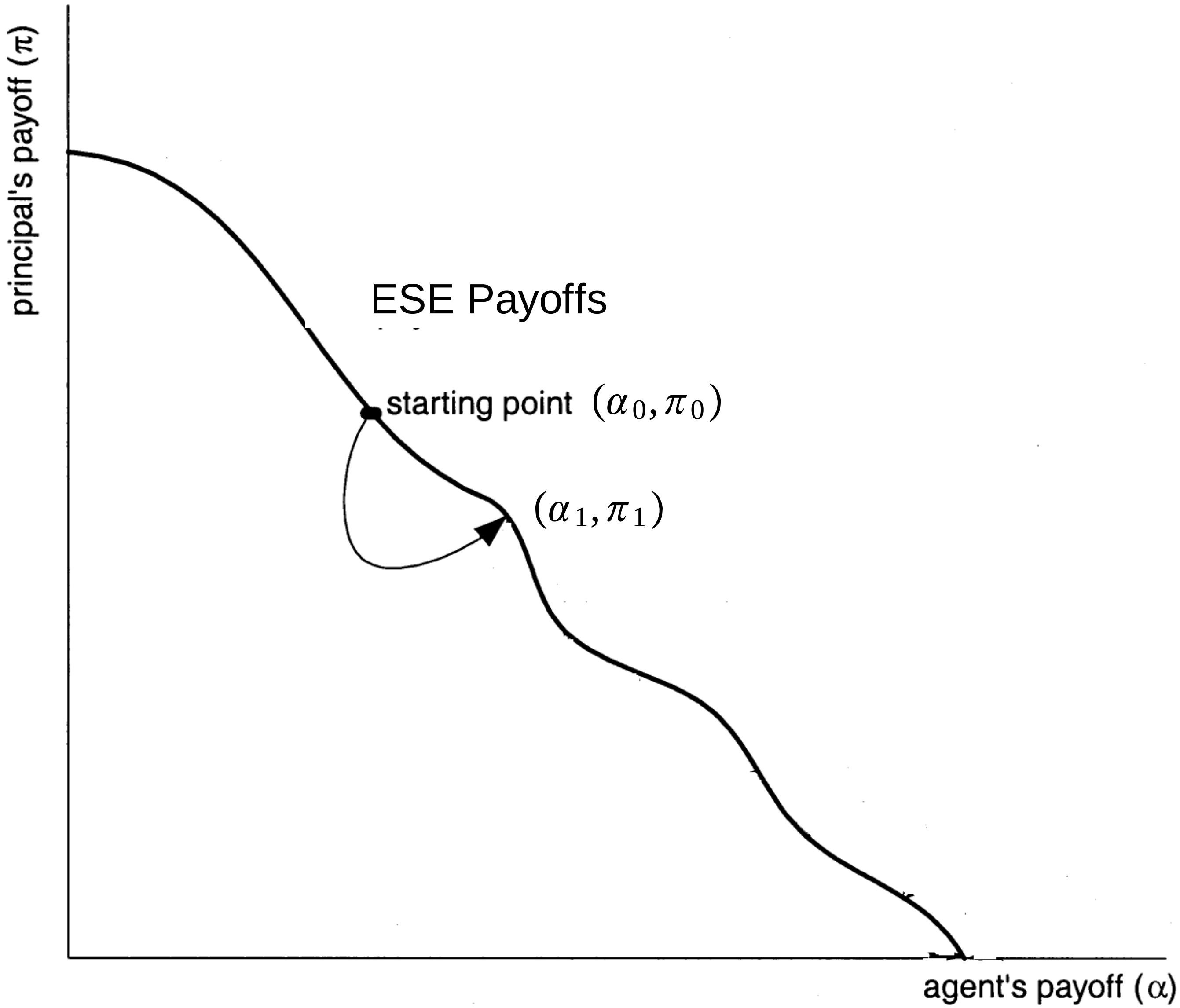
An Example: Dynamics of ESE Continuation Payoffs



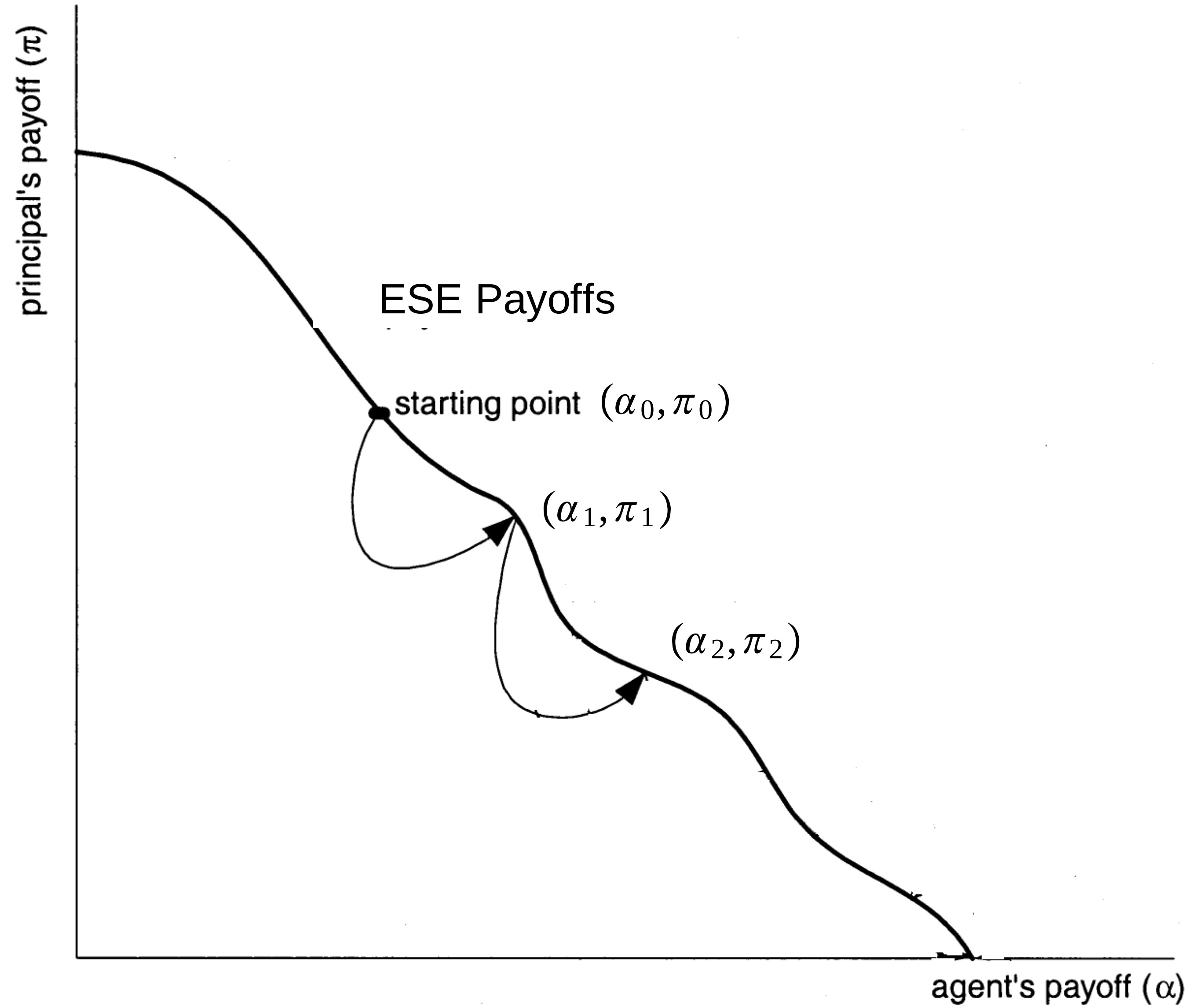
An Example: Dynamics of ESE Continuation Payoffs



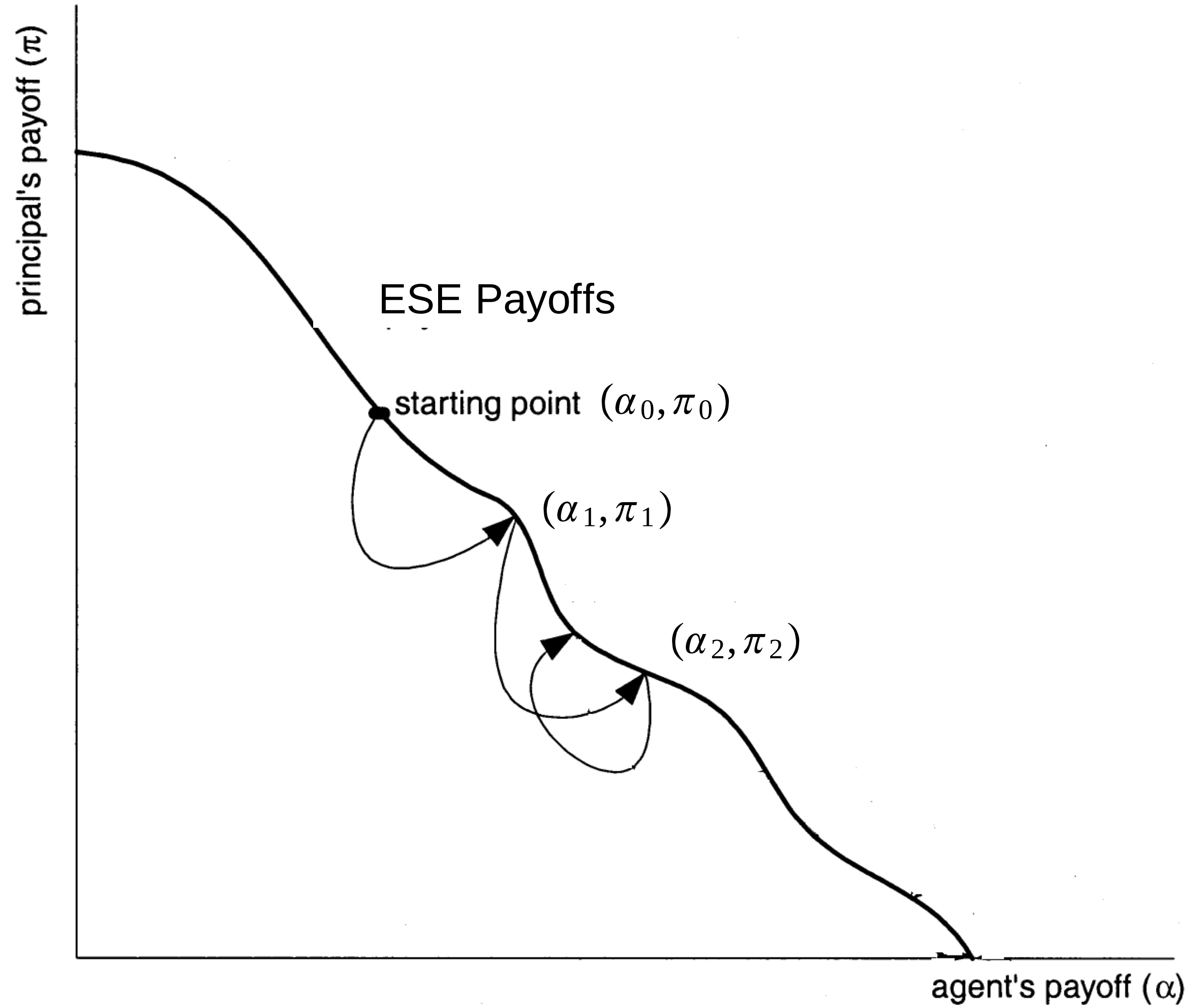
An Example: Dynamics of ESE Continuation Payoffs



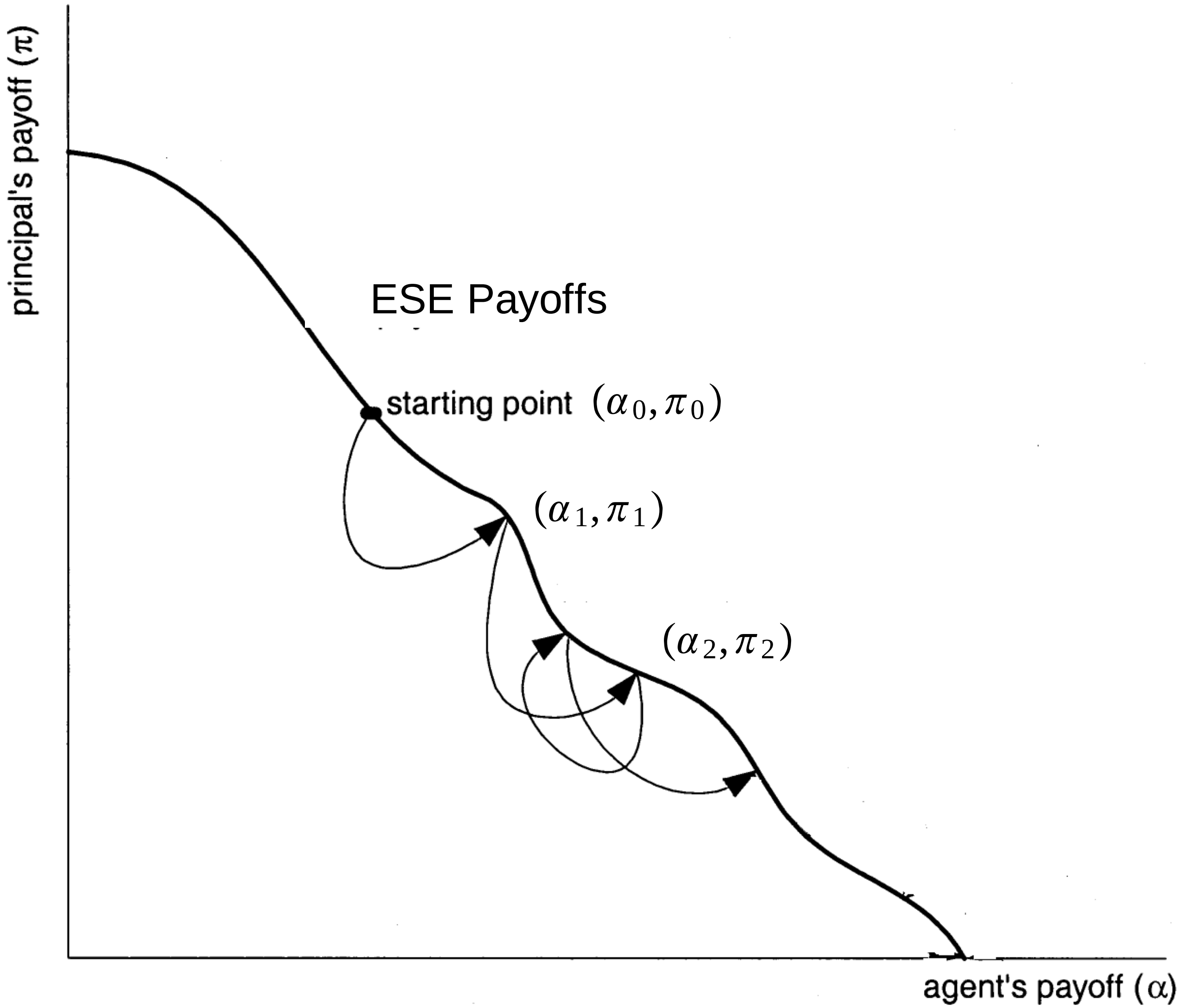
An Example: Dynamics of ESE Continuation Payoffs



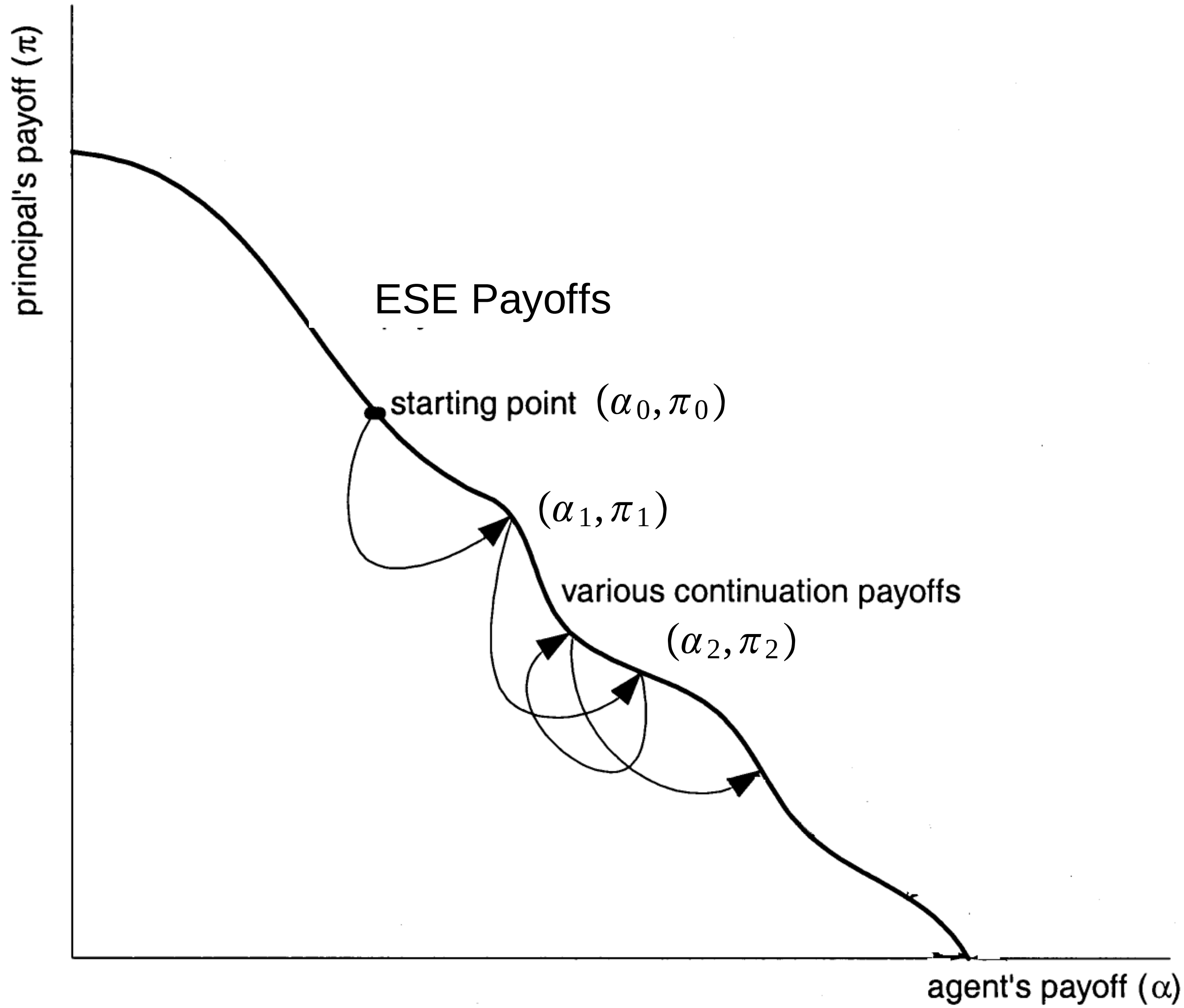
An Example: Dynamics of ESE Continuation Payoffs



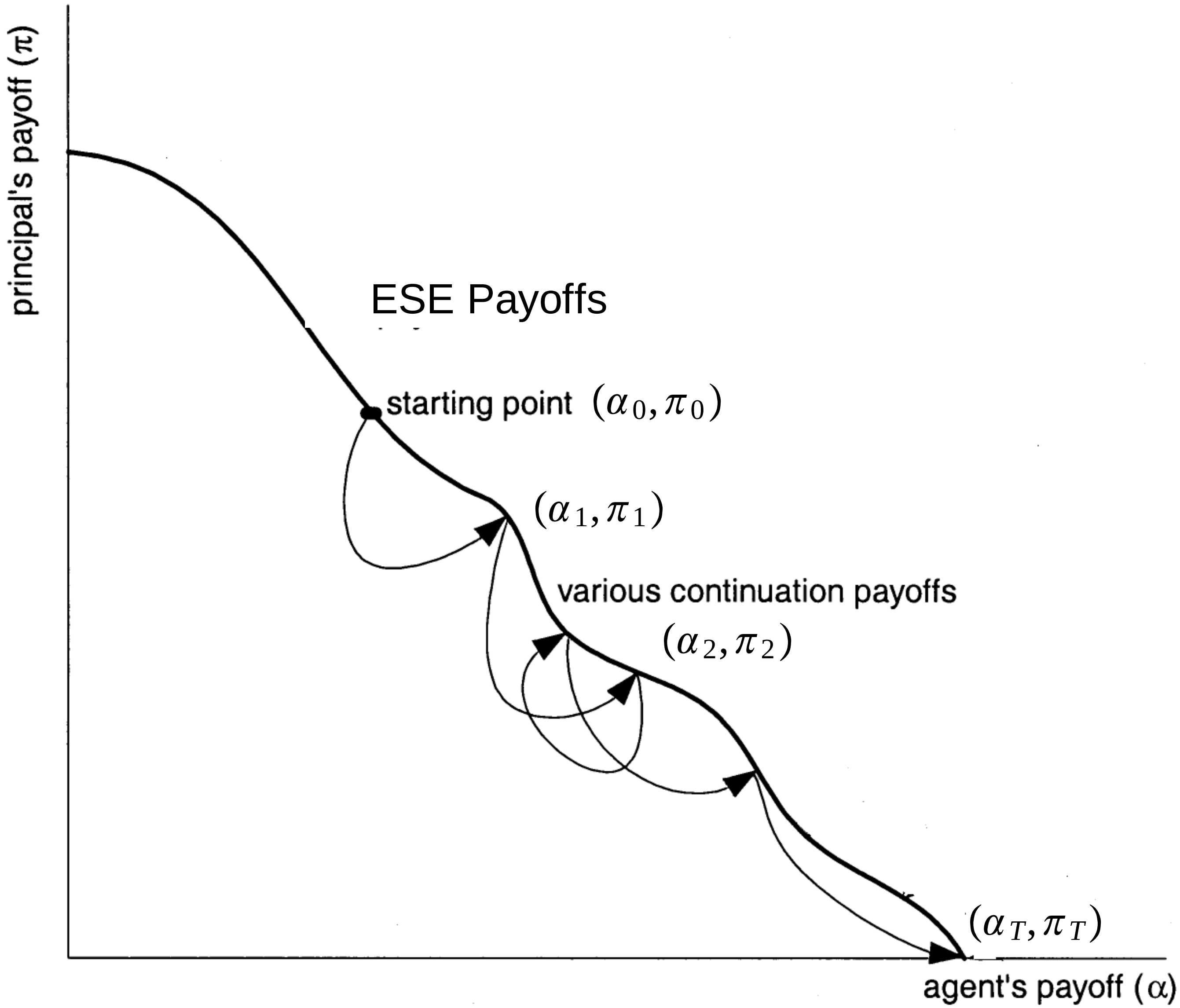
An Example: Dynamics of ESE Continuation Payoffs



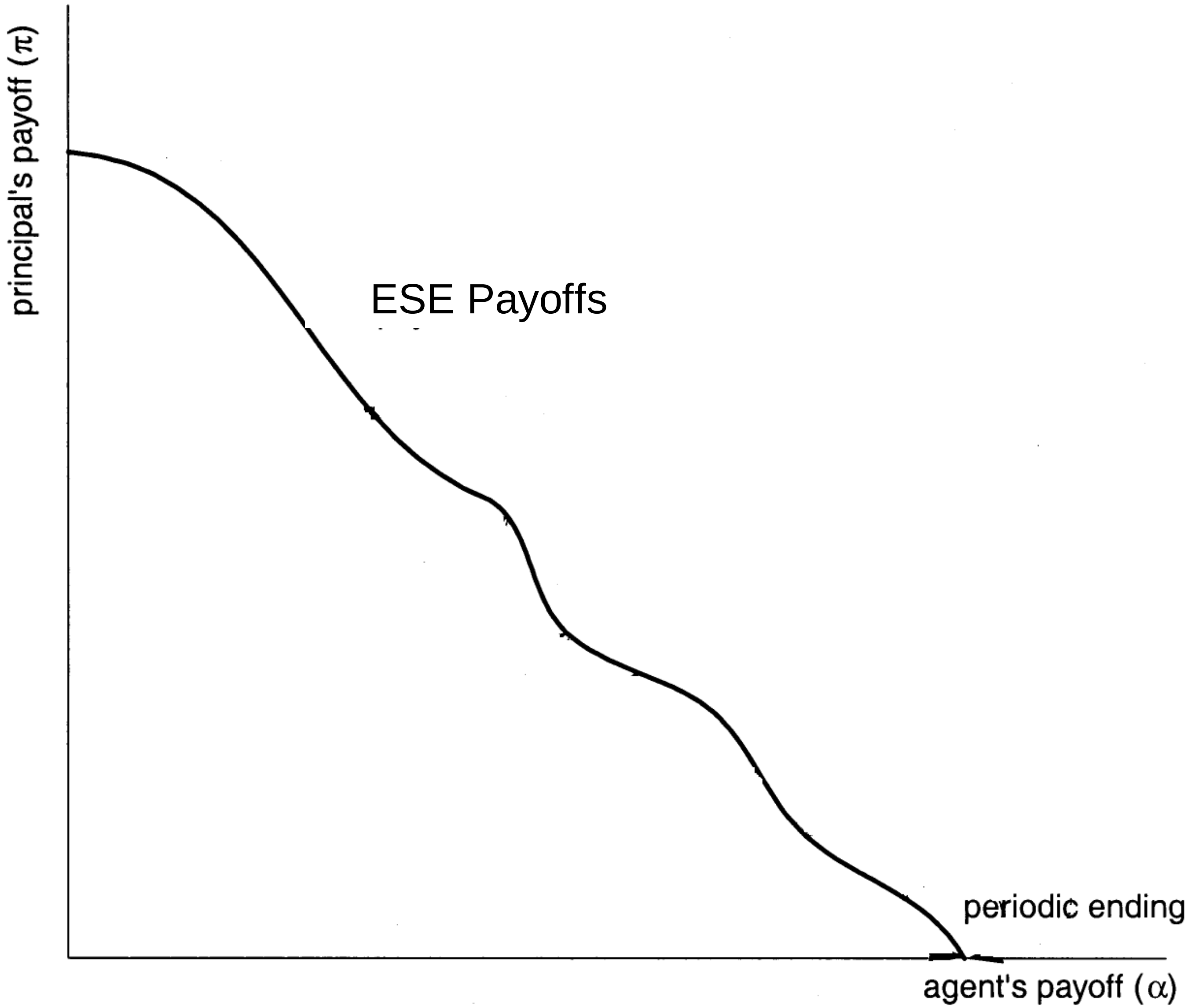
An Example: Dynamics of ESE Continuation Payoffs



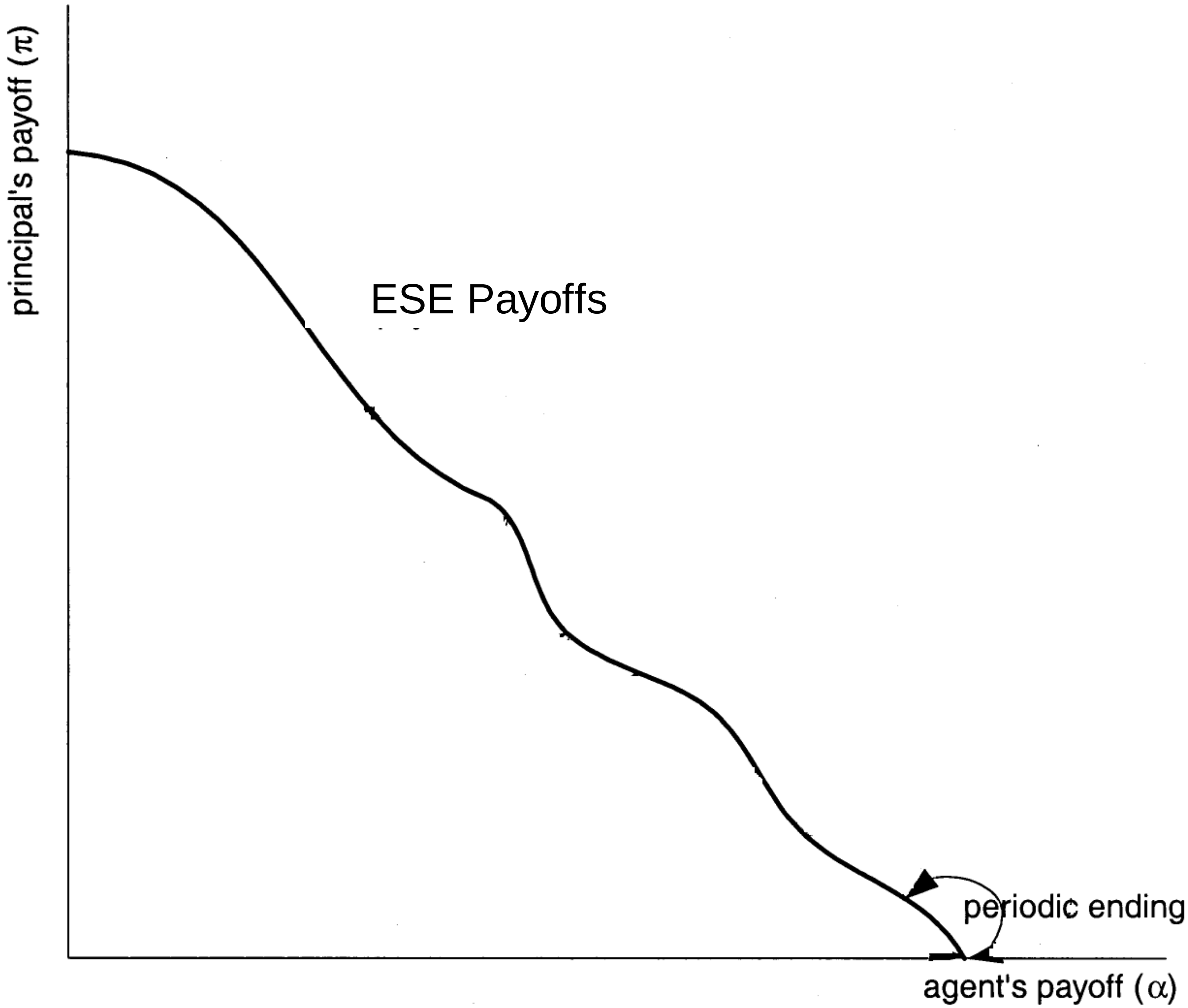
An Example: Dynamics of ESE Continuation Payoffs



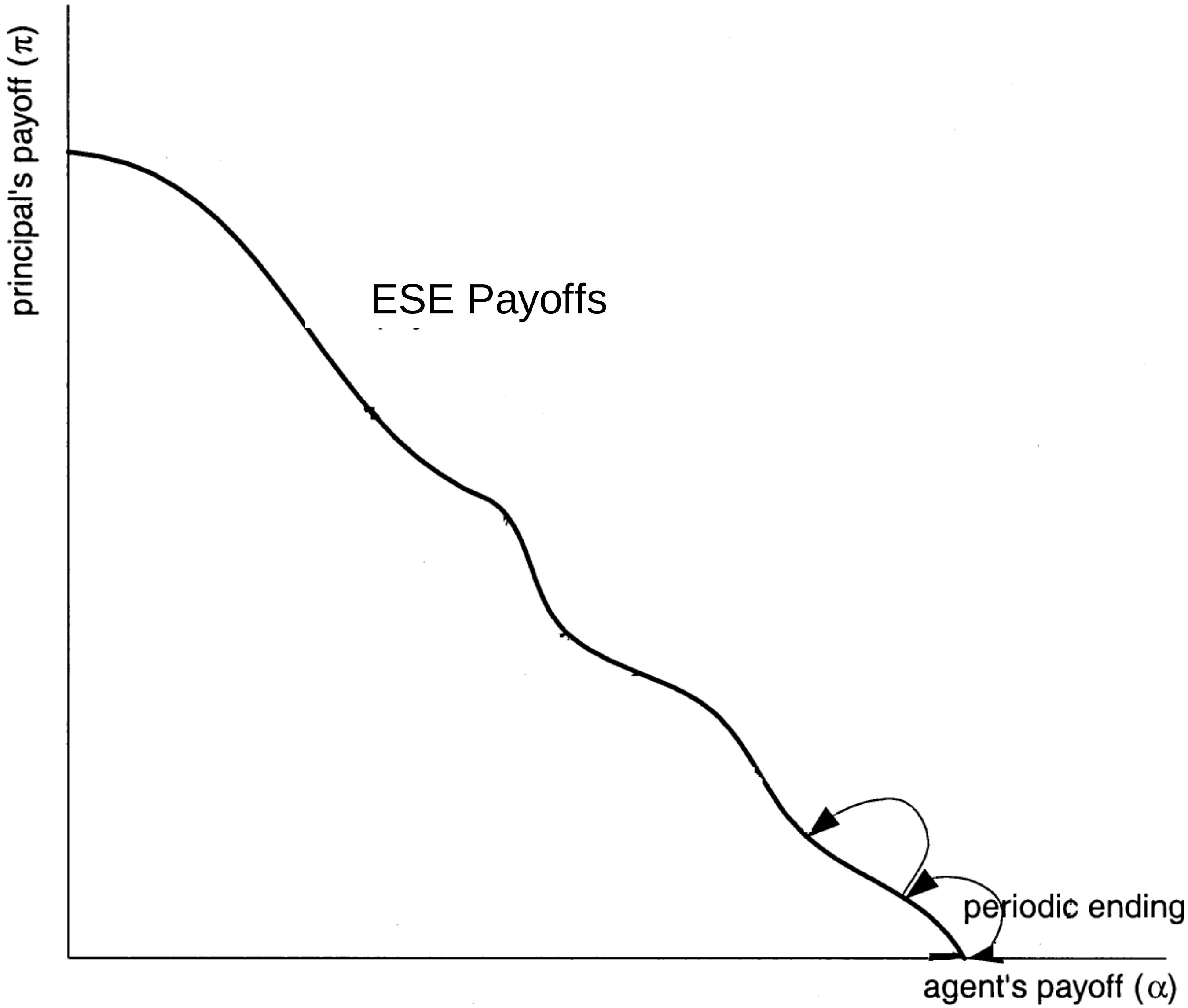
An Example: Dynamics of ESE Continuation Payoffs



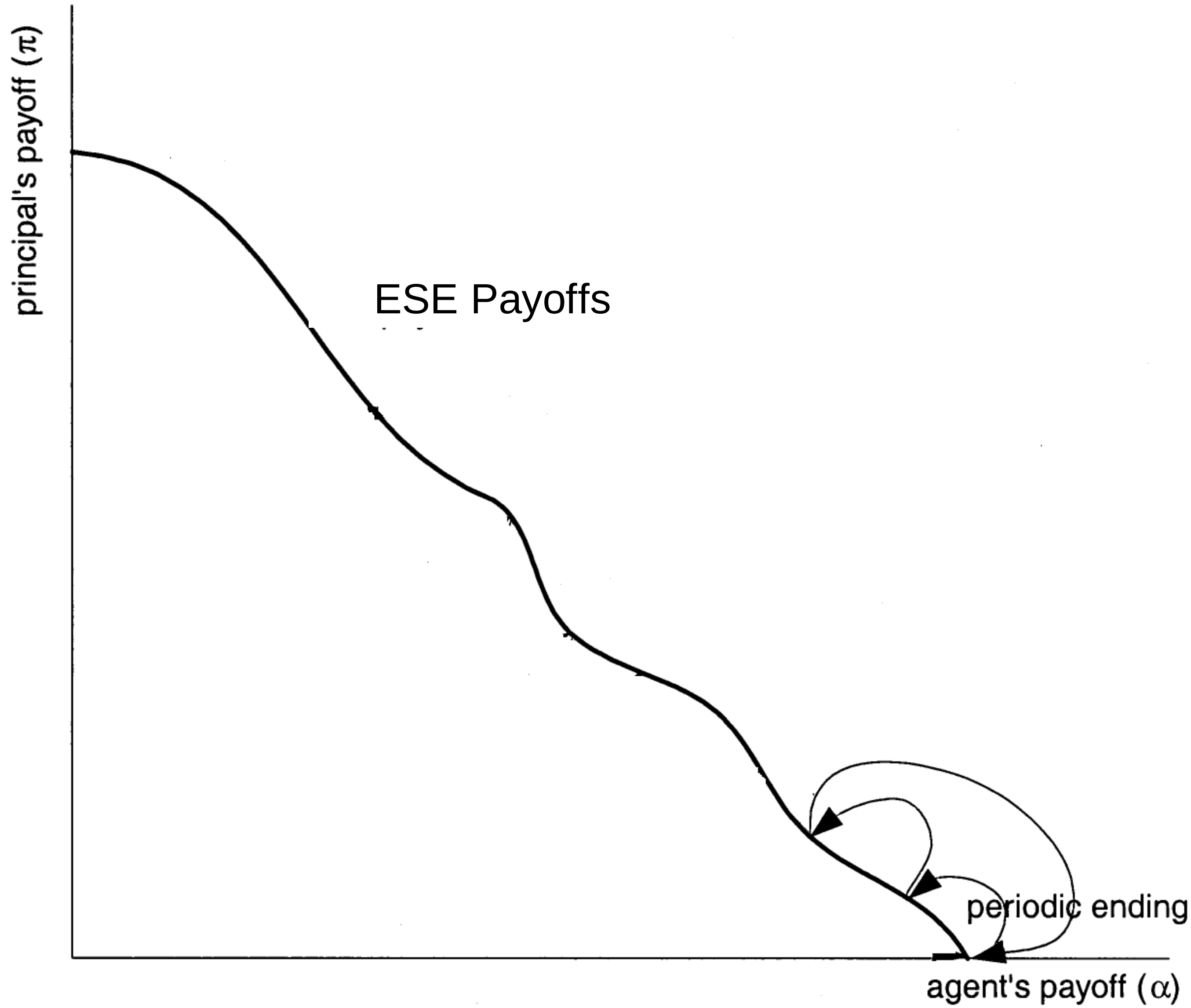
An Example: Dynamics of ESE Continuation Payoffs



An Example: Dynamics of ESE Continuation Payoffs



An Example: Dynamics of ESE Continuation Payoffs



The Structure of ESE Sequences in Quasi-Linear Environments

The Structure of ESE Sequences in Quasi-Linear Environments

Assume: $A(\xi) = A(c,m) = A(c) + m$ and $P(\xi) = P(c,m) = P(c) - m$.

The Structure of ESE Sequences in Quasi-Linear Environments

Assume: $A(\xi) = A(c,m) = A(c) + m$ and $P(\xi) = P(c,m) = P(c) - m$.

Theorem 3: Let $\{\xi_t\}$ be an ESE agreement sequence. Then there exists a finite date T such that an agent's best SE sequence is followed thereafter and for all $t \geq T$, $\xi_t = (c^*, m^*)$ (and $P(\xi_t) = 0$).

The Structure of ESE Sequences in Quasi-Linear Environments

Assume: $A(\xi) = A(c,m) = A(c) + m$ and $P(\xi) = P(c,m) = P(c) - m$.

Theorem 3: Let $\{\xi_t\}$ be an ESE agreement sequence. Then there exists a finite date T such that an agent's best SE sequence is followed thereafter and for all $t \geq T$, $\xi_t = (c^*, m^*)$ (and $P(\xi_t) = 0$).

Theorem 4: For any ESE sequence, consider the "initial phase" up to date T-1, before (c^*, m^*) is applied. Then:

The Structure of ESE Sequences in Quasi-Linear Environments

Assume: $A(\xi) = A(c,m) = A(c) + m$ and $P(\xi) = P(c,m) = P(c) - m$.

Theorem 3: Let $\{\xi_t\}$ be an ESE agreement sequence. Then there exists a finite date T such that an agent's best SE sequence is followed thereafter and for all $t \geq T$, $\xi_t = (c^*, m^*)$ (and $P(\xi_t) = 0$).

Theorem 4: For any ESE sequence, consider the "initial phase" up to date T-1, before (c^*, m^*) is applied. Then:

(i) for all $t = 0, .., T-1$ $\alpha_{t+1} > \alpha_t$,

The Structure of ESE Sequences in Quasi-Linear Environments

Assume: $A(\xi) = A(c,m) = A(c) + m$ and $P(\xi) = P(c,m) = P(c) - m$.

Theorem 3: Let $\{\xi_t\}$ be an ESE agreement sequence. Then there exists a finite date T such that an agent's best SE sequence is followed thereafter and for all $t \geq T$, $\xi_t = (c^*, m^*)$ (and $P(\xi_t) = 0$).

Theorem 4: For any ESE sequence, consider the "initial phase" up to date T-1, before (c^*, m^*) is applied. Then:

(i) for all $t = 0,..,T-1$ $\alpha_{t+1} > \alpha_t$,

(ii) for all $t = 0,..,T-2$ $P(c_{t+1}) - m_{t+1} > P(c_t) - m_t$.

Discussion and Extensions

Discussion and Extensions

- If one whishes to empirically study the allocation of bargaining “power” it is important to have a data over the duration of relationship.

Discussion and Extensions

- If one whishes to empirically study the allocation of bargaining “power” it is important to have a data over the duration of relationship.
- In particular, observation of the “mature” phase alone will say little or nothing about the allocation of bargaining “power”.

Discussion and Extensions

- If one whishes to empirically study the allocation of bargaining “power” it is important to have a data over the duration of relationship.
- In particular, observation of the “mature” phase alone will say little or nothing about the allocation of bargaining “power”.
- Some extensions:

Discussion and Extensions

- If one whishes to empirically study the allocation of bargaining “power” it is important to have a data over the duration of relationship.
- In particular, observation of the “mature” phase alone will say little or nothing about the allocation of bargaining “power”.
- Some extensions:

  - heterogeneous discount factors

Discussion and Extensions

- If one whishes to empirically study the allocation of bargaining “power” it is important to have a data over the duration of relationship.
- In particular, observation of the “mature” phase alone will say little or nothing about the allocation of bargaining “power”.
- Some extensions:

  - heterogeneous discount factors

  - GE with endogenous outside options