Relational Incentive Contracts

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Presented by Tomek Piskorski

Motivation and Question

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What is the structure of optimal self-enforcing contracts?

How the structure of optimal self-enforcing contracts varies across different environments (e.g. hidden action, hidden information)?

Setup

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Have common discount factor $0<\delta<1$.

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Principal
proposes:

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Principal proposes:

w_t

Setup: At Any Date *t*

Principal proposes:

w_t +

Setup: At Any Date *t*

Principal proposes:

w_t +

$b_t(\varphi_t)$

Setup: At Any Date *t*

Principal proposes: →

w_t +

$b_t(\varphi_t)$

Setup: At Any Date *t*

Principal proposes: → Agent

w_t +

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Principal proposes: $w_t + b_t(\varphi_t)$ → Agent → ***reject***

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Principal gets utility: **π**.
Agent gets utility: **u**.

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→ $t+1$

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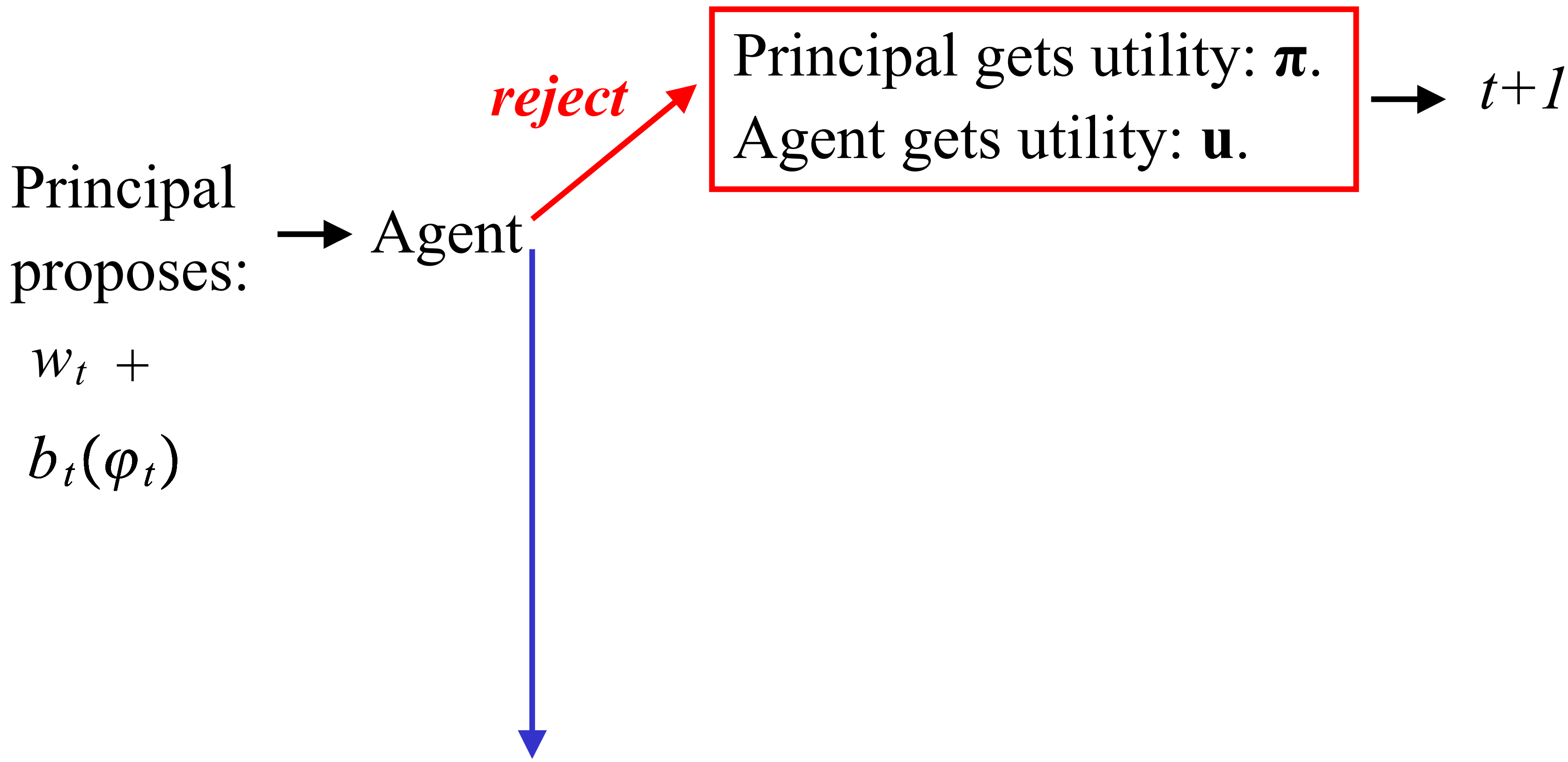

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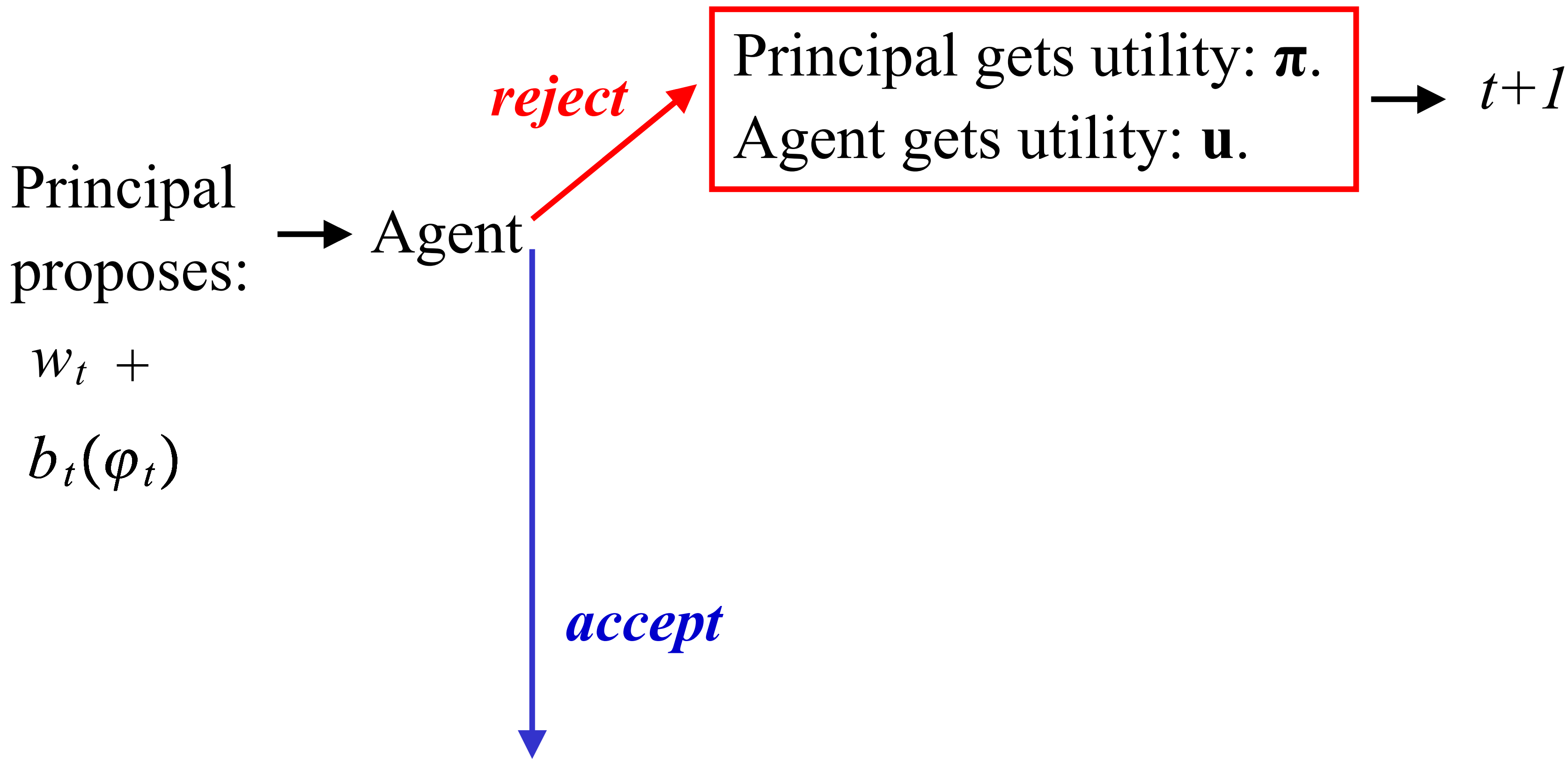

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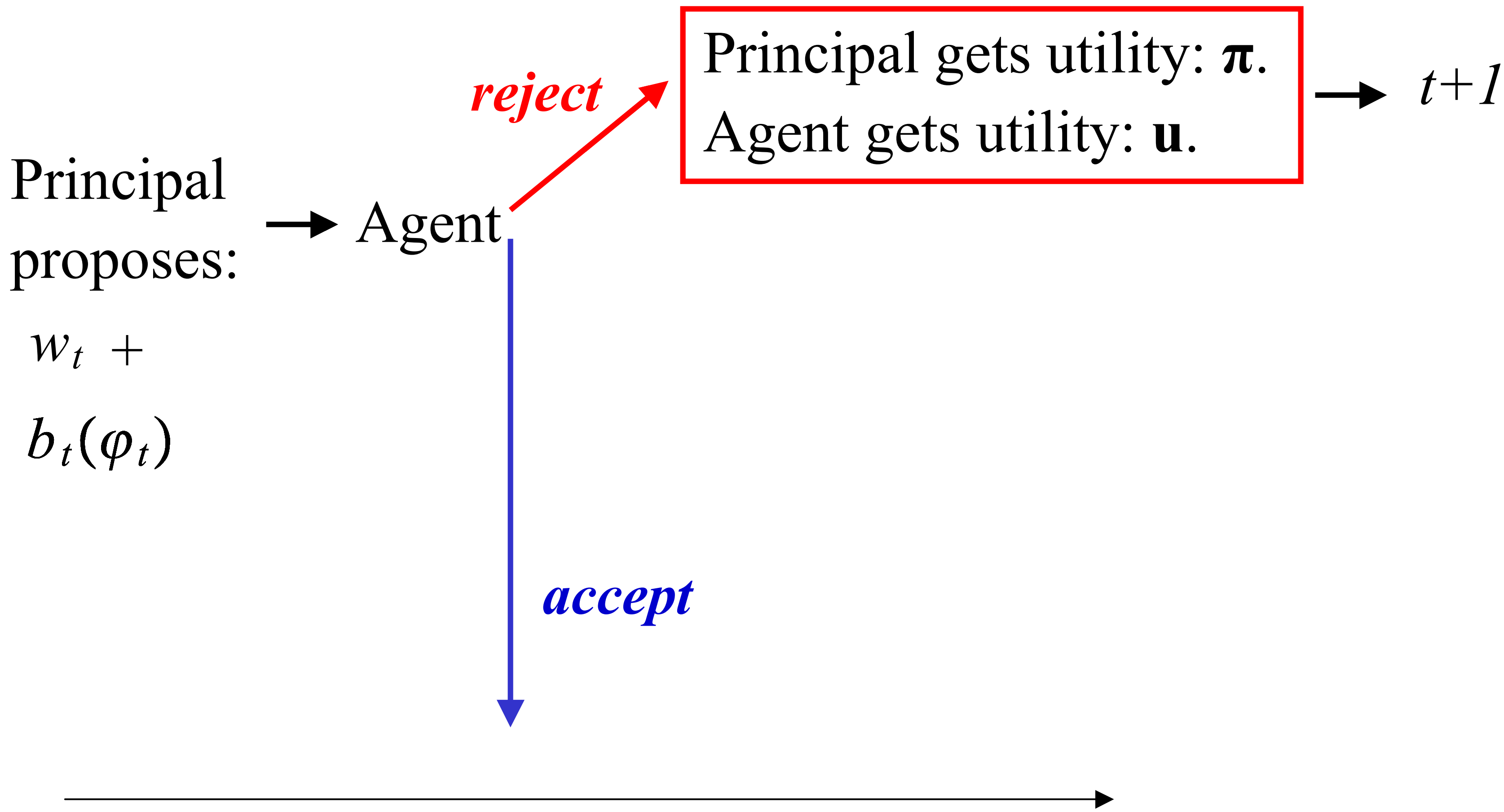

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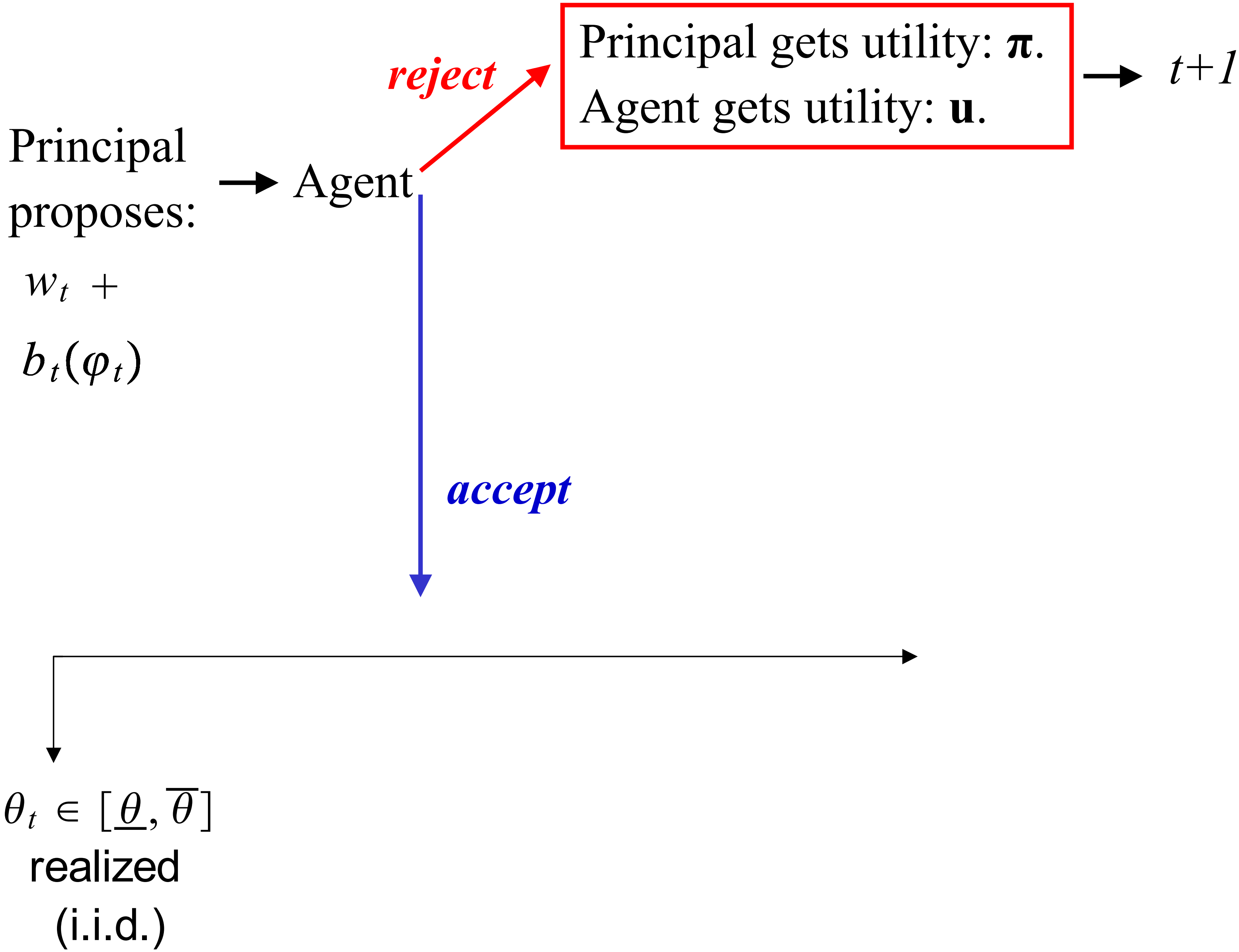

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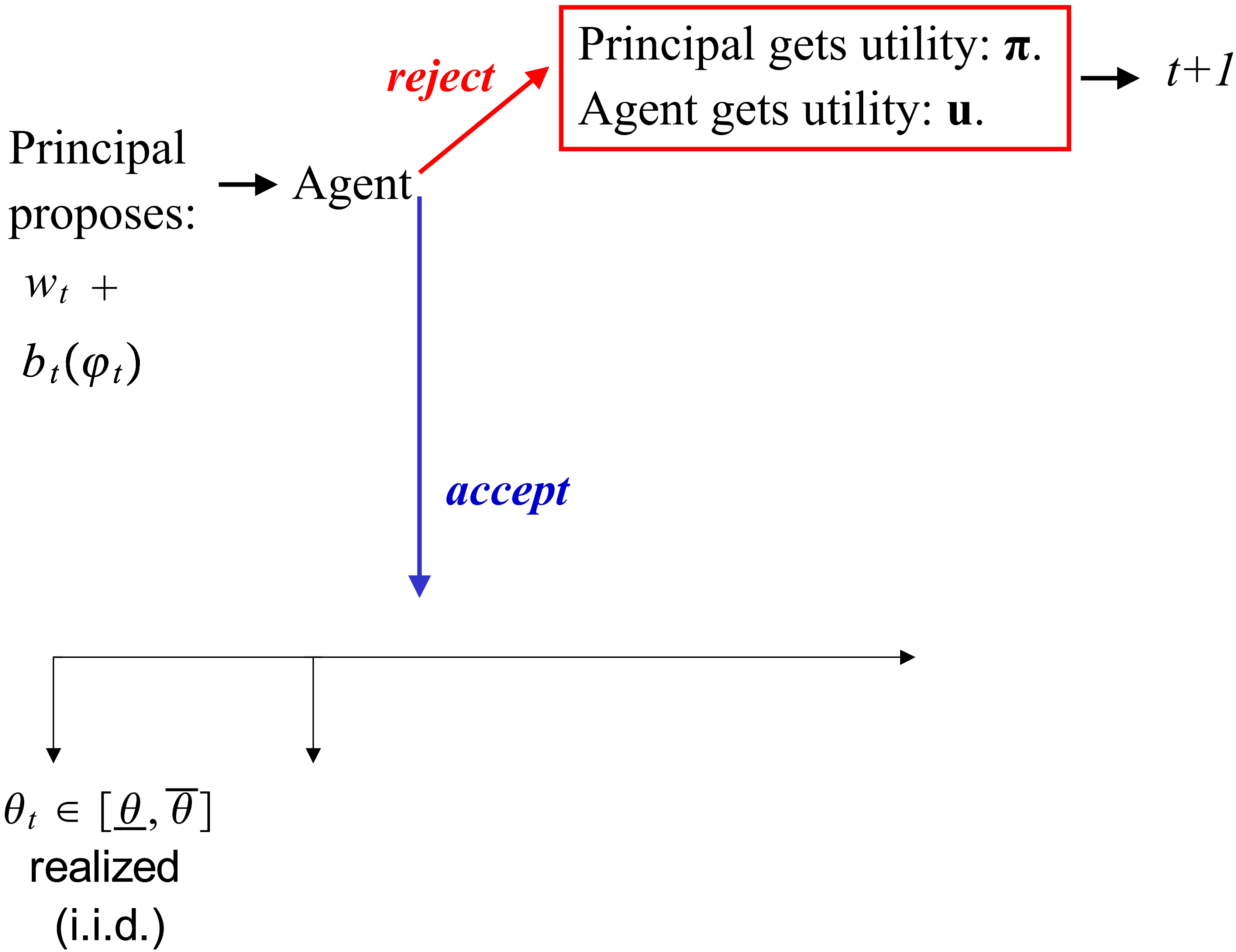

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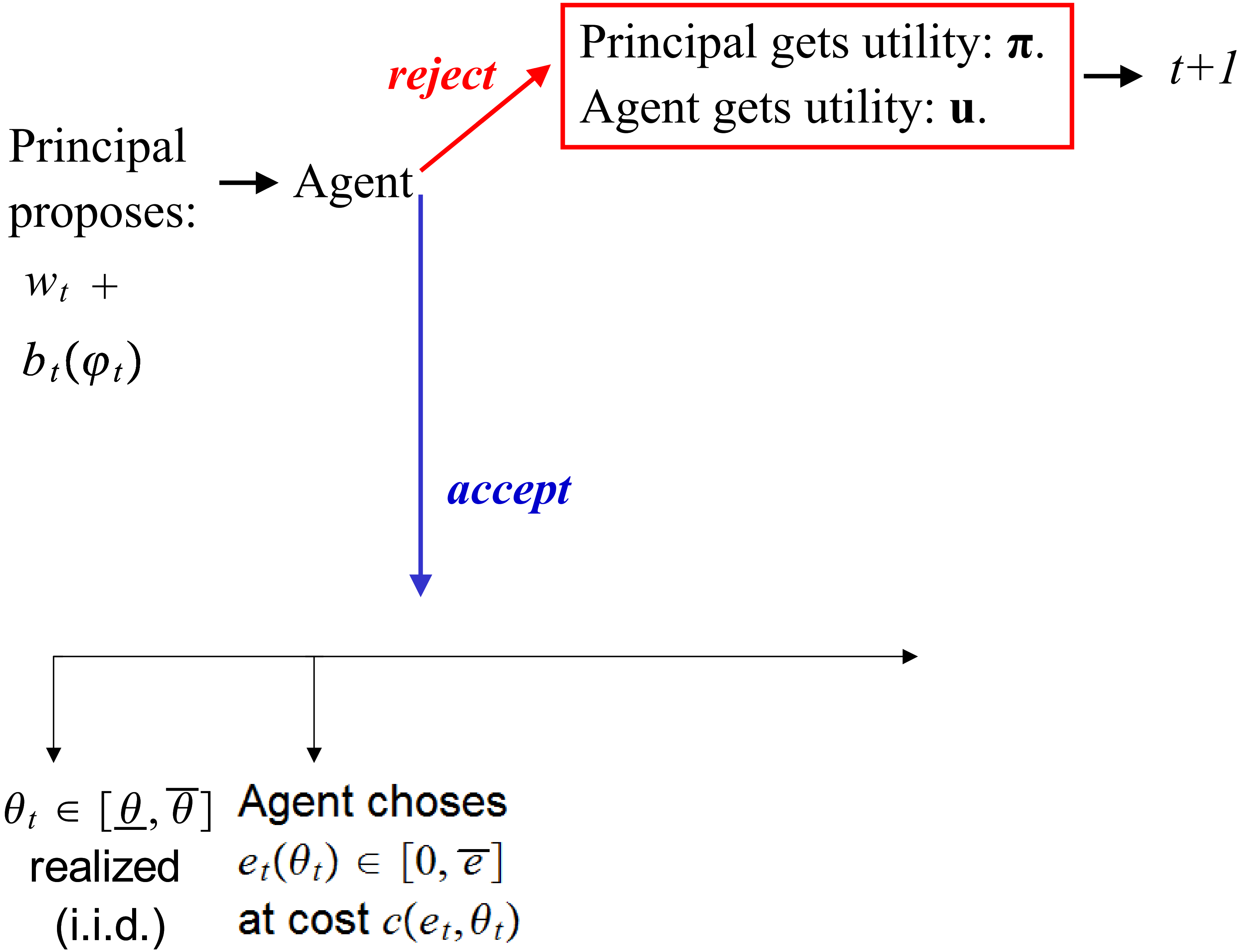

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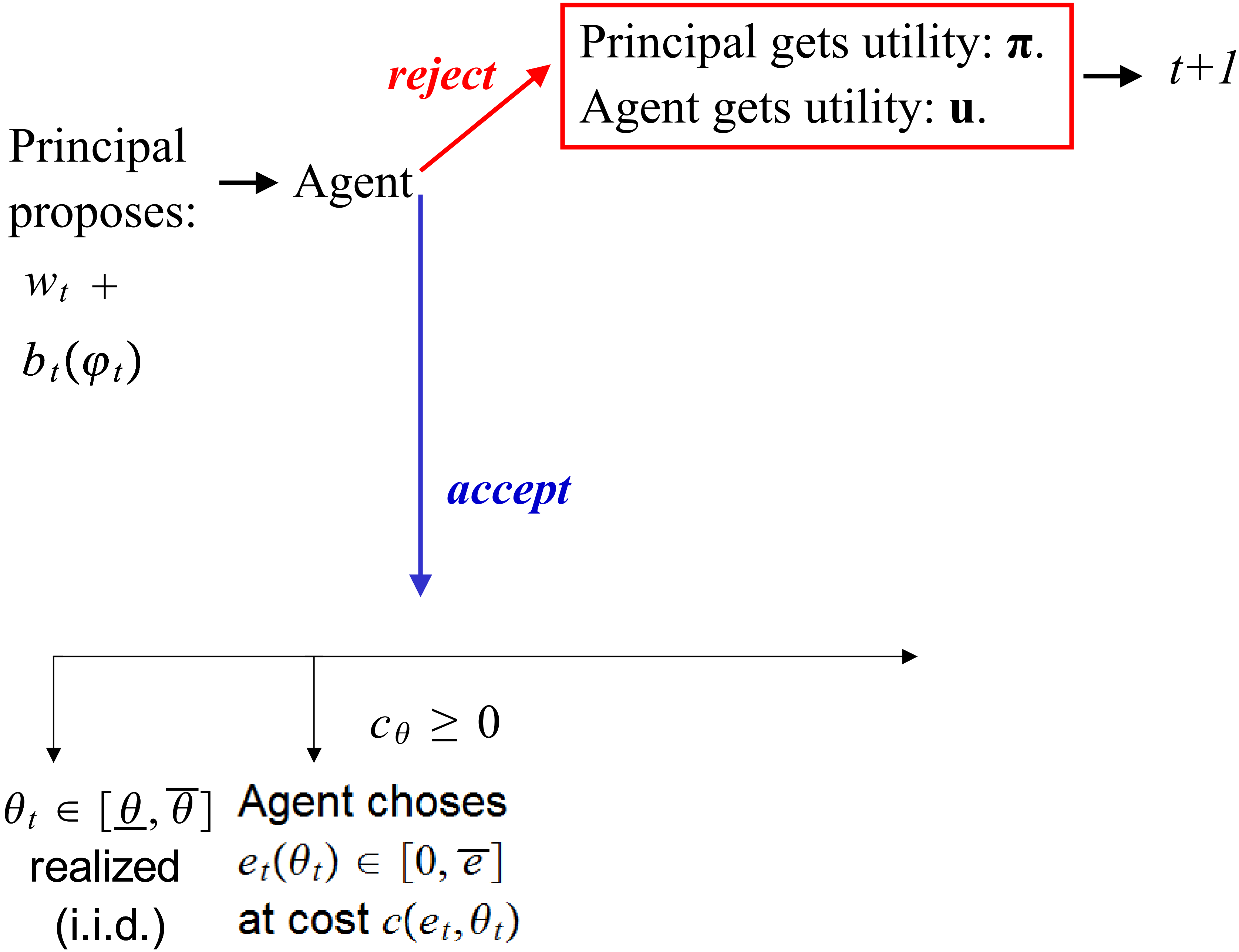

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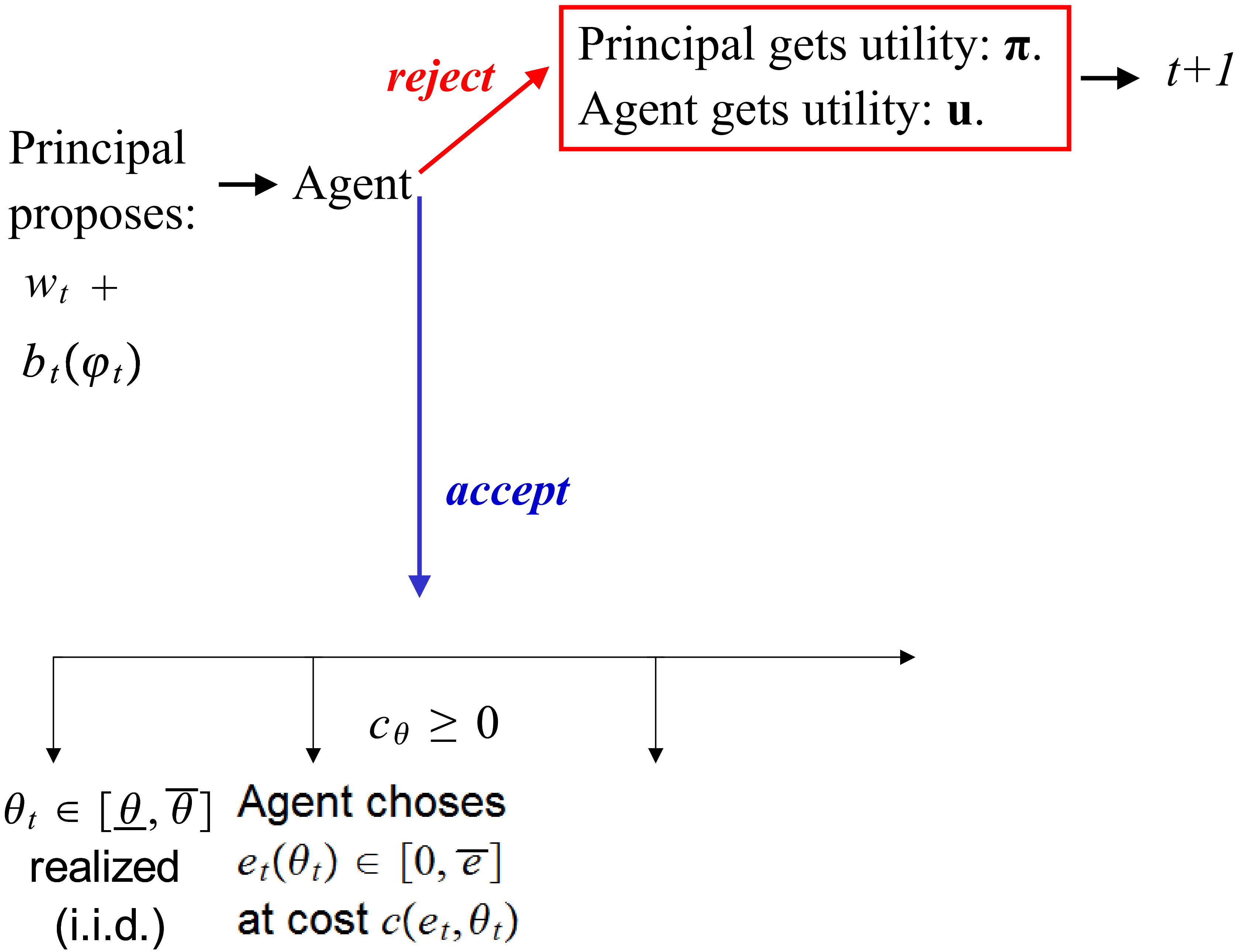

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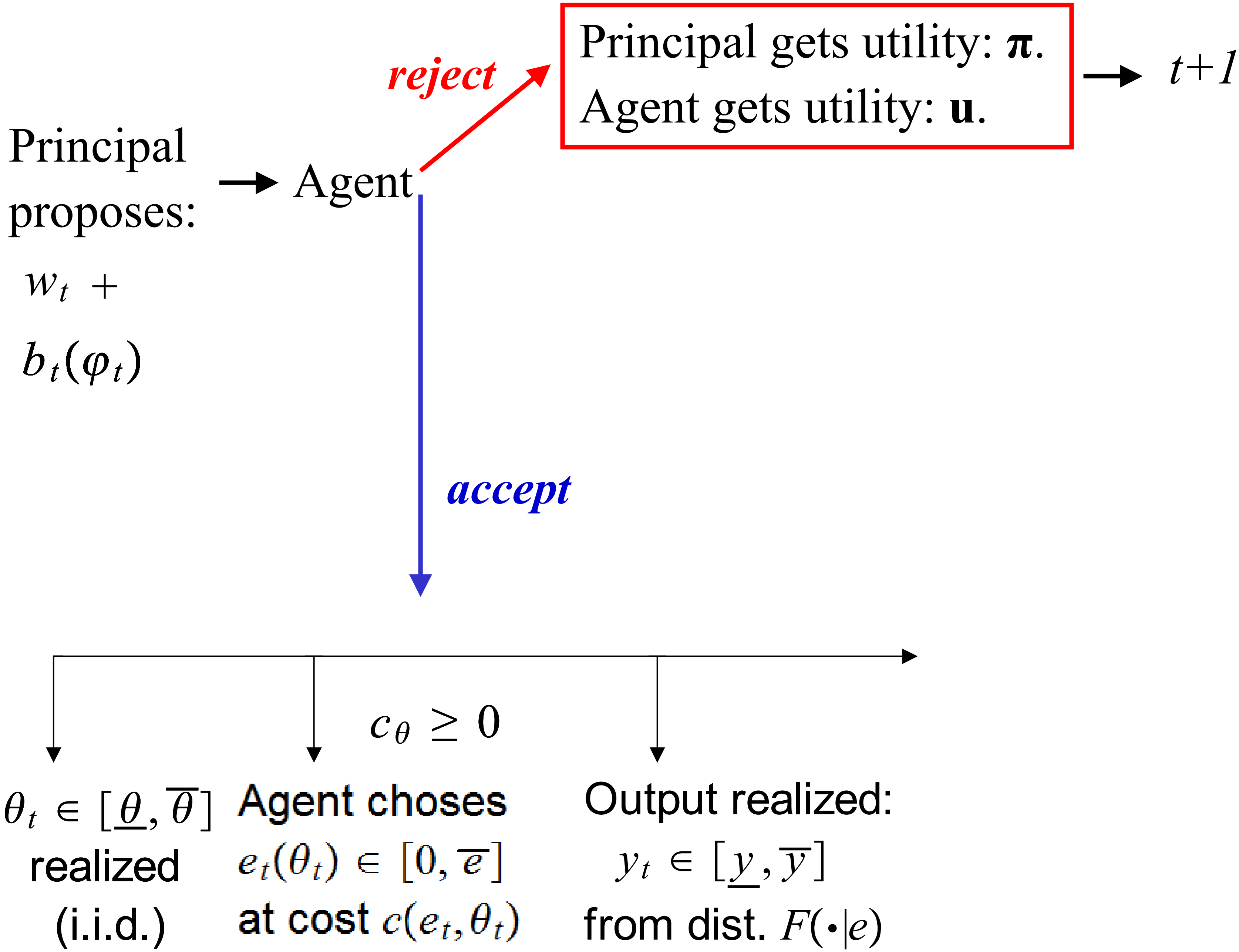

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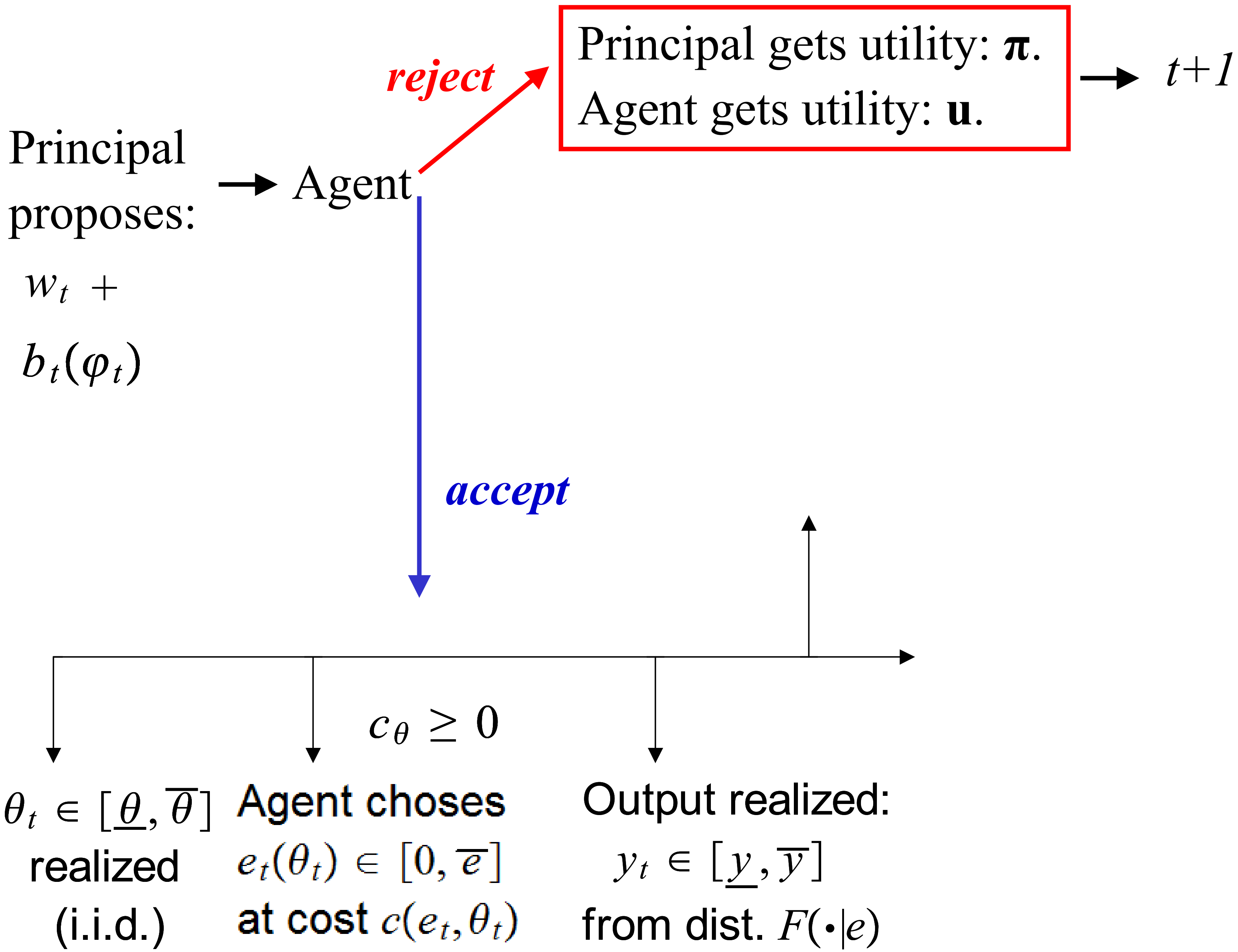

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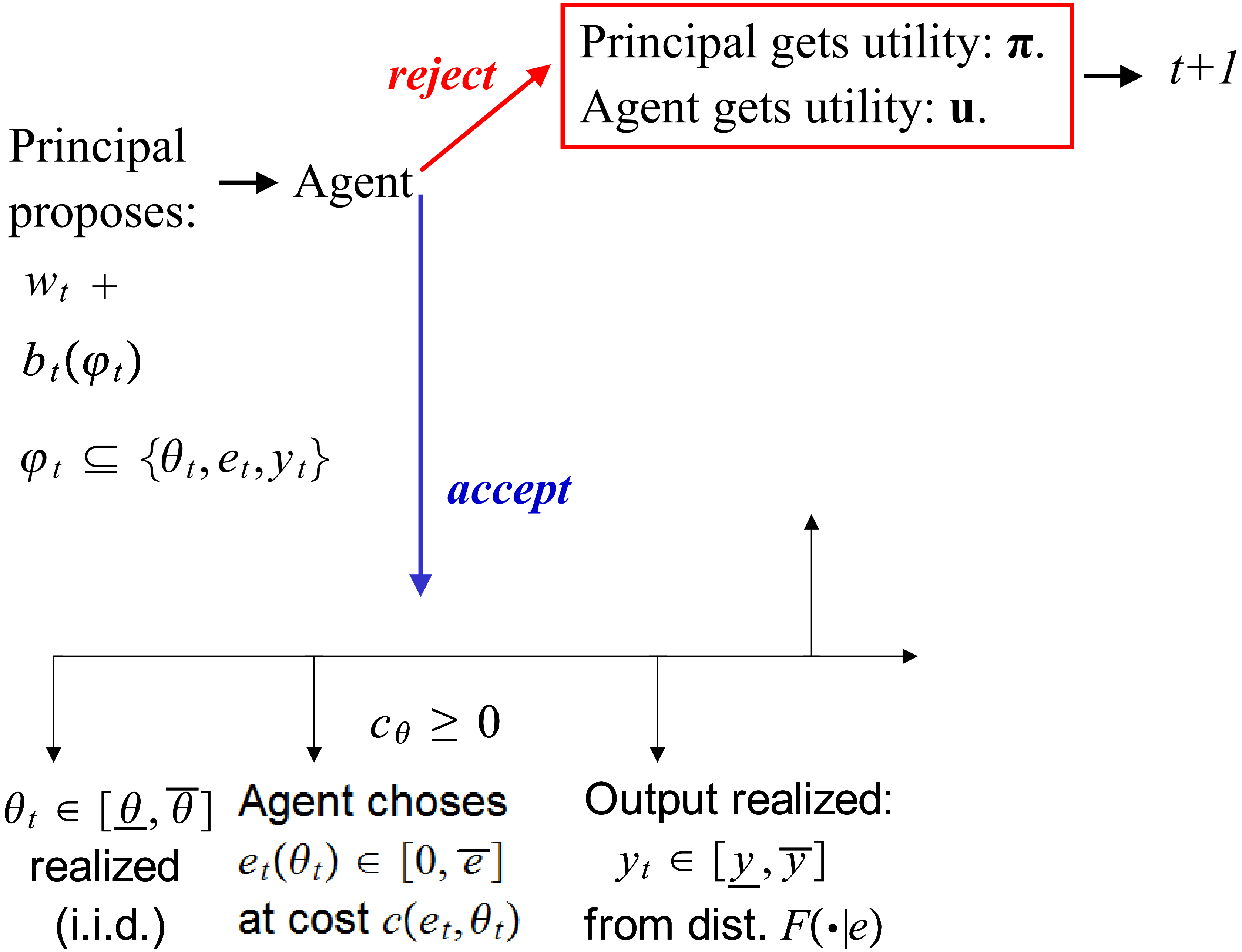

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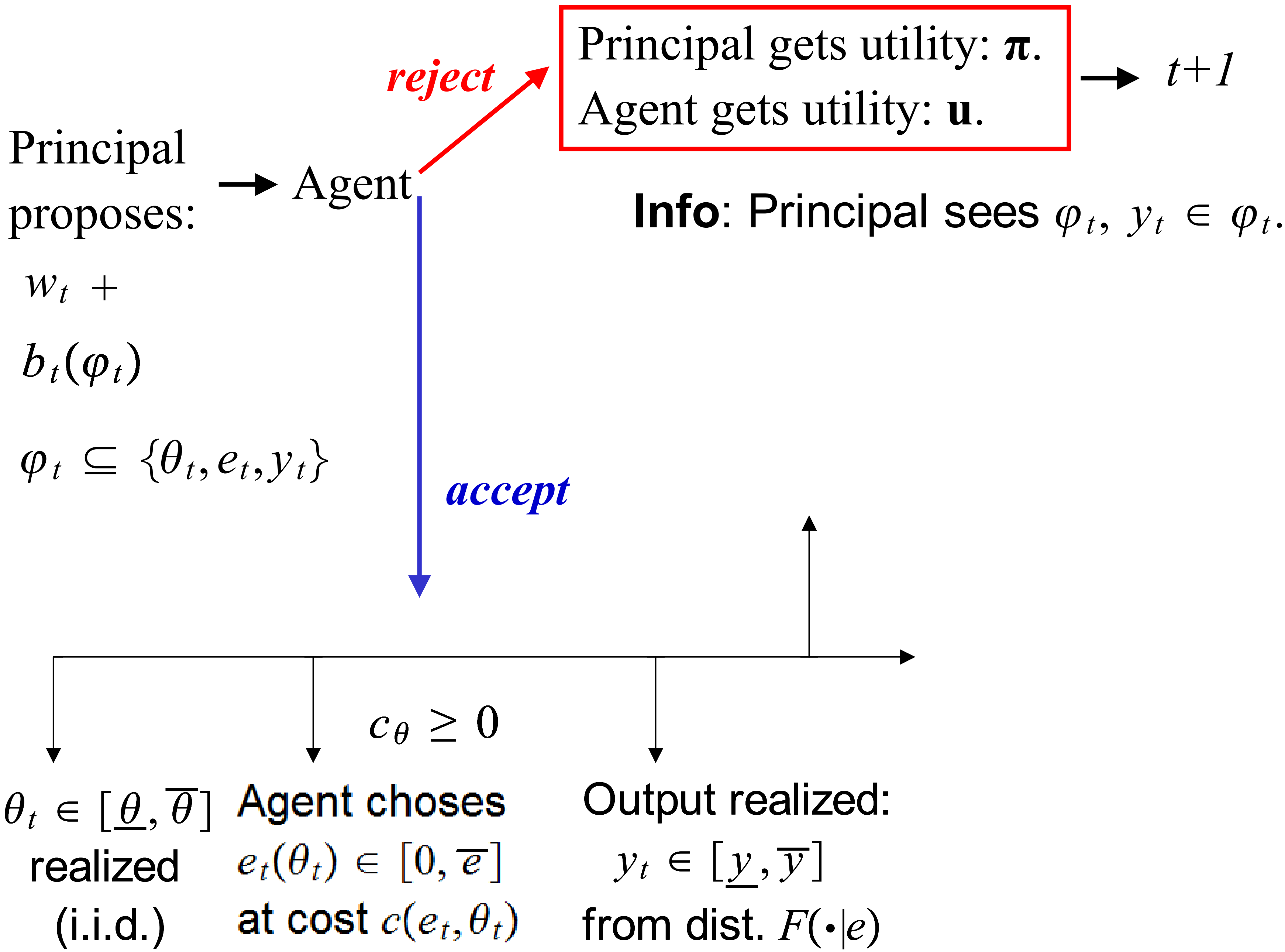

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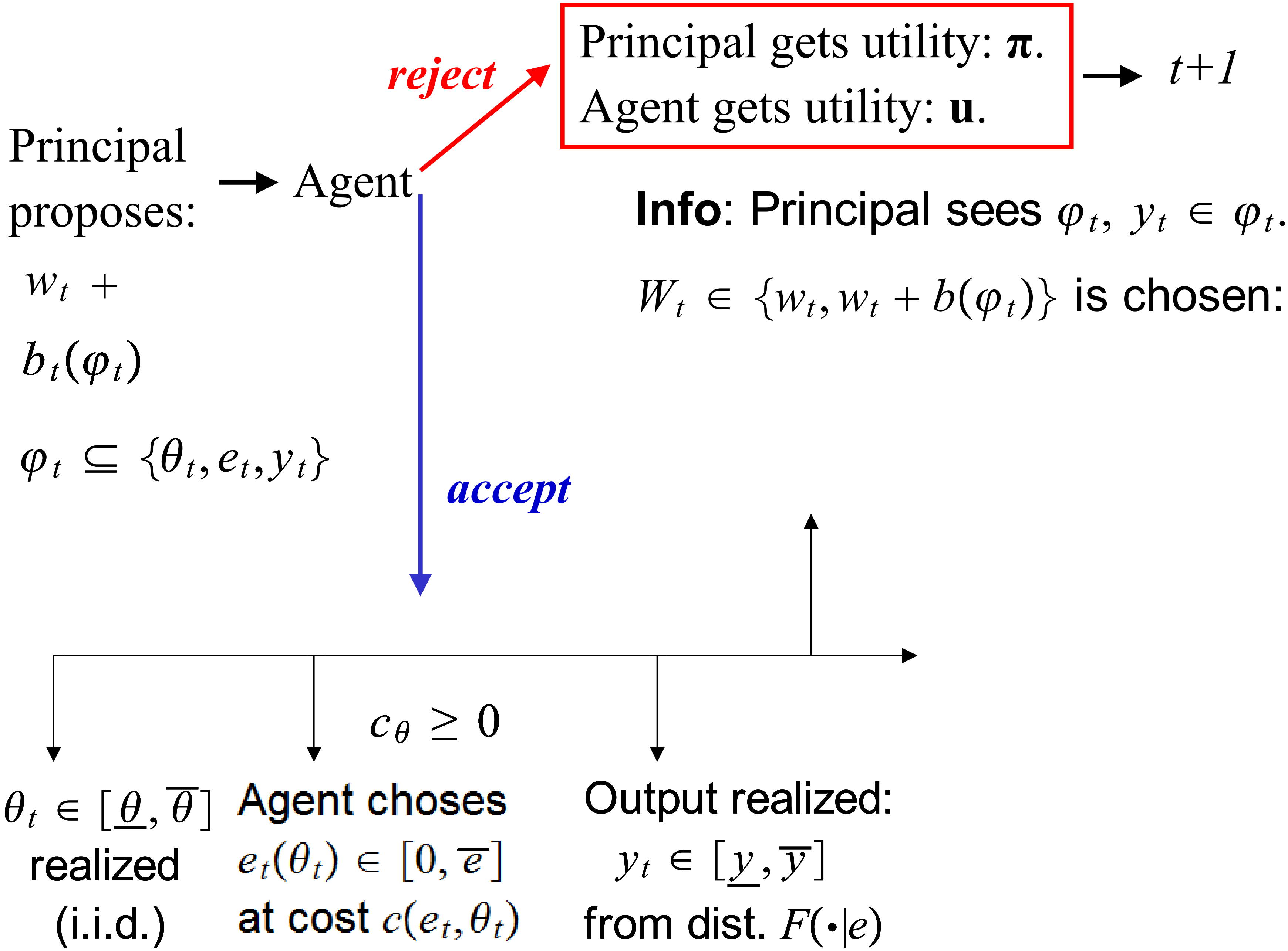

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Principal proposes: $w_t + b_t(\varphi_t)$

$\varphi_t \subseteq \{\theta_t, e_t, y_t\}$

→ Agent

reject → Principal gets utility: **π**. Agent gets utility: **u**. → $t+1$

Info: Principal sees φ_t, $y_t \in \varphi_t$.

$W_t \in \{w_t, w_t + b(\varphi_t)\}$ is chosen:

by the principal if $b(\varphi_t) > 0$,

accept

$\theta_t \in [\underline{\theta}, \overline{\theta}]$ realized (i.i.d.)

Agent choses $e_t(\theta_t) \in [0, \overline{e}]$ at cost $c(e_t, \theta_t)$

$c_\theta \geq 0$

Output realized: $y_t \in [\underline{y}, \overline{y}]$ from dist. $F(\bullet|e)$

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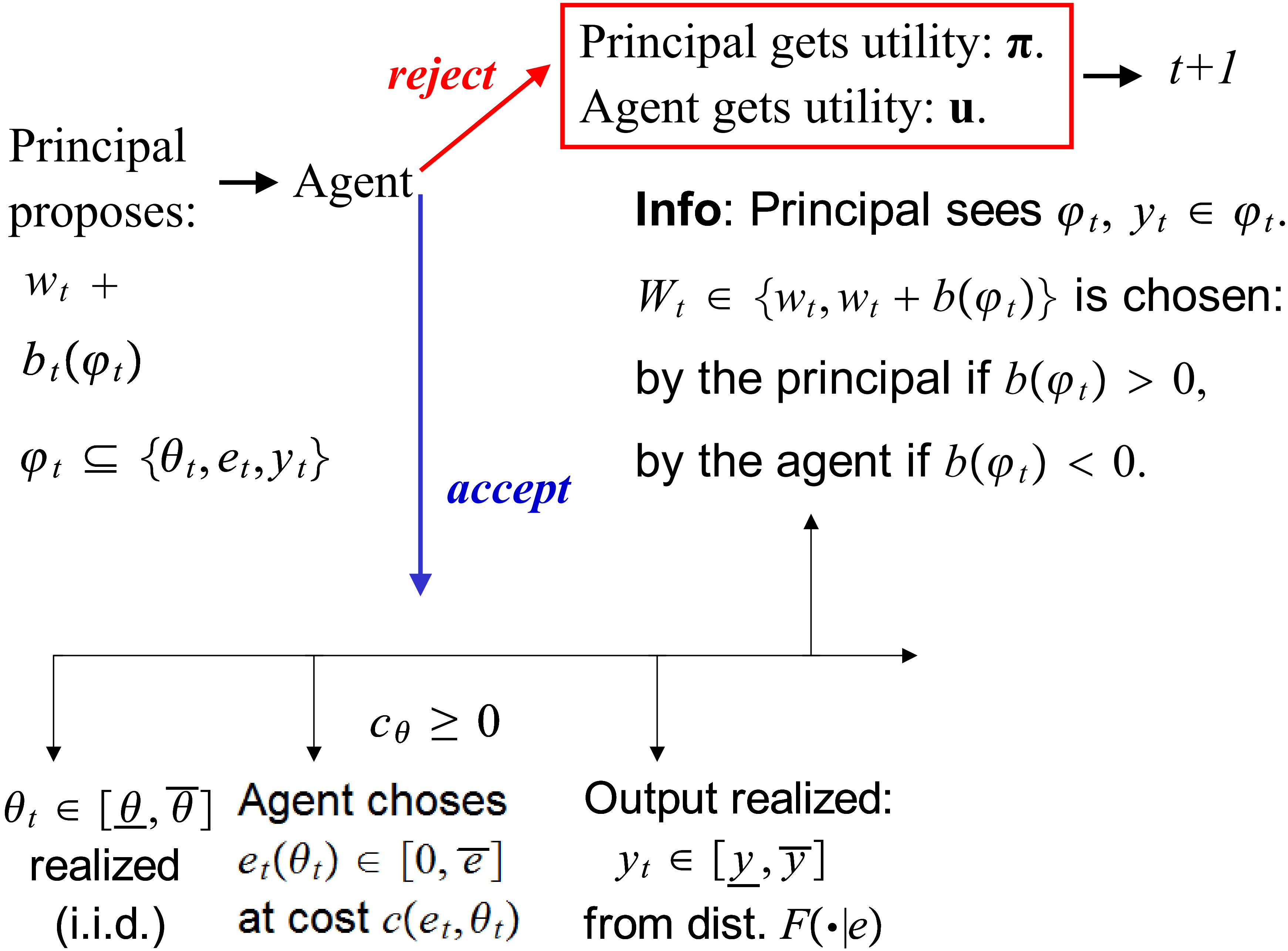

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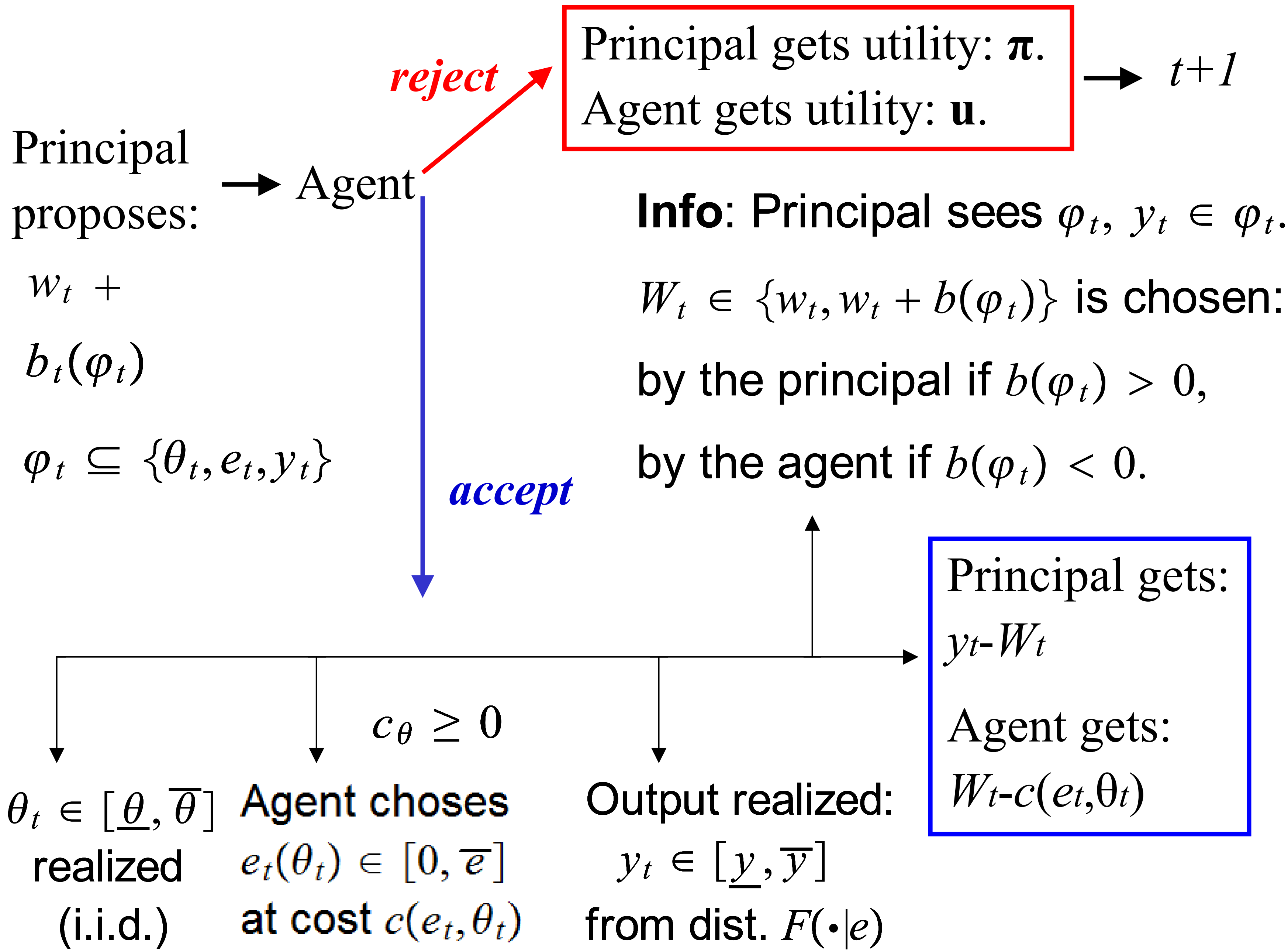

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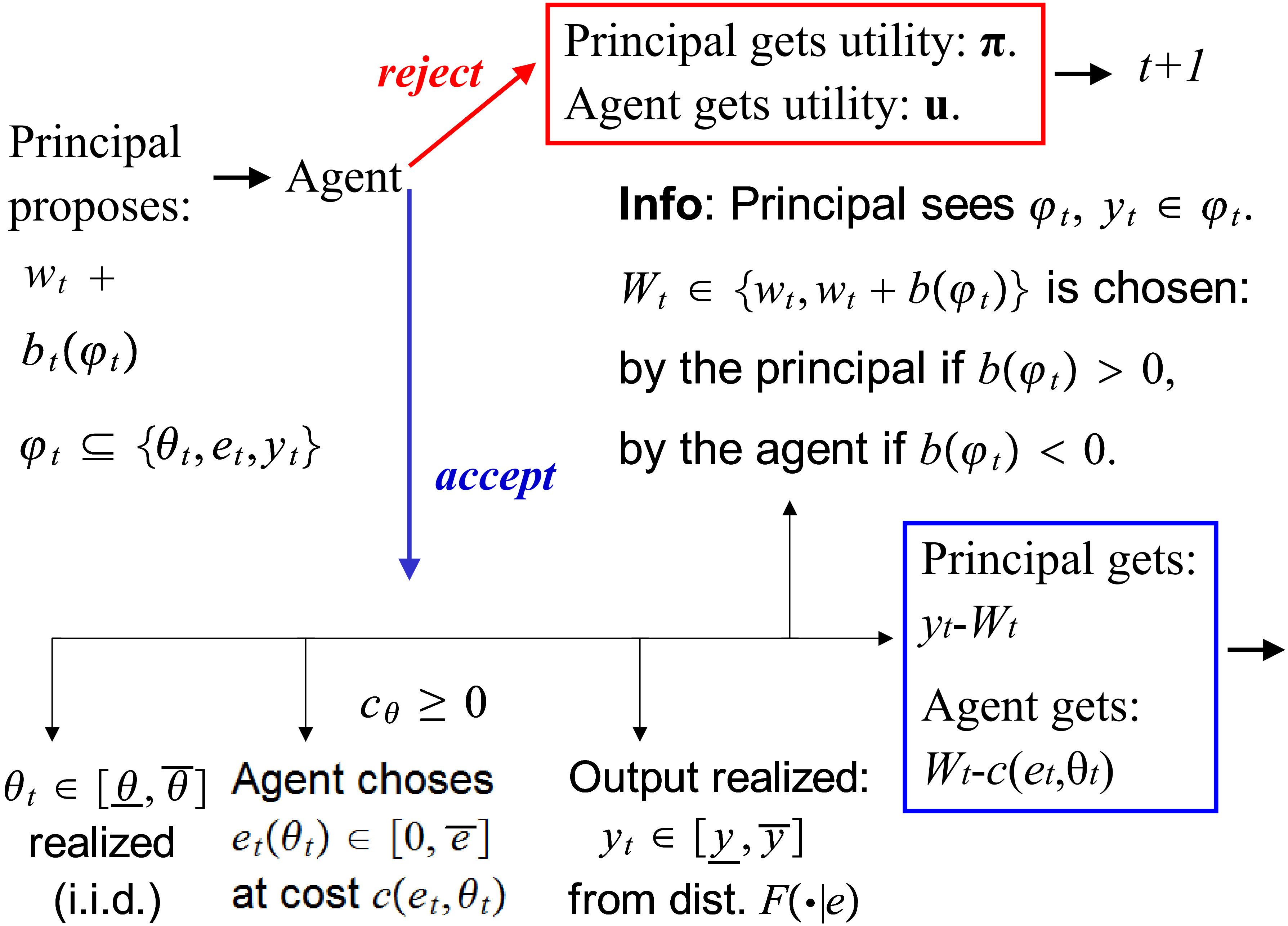

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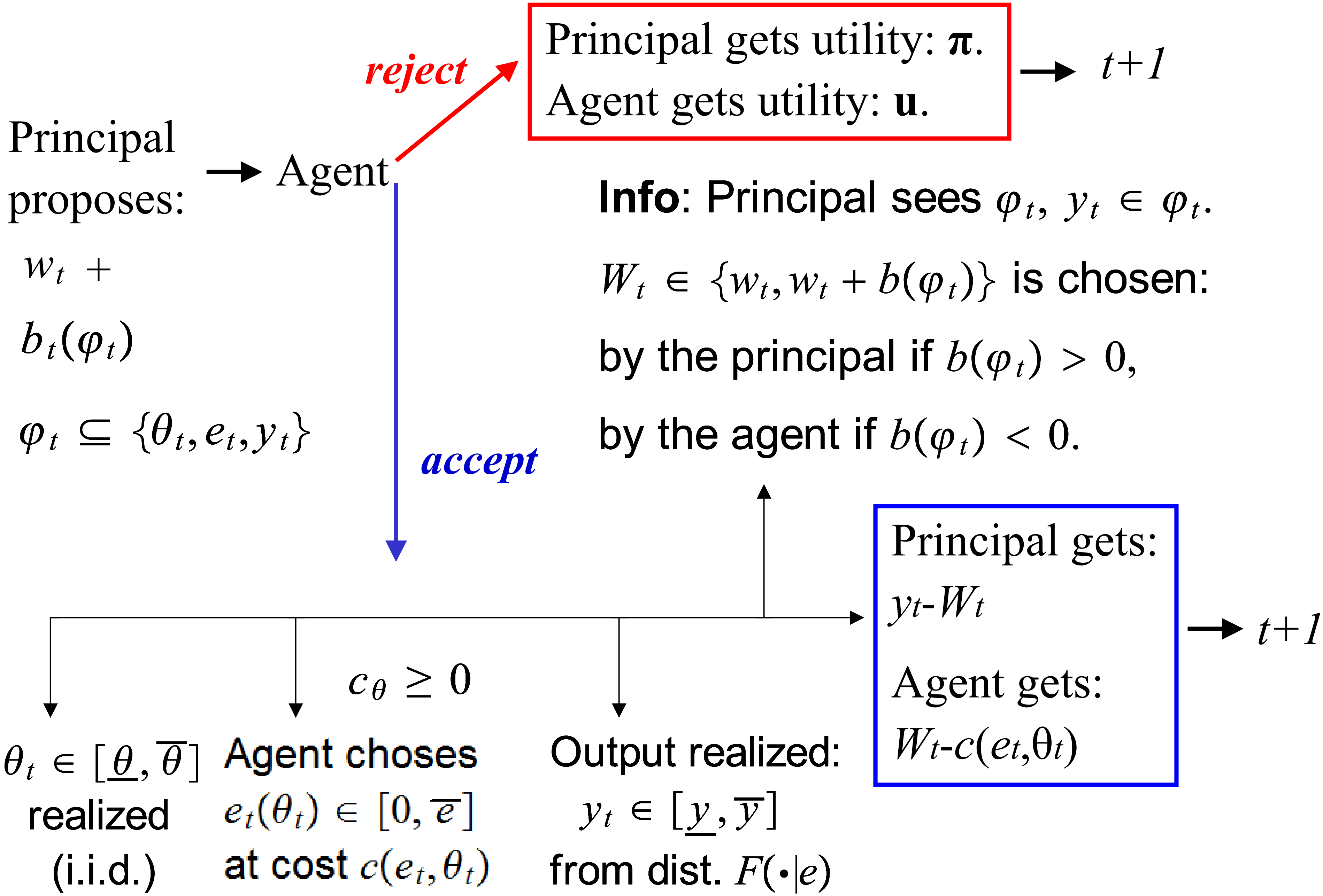

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Starting from date t payoffs for principal (π_t) and agent (u_t) are:

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(iii) the agent's action as a function of θ_t.

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Theorem 1: If there is a self-enforcing contract that generates expected surplus s greater then ($\mathbf{u}+\boldsymbol{\pi}$), then there are self-enforcing contracts that give as expected payoffs any pair (u, π) such that:

$u \geq \mathbf{u}$, $\pi \geq \boldsymbol{\pi}$, and $(u+\pi) \leq s$.

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Follows from risk neutrality.

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If (DE) binds at the optimal contract → inefficiency.

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In cases: (i) and (ii) $e(\theta) < e^{FB}(\theta)$ for all θ (DE binds).

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(ii) $e(\theta) < e^{FB}(\theta)$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$, $W(\theta, y) = \begin{cases} \underline{W} \text{ for all } y < \hat{y}(\theta) \\ \bar{W} \text{ for all } y \geq \hat{y}(\theta) \end{cases}$

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The principal will be willing to make distinct reports of output only if they give him the same total payoffs.

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Yes: Fuchs (AER, forthcoming).