Non-Expected Utility in Macroeconomics

Philippe Weil QJE 1990

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Objectives

- Introduce preferences that distinguish between risk aversion and intertemporal substitution

- How: generalize isoleastic utility using the K-P apparatus

- Understand the role of each parameter in the classic consumption/savings problem

Kreps-Porteus Preferences (1978)

- Abandon the axiom of reduction of compound lotteries in order to distinguish between early and late resolution of uncertainty

$$V_t = U\left[c_t, E_t V_{t+1}\right]$$

- $U_2\left(c_t, E_t V_{t+1}\right)$:discount factor

- vN-M case: U linear in the second argument, $U_2\left(c_t, E_t V_{t+1}\right) = \beta$

Generalized Isoelastic Preferences

- Weil

$$V_t = \frac{\left\{ (1-\beta)\, c_t^{1-\rho} + \beta \left[1 + (1-\beta)(1-\gamma) E_t V_{t+1} \right]^{\frac{1-\rho}{1-\gamma}} \right\}^{\frac{1-\gamma}{1-\rho}} - 1}{(1-\beta)(1-\gamma)}$$

- γ :coefficient of relative risk aversion,

- $\frac{1}{\rho}$: elasticity of intertemporal substitution for deterministic consumption paths,i.e. $\sigma = \frac{d \ln \frac{c_t}{c_{t+1}}}{d \ln MRS} = \frac{1}{\rho}$.Reduces to vN-M for $\gamma = \rho$

- $\gamma \to 1$, log risk preferences, $\rho \to 1$ log intertemporal preferences

More familiar version : E-Z

- We can rewrite the preferences as

$$v_t = \left\{ c_t^{1-\rho} + \beta \underbrace{\left[\left(E_t v_{t+1}^{1-\gamma} \right)^{1-\gamma} \right]^{1-\rho}}_{C.E.\text{ of future utility}} \right\}^{\frac{1}{1-\rho}}$$

- where $v_t \equiv \frac{(1+(1-\beta)(1-\gamma)V_t)^{\frac{1}{1-\gamma}}}{(1-\beta)^{\frac{1}{1-\rho}}}$

- the 'problem' with this representation is that we have to redefine for the case of $\gamma = 1, \rho = 1$

A simple illustration : Optimal consumption

- Assume iid interest rate $\tilde{R}$ (wealth becomes the only state variable) and non random income (normalize to zero)

- Bellman equation

$$\begin{aligned} V(w_t) &= \max_{c_t} U(c_t, E_t V(w_{t+1})) \\ \text{s.t. } w_{t+1} &= \tilde{R}(w_t - c_t) \end{aligned}$$

Euler equation

$$U_1\left(c_t, E_t V\left(w_{t+1}\right)\right) = \underbrace{U_2\left(c_t, E_t V\left(w_{t+1}\right)\right)}_{\text{discount factor}} E_t V'\left(w_{t+1}\right)\tilde{R}$$

- In our case

$$1 = E \underbrace{\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\rho}}_{vNM-\text{sdf}} \left[\frac{\tilde{V}_{t+1}^{\frac{1}{1-\rho}}}{E_t \tilde{V}_{t+1}^{\frac{1}{1-\gamma}}}\right]^{\rho-\gamma} \cdot \tilde{R}$$

where $\tilde{V}_t \equiv 1 + (1-\beta)(1-\gamma) V_t$

Guess

- $V(w) = \frac{(\psi w)^{1-\gamma} - 1}{(1-\beta)(1-\gamma)}$ and a linear consumption rule $c_t = \mu w_t$

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$$MPC = \mu = 1 - \beta^{\frac{1}{\rho}} \left[\underbrace{\left(E\tilde{R}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}}_{\text{certainty equivalent of the } \tilde{R} \text{ distribution}} \right]^{\frac{1-\rho}{\rho}}$$

and

$$\psi = \left[(1-\beta)\, \mu^{-\rho} \right]^{\frac{1}{1-\rho}}$$

Results I

- The value function is a power function of wealth. Curvature governed from risk aversion γ. Generalization of Samuelson (69) and Merton (73) results

- The MPC is a complicated function of risk aversion and EIS in general

- When $\rho = 1$, then $MPC = \mu = 1 - \beta \; \forall \; \gamma$

Results II: Consumption growth

- Expected consumption growth

$$E \ln\left(\frac{c_{t+1}}{c_t}\right) = \frac{1}{\rho}\left[\ln\beta + \frac{1-\rho}{1-\gamma}\ln\left[E\tilde{R}^{1-\gamma}\right]\right] + E\ln\tilde{R}$$

- For lognormal returns though $\ln\tilde{R} \sim N\left(r, 2\sigma^2\right)$ we have

$$E \ln\left(\frac{c_{t+1}}{c_t}\right) = \frac{1}{\rho}\left[r + \ln\beta\right] + \frac{(1-\rho)(1-\gamma)}{\rho}\sigma^2$$

- $\frac{\partial E \ln\left(\frac{c_{t+1}}{c_t}\right)}{\partial r} = \frac{1}{\rho}$, so the response of consumption growth to an increase in r depends on $\frac{1}{\rho}$ *only*.

Results III: When does more Uncertainty lead to more Savings?

- Let $\hat{R}$: certainty equivalent of the interest rate

$$MPC = \mu = 1 - \beta^{\frac{1}{\rho}} \left[\hat{R}\right]^{\frac{1-\rho}{\rho}}$$

- mean preserving spread *reduces* $\hat{R}$ and causes 2 effects: substitution (MPC↑) and an income effect (MPC↓)

- When $\rho > 1$ $(\sigma < 1)$, then $\frac{\partial MPC}{\partial \hat{R}} > 0$, So more uncertainty→*more* savings

- When $\rho < 1$ $(\sigma > 1)$,then $\frac{\partial MPC}{\partial \hat{R}} < 0$,So more uncertainty→*less* savings.So the *sign* depends on ρ, the *size* on γ (through $\hat{R}$)