Optimal Fiscal and Monetary Policy

Chari and Kehoe, Handbook of Macro

February 16, 2007

Outline

- This survey: How to use the *primal approach* in order to solve Ramsey Problems

- Develop some insights in a static public finance framework (mainly: Uniform Taxation Principle)

- Proceed to an application in dynamic fiscal policy under uncertainty: Lucas and Stokey (1983)

- Example of the Uniform Taxation Principle in LS.

Static Framework

- consumption goods $c_i, i = 1, ...n$ with prices p_i and tax rate τ_i.Labor l with $w = 1$.

- Utility of the agent

$$U(c_1, ..., c_n, l)$$

- Problem of the consumer

$$\max_{\{c,l\}} U(c_1, ..., c_n, l)$$

s.t.

$$\sum_{i=1}^{n} p_i(1 + \tau_i)c_i = l$$

- Representative firm produces x_i units of each good i with the input l with the *CRS* technology

$$F(x_1, ..., x_n, l) = 0$$

- Problem of the firm

$$\max_{\{x,l\}} \sum_i p_i x_i - l$$

s.t.

$$F(x_1, ..., x_n, l) = 0$$

- Market clearing

$$c_i + g_i = x_i, i = 1, ..., n$$

- Govenment Budget Constraint

$$\sum_{i=1}^{n} p_i g_i = \sum_{i=1}^{n} p_i \tau_i c_i$$

Definition: *CE with Taxes is a policy* $\pi \equiv \{\tau_i\}$; *a* (c, l, x) *allocation; and a price system* p *such that a) consumers maxamize utility b) producers maximize profits c) markets clear d)the government budget constraint holds.*

- *Ramsey Allocation Problem:* Choose the CE that maximizes the utility of the RA.

- *Primal approach:* Eliminate prices and taxes through the focs and work with *allocations*.

- Consumer's FOC

$$\frac{U_i}{U_l} = -p_i(1+\tau_i) \qquad \Leftrightarrow \qquad \underbrace{\frac{-U_l}{U_i}}_{\text{MRS leisure with good } i} = \underbrace{\frac{1}{p_i(1+\tau_i)}}_{\text{real wage}}$$

- Firm's FOC

$$p_i = -\frac{F_i}{F_l} \Rightarrow \frac{U_i}{U_l} = \frac{F_i}{F_l}(1+\tau_i)$$

Thus we get the *inefficiency wedge*

$$\underbrace{\frac{U_i}{U_j}}_{\text{MRS } i \text{ with } j} = \underbrace{\frac{F_i}{F_j}}_{\text{MRT} i \text{ with } j} \cdot \underbrace{\frac{1+\tau_i}{1+\tau_j}}_{\text{wedge}}$$

- Replace prices and taxes in consumer's budget constaint to get the *Implementability Constraint*

$$\sum_i U_i c_i + U_l l = 0$$

- *Resource Constraint*

$$F(c_1 + g_1, ..., c_n + g_n, l) = 0$$

- *Ramsey Problem*

$$\max_{\{c,l\}} U(c, l)$$

s.t.

$$\begin{aligned} \sum_i U_i c_i + U_l l &= 0, \quad (\lambda) \\ F(c_1 + g_1, ..., c_n + g_n, l) &= 0 \quad (\gamma) \end{aligned}$$

- FOC

$$\begin{aligned}(1+\lambda)U_i - \lambda U_i H_i &= \gamma F_i, i = 1, ..., n \\ (1+\lambda)U_l - \lambda U_l H_l &= \gamma F_l\end{aligned}$$

where

$$\begin{aligned}H_i &\equiv -\frac{\sum_j U_{ji}c_j + U_{li}l}{U_i} \\ H_l &\equiv -\frac{\sum_j U_{lj}c_j + U_{ll}l}{Ul}\end{aligned}$$

- *Remark:* if $\lambda = 0 \Rightarrow$ first best.

- Combining FOCS we get

$$\frac{\tau_i}{1+\tau_i} = \frac{\lambda}{\gamma}\frac{U_l}{F_l}(H_i - H_l) = \frac{\lambda(H_i - H_l)}{1+\lambda-\lambda H_l}$$

So

$$\frac{\frac{\tau_i}{1+\tau_i}}{\frac{\tau_j}{1+\tau_j}} = \frac{H_i - H_l}{H_j - H_l}$$

- Thus if $H_i > H_j \Rightarrow \tau_i > \tau_j$

- *Case 1*: Assume that utility is *additively separable* in consumption goods *and* labor. Then

$$\frac{H_i}{H_j} = \frac{\eta_j}{\eta_i}$$

where $\eta_j \equiv \frac{\partial c_j}{\partial m}\frac{m}{c_j}$: income elasticity of good i $\Rightarrow$ Tax *more the necessity* than the *luxury.*

Case 2: Utilility is *additively separable* and *linear* in labor

$$U(c,l) = \sum_i V^i(c_i) - l$$

Then

$$\frac{H_i}{H_j} = \frac{\varepsilon_j}{\varepsilon_i}$$

where ε_i: price elasticity of $i \Rightarrow$ Tax more *inelastic* goods (P.E. result)

*Case 3:*Utility is weakly *separable* and *homothetic*

$$U(c,l) = W(\varphi(c_1, ..., c_n), l)$$

φ: homothetic. Then

$$H_i = H_j \Rightarrow \tau_i = \tau_j$$

This is the *Uniform Commodity Taxation* Result.

Application of the Primal approach in Dynamic Fiscal Policy: Lucas and Stokey (1983)

- Representative agent/only labor/*complete* markets. State $s^t \sim \pi_t$

- Agent consumes c_t, works h_t and is endowed with one unit of leisure.

- Linear Technology ($\Rightarrow w_t = 1$)
$$c_t + g_t = h_t.$$

- Random *exogenous* government expenditures g_t. Dynamic budget constraint
$$b_t(s^t) + g_t(s^t) = \tau(s^t)h_t(s^t) + \sum_{s_{t+1}} p_t(s_{t+1}, s^t)b_{t+1}(s_{t+1}, s^t).$$

- Agent's Problem with *time-separable expected utility*

$$\max_{c,h} E_0 \sum_{t=0} \beta^t U\left(c_t, 1-h_t\right)$$

s.t.

$$\begin{aligned} \sum_{t,s^t} q_t(s^t) c_t(s^t) &= \sum_{t,s^t} q_t(s^t)\left(1-\tau_t(s^t)\right) h_t(s^t) + b_0. \\ c_t(s^t) &\geq 0 \end{aligned}$$

- FOCs.

$$\frac{U_l}{U_c} = 1-\tau_t \quad \text{and} \quad q_t(s^t) = \beta^t \pi_t(s^t) \frac{U_c(s^t)}{U_c(s^0)}$$

- *Remark*: Compete markets + Absence of Capital$\Rightarrow$Make the problem *static*

- *Ramsey Problem:* Choose $\{c, h\}$ to max

$$E_t \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - h_t)$$

s.t.

$$\begin{aligned} \sum_{t,s^t} \beta^t \pi_t(s^t) \left[U_c(s^t) c_t(s^t) - U_l(s^t) h_t(s^t) \right] - U_c(0) b_0 &= 0 \qquad (\Phi) \\ c_t + g_t &= h_t \quad (\beta^t \pi_t \lambda_t) \end{aligned}$$

FOCs ($b_0 = 0$):

$$\begin{aligned} c_t &: (1+\Phi)U_c - \Phi U_c H_c = \lambda_t \\ h_t &: -(1+\Phi)U_l + \Phi U_l H_l = -\lambda_t \end{aligned}$$

with $H_c \equiv -\frac{U_{cc}c - U_{cl}h}{U_c}$ and $H_l \equiv \frac{U_{ll}h - U_{cl}c}{U_l}$

- Combining the two conditions we get the "standard" foc:

$$(1+\Phi)\left(U_{lt} - U_{ct}\right) = \Phi[U_{cc}c_t - U_{lc}(c_t + h_t) + U_{ll}h_t]$$

- Static equation$\Rightarrow$ Use RC to solve for consumption

$$\begin{aligned} c_t &= c(g_t; \Phi) \\ h_t &= h(g_t; \Phi) \end{aligned}$$

Allocation and tax rates inherit stochastic properties of g_t.

Explorations of Uniform Taxation

- Proceed now like in the Static Setup: Tax rate is

$$1 - \frac{U_{lt}}{U_{ct}} = \tau_t = \frac{\Phi(H_{ct} - H_{lt})}{1 + \Phi(1 - H_{lt})}$$

Taxation depends on H_{ct} *and* H_{lt}.

- *Case 1: Separable* and *linear* in labor (no income effects in c)

$$U = u(c) - h$$

Then $H_{lt} = 0 \Rightarrow c_t = \bar{c} \Rightarrow \tau_t = \bar{\tau}$ $(\varepsilon_i = \varepsilon_j)$ and labor is

$$h_t = \bar{c} + g_t$$

- *Case 2*: *Separable* and *linear* in consumption (no income effects in labor)

$$U = c - v(h)$$

Then $H_{ct} = 0 \Rightarrow h_t = \bar{h} \Rightarrow \tau_t = \bar{\tau}$ and consumption is a "*residual*" $c_t = \bar{h} - g_t$

- *Case 3*: *Separable*, *homothetic* in concumption and *homothetic* in labor

$$U = \frac{c^{1-\gamma}}{1-\gamma} - \frac{h^{1+\varphi_n}}{1+\varphi_n}$$

Then $H_c = \gamma, H_l = -\varphi_n$. So the tax rate is

$$\bar{\tau} = \frac{\Phi(\gamma + \varphi_n)}{1 + \Phi(1 + \varphi_n)} \; \forall t, s^t$$

For example, if γ high$\Rightarrow$IES low$\Rightarrow$ tax high.