A Multiplier Approach to Understanding the Macro Implications of Household Finance

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Overview of the paper

Taking heterogeneity in households' trading technology as a given, what are the implications for...

1. ...the moments of asset prices? What role does non-participation play?
2. ...households' portfolios?
3. ...consumption moments?
4. ...the wealth distribution?

Methodological contribution

- A solution method based on multipliers, rather than moments of the wealth distribution

Related papers

- Empirical household finance: Calvet, Campbell and Sodini (2007)
- Models with non-participation: Basak and Cuoco (1998), Guvenen (2005)
- Wealth distribution: Krusell and Smith (1997), Favilukis (2007)

Notation

- Aggregate shock: z^t
- Idiosyncratic shock: η^t (iid across HH's)
- $\pi(z^{t+1}, \eta^{t+1} | z^t, \eta^t) = \pi(z_{t+1} | z_t) \pi(\eta_{t+1} | z_{t+1}, \eta_t)$
- Successor nodes: $z^{t+1} \succ z^t$; set of successor histories: $\{z^\tau \succ z^t\}$
- History from zero to $t-1$ contained in η^t: $\eta^{t-1}(\eta^t)$

Endowment

- $Y(z^t) = exp\{z_t\} Y(z^{t-1})$
- Diversifiable income (dividend on the stock mkt): $(1-\gamma)Y(z^t)$
- Non-diversifiable income: $\gamma Y(z^t)\eta_t$

Preferences

- $U(c) = E\left\{\sum_{t\geq 1}^{\infty} \beta^t \pi(z^t, \eta^t) \frac{c(z^t,\eta^t)^{1-\alpha}}{1-\alpha}\right\}$

4 types of HH's

- Active traders: Complete, z-complete
- Passive traders: Diversified, Non-participants

Prices

- Aggregate state prices: $q\left[\left(z^{t+1},\eta^{t+1}\right),\left(z^{t},\eta^{t}\right)\right]=\pi(\eta^{t+1}|z^{t+1},\eta^{t})q(z_{t+1},z^{t})$
- Arrow-Debreu prices: $\widetilde{P}(z^{t},\eta^{t})=\pi(z^{t},\eta^{t})P(z^{t},\eta^{t})=q(z_{t},z^{t-1})q(z_{t-1},z^{t-2})..q(z_{1},z^{0})q(z^{0})$
- $m(z^{t+1}|z^{t})=P(z^{t+1})/P(z^{t})$ (*one* SDF if $\mu_{c}=0$)

Budget constraints

1. **Complete traders (c)**

$$\begin{aligned} & \gamma Y(z^t)\eta_t + a_{t-1}(z^t, \eta^t) + \sigma(z^{t-1}, \eta^{t-1})\left[(1-\gamma)Y(z^t) + \overline{\omega}(z^t)\right] - c(z^t, \eta^t) \\ \geq & \sum_{z^{t+1} \succ z^t} q(z_{t+1}, z^t) \sum_{\eta^{t+1} \succ \eta^t} a_t(z^{t+1}, \eta^{t+1}) \pi(\eta^{t+1} | z^{t+1}, \eta^t) + \sigma(z^t, \eta^t)\overline{\omega}(z^t) \end{aligned}$$

2. **z-complete traders (z)**

$$\begin{aligned} & \gamma Y(z^t)\eta_t + a_{t-1}(z^t, \eta^{t-1}) + \sigma(z^{t-1}, \eta^{t-1})\left[(1-\gamma)Y(z^t) + \overline{\omega}(z^t)\right] - c(z^t, \eta^t \\ \geq & \sum_{z^{t+1} \succ z^t} q(z_{t+1}, z^t) a_t(z^{t+1}, \eta^t) + \sigma(z^t, \eta^t)\overline{\omega}(z^t) \end{aligned}$$

3. Diversified traders (div)

$$\gamma Y(z^t)\eta_t + \sigma(z^{t-1},\eta^{t-1})\left[(1-\gamma)Y(z^t) + \overline{\omega}(z^t)\right] - c(z^t,\eta^t) \geq \sigma(z^t,\eta^t)\overline{\omega}(z^t)$$

4. Non-participants (np)

$$\gamma Y(z^t)\eta_t + a_{t-1}(z^{t-1},\eta^{t-1}) - c(z^t,\eta^t) \geq \sum_{z^{t+1}\succ z^t} q(z_{t+1},z^t)a_t(z^t,\eta^t)$$

Market clearing

$$\sum_{\eta^t}\left[\begin{array}{c}\mu_1 a^c_{t-1}(z^t,\eta^t) + \mu_2 a^z_{t-1}(z^t,\eta^{t-1}(\eta^t)) \\ +\mu_4 a^{np}_{t-1}(z^{t-1}(z^t),\eta^{t-1}(\eta^t))\end{array}\right]\pi(\eta^t|z^t) = 0$$

$$\sum_{\eta^t}\left[\mu_1\sigma^c(z^t,\eta^t) + \mu_2\sigma^z(z^t,\eta^t) + \mu_3\sigma^{div}(z^t,\eta^t)\right]\pi(\eta^t|z^t) = 1$$

Net wealth

$$\widehat{a}_{t-1}(z^t, \eta^t) \equiv a_{t-1}(z^t, \eta^t) + \sigma(z^{t-1}, \eta^{t-1}) \left[(1-\gamma) Y(z^t) + \varpi(z^t)\right]$$

2. z-complete traders

$$\widehat{a}_{t-1}(z^t, \left[\eta^{t-1}, \eta_t\right]) = \widehat{a}_{t-1}(z^t, \left[\eta^{t-1}, \widetilde{\eta}_t\right]) \ \forall z^t, \eta^{t-1} \text{ and } \eta_t, \widetilde{\eta}_t \in N$$

3. Diversified traders

$$\frac{\widehat{a}_{t-1}(\left[z^{t-1}, z_t\right], \left[\eta^{t-1}, \eta_t\right])}{(1-\gamma) Y(z^{t-1}, z_t) + \varpi(z^{t-1}, z_t)} = \frac{\widehat{a}_{t-1}(\left[z^{t-1}, \widetilde{z}_t\right], \left[\eta^{t-1}, \widetilde{\eta}_t\right])}{(1-\gamma) Y(z^{t-1}, \widetilde{z}_t) + \varpi(z^{t-1}, \widetilde{z}_t)}$$

$$\forall z^{t-1}, \eta^{t-1} \text{ and } z_t, \widetilde{z}_t \in Z \text{ and } \eta_t, \widetilde{\eta}_t \in N$$

4. Non-participants

$$\widehat{a}_{t-1}(\left[z^{t-1}, z_t\right], \left[\eta^{t-1}, \eta_t\right]) = \widehat{a}_{t-1}(\left[z^{t-1}, \widetilde{z}_t\right], \left[\eta^{t-1}, \widetilde{\eta}_t\right])$$

$$\forall z^{t-1}, \eta^{t-1} \text{ and } z_t, \widetilde{z}_t \in Z \text{ and } \eta_t, \widetilde{\eta}_t \in N$$

Lagrangian

$$
\begin{aligned}
L = & \min_{\{\chi,\nu,\varphi\}} \max_{\{c,\widehat{a}\}} \sum_{t=1}^{\infty} \beta^t \sum_{(z^t,\eta^t)} u(c(z^t,\eta^t))\pi(z^t,\eta^t) \\
& +\chi \{\text{present value budget constr.}\} \\
& + \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \nu(z^t,\eta^t) \{\text{measurability constr. in } (z^t,\eta^t)\} \\
& + \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \varphi(z^t,\eta^t) \{\text{borrowing constr. in } (z^t,\eta^t)\}
\end{aligned}
$$

Lagrangian a la Marcet and Marimon (1999)

$$
\zeta_0 = \chi; \quad \zeta(z^t,\eta^t) = \zeta(z^{t-1},\eta^{t-1}) + \nu(z^t,\eta^t) - \varphi(z^t,\eta^t)
$$

$$
\begin{aligned}
L^{z} = & \min_{\{\chi,\nu,\varphi\}} \max_{\{c,\widehat{a}\}} \sum_{t=1}^{\infty} \beta^t \sum_{(z^t,\eta^t)} u(c(z^t,\eta^t))\pi(z^t,\eta^t) \\
& + \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \{\zeta(z^t,\eta^t)\left(\gamma Y(z^t)\eta_t - c(z^t,\eta^t)\right)\} \\
& + \sum_{t\geq 1} \sum_{(z^t,\eta^t)} \left\{ \begin{array}{c} \nu(z^t,\eta^t)\widehat{a}_{t-1}(z^t,\eta^{t-1}) \\ -\varphi(z^t,\eta^t)\cdot\{\text{borrowing constr. in } (z^t,\eta^t)\} \end{array} \right\}
\end{aligned}
$$

The aggregate multiplier and the SDF

FOC for consumption, irrespective of trading tech.

$$\frac{\beta^t u'(c(z^t,\eta^t))}{P(z^t)} = \zeta(z^t,\eta^t)$$

Consumption sharing rule

$$\frac{c(z^t,\eta^t)}{C(z^t)} = \frac{\zeta(z^t,\eta^t)^{\frac{-1}{\alpha}}}{h(z^t)}, \quad h(z^t) = \sum_{\eta^t} \zeta(z^t,\eta^t)^{\frac{-1}{\alpha}} \pi(\eta^t|z^t)$$

Stochastic discount factor

$$m(z^{t+1}|z^t) = \beta \left(\frac{C(z^{t+1})}{C(z^t)}\right)^{-\alpha} \left(\frac{h(z^{t+1})}{h(z^t)}\right)^{\alpha}$$

HH's net savings functions, $\mathbf{j} \in \{\mathbf{c}, \mathbf{z}, \mathbf{div}, \mathbf{np}\}$:

$$\begin{aligned} S^j(\zeta(z^t,\eta^t); z^t, \eta^t) &= \left[\gamma\eta_t - \frac{\zeta(z^t,\eta^t)^{\frac{-1}{\alpha}}}{h(z^t)}\right] C(z^t) \\ &+ \sum_{z^{t+1},\eta^{t+1}} \frac{\pi(z^{t+1},\eta^{t+1})P(z^{t+1})}{\pi(z^t,\eta^t)P(z^t)} \cdot \\ &S(\zeta(z^{t+1},\eta^{t+1}); z^{t+1}, \eta^{t+1}) \end{aligned}$$

Ratio of savings to aggr. consumption

$$\widetilde{S}^j(\zeta(z^t,\eta^t); z^t, \eta^t) = \frac{S^j(\zeta(z^t,\eta^t); z^t, \eta^t)}{Y(z^t)}$$

Without non-participants, the (conditional) equity risk premium is the Breeden-Lucas one

Conditions

- $\phi(z_{t+1}|z_t) = \phi(z_{t+1})$
- $\pi(\eta_{t+1}, z_{t+1}|\eta_t, z_t) = \varphi(\eta_{t+1}|\eta_t)\phi(z_{t+1}|z_t)$

Given the conditions and without non-participants, ζ and the consumption share are independent of z^t

- $\frac{h_{t+1}}{h_t}$ non-random
- $m(z^{t+1}|z^t) = \beta \left(\frac{C(z^{t+1})}{C(z^t)}\right)^{-\alpha} \left(\frac{h(z^{t+1})}{h(z^t)}\right)^{\alpha}$

The role of non-participants

- np's measurability condition in terms of net savings is given by:

$$\frac{\widetilde{S}^{np}_{t+1}(\zeta(z^{t+1}, \eta^{t+1}); z^{t+1}, \eta^{t+1})}{e^{z_{t+1}}} = \frac{\widetilde{S}^{np}_{t+1}(\zeta(\widetilde{z}^{t+1}, \widetilde{\eta}^{t+1}); \widetilde{z}^{t+1}, \widetilde{\eta}^{t+1})}{e^{\widetilde{z}_{t+1}}}$$

- np "spill" risk that depends on z_{t+1} *and* η_{t+1} to other household types
- In the presence of np, $\{h_{t+1}/h_t\}$ depends on z^{t+1}

Shifting residual aggregate risk to active traders

Div. do not bear any of the residual aggregate risk created by np

- Div's measurability constraint

$$\frac{S_a^{div}(z^t, z_{t+1})}{[(1-\gamma)Y(z^t, z_{t+1}) + \overline{\omega}(z^t, z_{t+1})]} = \frac{S_a^{div}(z^t, \widetilde{z}_{t+1})}{[(1-\gamma)Y(z^t, \widetilde{z}_{t+1}) + \overline{\omega}(z^t, \widetilde{z}_{t+1})]}$$

Calibration

Endowment process

- $Y(z^t)$ as in Mehra and Prescott (1985)
- η as in STY (2003)
- $(1-\gamma) = 0.1$

Households

- $\mu_{np} = 0.7$
- $\alpha = 5$
- $\beta = 0.95$

Asset pricing (Table 3)

	RA Economy	HTT Economy			Data
		Case 1	*Case 2*	*Case 3*	
complete		0%	5%	10%	
z-complete		10%	5%	0%	
diversified		20%	20%	20%	
non-part		70%	70%	70%	
$E[R_f]$	12.96	1.737	1.922	2.185	1.049
$\sigma[R_f]$	0.000	0.066	0.237	0.292	1.560
$\sigma[m]/E[m]$	0.193	0.440	0.467	0.510	
$Std[\sigma_t[m]/E_t[m]]$	0.000	0.033	0.045	0.058	
$E[R_{lc} - R_f]$	3.081	6.702	6.435	6.874	7.531
$\sigma[R_{lc} - R_f]$	15.94	15.27	13.89	13.69	16.94
$E[R_{lc} - R_f]/\sigma[R_{lc} - R_f]$	0.193	0.438	0.463	0.502	0.444
$E[W^{Coll}/C]$	0.855	5.960	4.889	6.458	3.870
$E[PD_{lc}]$	7.936	20.98	18.02	23.16	33.87
$\sigma[PD_{lc}]$	13.09	15.92	15.59	15.20	16.78
$E[R_b - R_f]$	0.000	-0.271	-0.046	-0.437	1.070
$\sigma[R_b - R_f]$	0.000	0.604	0.143	0.935	9.366
$E[R_b - R_f]/\sigma[R_b - R_f]$	/	-0.324	-0.467	-0.449	.1145
$\rho[R_{lc}(t), R_{lc}(t-1)]$	0.000	-0.015	-0.010	-0.010	-0.191
$\rho[R_{lc}(t), R_{lc}(t-1)]$	0.000	0.003	0.012	-0.005	-0.191
$\rho[R_{lc}, R_f]$	0.000	-0.024	-0.014	-0.020	0.272

Consumption (Table 5)

	Case 1	Case 2	Case 3		Case 1	Case 2	Case 3
complete	0%	5%	10%		0%	5%	10%
z-complete	10%	5%	0%		10%	5%	0%
diversified	20%	20%	20%		20%	20%	20%
non-part	70%	70%	70%		70%	70%	70%
	Household Consumption				**Aggregate Consumption**		
$\sigma[\Delta\log(c_c)]$	NA	5.641	5.417	$\sigma[\Delta\log(\widehat{C}_c)]$	NA	3.840	4.873
$\sigma[\Delta\log(c_z)]$	7.892	7.131	NA	$\sigma[\Delta\log(\widehat{C}_z)]$	3.972	4.402	NA
$\sigma[\Delta\log(c_{div})]$	11.44	11.35	11.07	$\sigma[\Delta\log(\widehat{C}_{div})]$	0.334	0.329	0.368
$\sigma[\Delta\log(c_{np})]$	12.62	12.50	12.35	$\sigma[\Delta\log(\widehat{C}_{np})]$	1.062	1.037	1.071
$\rho[R_s,(\Delta\log(\widehat{c}_p)]$	0.163	0.204	0.283				
$\rho[R_s,(\Delta\log(\widehat{c}_c)]$	NA	0.649	0.857	$\rho[R_s,\Delta\log(\widehat{C}_c)]$	NA	0.949	0.951
$\rho[R_s,(\Delta\log(\widehat{c}_z)]$	0.482	0.588	NA	$\rho[R_s,\Delta\log(\widehat{C}_z)]$	0.965	0.956	NA
$\rho[R_s,(\Delta\log(\widehat{c}_{div})]$	0.003	−0.002	−0.003	$\rho[R_s,\Delta\log(\widehat{C}_{div})]$	0.119	−0.083	−0.117
$\rho[R_s,(\Delta\log(\widehat{c}_{np})]$	−0.071	−0.070	−0.073	$\rho[R_s,\Delta\log(\widehat{C}_{np})]$	−0.965	−0.959	−0.948

THE EQUITY SHARE ALONG THE WEALTH DISTRIBUTION (TABLE 9)

	Data		Model	
Percentile	2001	2004	Standard	Twisted
15%	4.512	2.633	5.694	3.942
25%	15.40	6.797	6.617	3.293
35%	6.057	6.669	7.331	3.722
50%	8.077	2.762	6.817	3.115
65%	11.09	10.16	6.572	8.207
75%	19.04	10.12	7.962	11.02
80%	14.45	17.34	9.204	10.08
85%	24.16	16.56	13.11	9.263
90%	32.59	18.94	27.50	12.78
95%	34.30	25.37	52.02	41.86
100%	42.67	34.19	59.02	59.80

Active traders is a very happy family

They hold leveraged portfolios, earn high returns, trade a lot, and face little consumption volatility!

	c	z	div	np
$E\left[R^W - R^f\right]$	0.125	0.062	0.008	-0.012
$\sigma\left[\Delta \log(c)\right]$	5.64	7.13	11.4	12.5

Portfolio positions

- In model: c holds leveraged equity portfolios (160%), z: 93%
- Average returns are increasing in the degree of insurance
- Empirical research (Vissing-Jorgensen, Guvenen) reject the hypothesis of complete markets for stockholders, but not for non-stockholders
- Would their tests reject complete markets among active traders?
- Again, who are c and z?

Trading frequency and volume

- Empirically, the evidence of households trading frequency and trading volumes is mixed
- std of fraction invested in stocks for c is 60%
- std of fraction invested in stocks for z is 30%

Is there a mapping between CCL and Calvet, Campbell, and Sodini (2007)?

Is there no financial income in non-diversifiable income?

- Calvet et al (2007) focus on households' financial portfolios and argue that many are subject to idiosyncratic risk
- Would Calvet et al (2007) agree that a diversified trader is somebody who is afraid of making investment mistakes (CCL p. 5)?

How categorize households in the data?

- Portfolio holdings would be natural!
  - np=easy
  - div=well-diversified stock-holders?
  - c=?
  - z=?
  - without c or z there is no equilibrium!
- What about HH's Sharpe ratios?
- What about HH's trading volume or trading pattern?

Where are entrepreneurs and their capital?

Where does an entrepreneur with little non-proprietary wealth belong?

- Moskowitz and Vissing-Jorgenssen (AER, 2004)
- Entrepreneurs as a group own a lot of capital
- They face a lot of uninsurable risk (they are not c or z)
- They are also poorly diversified (not div)
- Are they np? But np in the strict sense are typically poor!