Intertemporal Disturbances

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Motivation

- Given the following problem

$$\max E_t \sum_{s=0}^{\infty} \beta^s \left[\frac{C_{t+s}^{1-\theta}}{1-\theta} \right]$$

$$s.t. \quad C_{t+s} + I_{t+s} + B_{t+s} \leq (1 + r_{t+s-1}) B_{t+s-1} + r_{t+s}^k K_{t+s}$$

$$K_{t+1} = (1-\delta) K_t + I_t$$

- First order conditions

$$\begin{aligned} 1 &= E_t \left[M_{t+1} (1 + r_t) \right] \\ 1 &= E_t \left[M_{t+1} \left(r_{t+1}^k + (1-\delta) \right) \right] \\ M_{t+1} &= \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\theta} \end{aligned}$$

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s.t. \quad & C_{t+s} + I_{t+s} + B_{t+s} \leq (1 + r_{t+s-1}) B_{t+s-1} + r_{t+s}^k K_{t+s} \\
& K_{t+1} = (1-\delta) K_t + I_t
\end{aligned}
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\begin{aligned}
1 &= E_t \left[M_{t+1} (1 + r_t) \right] \\
1 &= E_t \left[M_{t+1} \left(r_{t+1}^k + (1-\delta) \right) \right] \checkmark \\
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- Can we satisfy both Euler equations?

Motivation

- Given the following problem

$$\max E_t \sum_{s=0}^{\infty} \beta^s b_{t+s} \left[\frac{C_{t+s}^{1-\theta}}{1-\theta} \right]$$

$$s.t. \quad C_{t+s} + I_{t+s} + B_{t+s} \leq (1 + r_{t+s-1}) B_{t+s-1} + r_{t+s}^k K_{t+s}$$

$$K_{t+1} = (1-\delta) K_t + \mu_t I_t$$

  with μ_t and b_t AR(1) process.

- First order conditions

$$1 = E_t \left[M_{t+1} (1 + r_t) \right]$$

$$1 = E_t \left[M_{t+1} \mu_t \left(r_{t+1}^k + \frac{1-\delta}{\mu_{t+1}} \right) \right]$$

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\theta} \frac{b_{t+1}}{b_t}$$

- What is the role of μ_t and b_t in explaining business cycle fluctuations?

Roadmap

- Write down a ‘realistic’ RBC model
- Log-linearization
- Bayesian Estimation
- Variance decomposition
- Shut down parts of the baseline model

An RBC model

- Final good prod.: $Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{p,t}}}\right]^{1+\lambda_{p,t}}$
- Interm. good prod.: $Y_t(i) = (A_{t-1}e^{z_t})^{1-\alpha} K_t(i)^\alpha L_t(i)^{1-\alpha} - A_{t-1}e^{z_t}F$
- Households: $\max E_t \sum_{s=0}^\infty \beta^s b_{t+s} \left[\log\left(C_{t+s}(j) - hC_{t+s-1}(j)\right) - \varphi_{t+s}\frac{L_{t+s}^{1+\nu}}{1+\nu}\right]$
- Variable Cap. Util.: $K_t(j) = u_t(j)\overline{K}_{t-1}(j)$
- Cap. law of mot.: $\overline{K}_t(j) = (1-\delta)\overline{K}_{t-1}(j) + \mu_t\left(1 - S\left(\frac{I_t(j)}{I_{t-1}(j)}\right)\right) I_t(j)$
- Mon. Auth.: $r_t = \rho_r r_{t-1} + (1-\rho_r)\left[\phi_\pi \pi_t + \phi_y(y_t - a_t)\right] + \varepsilon_{MP,t}$
- Fiscal Auth.: $G_t = \left(1 - \frac{1}{g_t}\right) Y_t$
- Price rigidities
- Wage rigidities
- Euler equations
  - $1 = E_t\left[M_{t+1}(1+r_t)\right]$
  - $1 = E_t\left[M_{t+1}\mu_t\left(r_{t+1}^k + \frac{1-\delta}{\mu_{t+1}}\right)\right]$

Variance decomposition

	Shocks						
Variables	M.P. (ε_t^{MP})	tech. (z_t)	Gov. (g_t)	i.s. tech (μ_t)	mark-up $(\lambda_{p,t})$	intra.pref. (φ_t)	inter.pref. (b_t)
$\Delta \log Y_t$	0.01	0.54	0.01	0.04	0.04	0.31	0.05
$\Delta \log C_t$	0.00	0.09	0.06	0.37	0.01	0.02	0.45
$\Delta \log I_t$	0.01	0.12	0.12	0.28	0.04	0.10	0.33
L_t	0.00	0.01	0.01	0.02	0.02	0.90	0.04
$\Delta \log \frac{W_t}{P_t}$	0.15	0.39	0.00	0.01	0.37	0.07	0.01
π_t	0.53	0.07	0.01	0.30	0.01	0.05	0.04
R_t	0.00	0.13	0.02	0.71	0.02	0.06	0.07

A growth model

- Final good prod.: $Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{p,t}}}\right]^{1+\lambda_{p,t}}$ ✓
- Interm. good prod.: $Y_t(i) = (A_{t-1}e^{z_t})^{1-\alpha} K_t(i)^{\alpha} L_t(i)^{1-\alpha} - A_{t-1}e^{z_t}F$ ✓
- HH: $\max E_t \sum_{s=0}^{\infty} \beta^s b_{t+s} \left[\log\left(C_{t+s} - hC_{t+s-1}\right) - \varphi_{t+s}\frac{L_{t+s}^{1+\nu}}{1+\nu}\right]$ $\longleftarrow$ (h=0,b=1)
- Variable Cap. Util.: $K_t(j) = u_t(j)\overline{K}_{t-1}(j)$ $\longleftarrow$ ($u_t = 1$)
- Cap. law of mot.: $\overline{K}_t(j) = (1-\delta)\overline{K}_{t-1}(j) + \mu_t\left(1 - S\left(\frac{I_t(j)}{I_{t-1}(j)}\right)\right) I_t(j)$ $\longleftarrow$ (S=0)
- ~~Mon. Auth.: $r_t = \rho_r r_{t-1} + (1-\rho_r)\left[\phi_\pi \pi_t + \phi_y(y_t - a_t)\right] + \varepsilon_{MP,t}$~~
- Fiscal Auth.: $G_t = \left(1 - \frac{1}{g_t}\right) Y_t$ ✓
- ~~Price rigidities~~
- ~~Wage rigidities~~
- Euler equations
  - ~~$1 = E_t\left[M_{t+1}(1+r_t)\right]$~~
  - $1 = E_t\left[M_{t+1}\mu_t\left(r_{t+1}^k + \frac{1-\delta}{\mu_{t+1}}\right)\right]$ ✓

Growth model: variance decomposition

	Shocks			
Variables	tech. (z_t)	Gov. (g_t)	i.s. tech (μ_t)	intra.pref. (φ_t)
$\Delta \log Y_t$	0.60	0.01	0.02	0.37
$\Delta \log C_t$	0.26	0.13	0.58	0.04
$\Delta \log I_t$	0.23	0.23	0.29	0.25
L_t	0.01	0.01	0.04	0.95

A model with real frictions

- Final good prod.: $Y_t = \left[\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_{p,t}}}\right]^{1+\lambda_{p,t}}$ ✓
- Interm. good prod.: $Y_t(i) = (A_{t-1}e^{z_t})^{1-\alpha} K_t(i)^\alpha L_t(i)^{1-\alpha} - A_{t-1}e^{z_t}F$ ✓
- HH: $\max E_t \sum_{s=0}^{\infty} \beta^s b_{t+s} \left[\log\left(C_{t+s} - hC_{t+s-1}\right) - \varphi_{t+s}\frac{L_{t+s}^{1+\nu}}{1+\nu}\right]$ $\longleftarrow$ (b=1)
- Variable Cap. Util.: $K_t(j) = u_t(j)\overline{K}_{t-1}(j)$ ✓
- Cap. law of mot.: $\overline{K}_t(j) = (1-\delta)\overline{K}_{t-1}(j) + \mu_t\left(1 - S\left(\frac{I_t(j)}{I_{t-1}(j)}\right)\right) I_t(j)$ ✓
- ~~Mon. Auth.: $r_t = \rho_r r_{t-1} + (1-\rho_r)\left[\phi_\pi \pi_t + \phi_y(y_t - a_t)\right] + \varepsilon_{MP,t}$~~
- Fiscal Auth.: $G_t = \left(1 - \frac{1}{g_t}\right) Y_t$ ✓
- ~~Price rigidities~~
- Wage rigidities ✓
- Euler equations
  - ~~$1 = E_t\left[M_{t+1}(1+r_t)\right]$~~
  - $1 = E_t\left[M_{t+1}\mu_t\left(r_{t+1}^k + \frac{1-\delta}{\mu_{t+1}}\right)\right]$ ✓

Real frictions: variance decomposition

	Shocks			
Variables	tech. (z_t)	Gov. (g_t)	i.s. tech (μ_t)	intra.pref. (φ_t)
$\Delta \log Y_t$	0.62	0.00	0.03	0.34
$\Delta \log C_t$	0.16	0.07	0.74	0.03
$\Delta \log I_t$	0.32	0.24	0.23	0.21
L_t	0.01	0.01	0.03	0.95

Conclusion and comment

- According to PST $\frac{Var b}{Var \Delta c}$ is high ($\approx 50\%$)

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$$\begin{aligned} \Delta c_{t+1} &= b_{t+1} + \varepsilon_{c,t+1} \\ b_{t+1} &= \underbrace{\rho_b}_{.832} b_t + \varepsilon_{b,t+1} \end{aligned}$$

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- Then

$$\sigma(\log M) \approx 10\% << 30\% - 40\%$$

$$\rho\left(\Delta c_t, \Delta c_{t-1}\right) \approx 0.4 >> 0.1$$

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- To increase the volatility of the SDF to the HJ bound, we would need to increase the autocorrelation of Δc to even more counterfactual levels.
- When it comes to reconciling quantities and asset prices, do we want intertemporal disturbances to explain the variability of consumption or the variability of returns?