Risks for the Long Run and the Real Exchange Rate

Riccardo Colacito - *New York University*

Mariano M. Croce - *New York University*

Puzzle

- By no arbitrage

$$E_t\left[M_{t+1}^{f} R_{t+1}^{f}\right] \quad = 1 = \quad E_t\left[M_{t+1}^{h} R_{t+1}^{h}\right]$$

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   - low volatility of depreciation rate
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3. Can we estimate this model?

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- Pricing kernels

$$\log M_{t+1}^i = \theta \log \delta - \frac{\theta}{\psi} \log\left(\frac{C_{t+1}^i}{C_t^i}\right) + (\theta - 1) \log R_{c,t+1}^i$$

Remainder of the economy

- Home country

$$\Delta c_t^h = \mu_c + x_{t-1}^h + \sigma \varepsilon_{c,t}^h$$
$$x_t^h = \rho x_{t-1}^h + \sigma \varphi_e \varepsilon_{x,t}^h$$

- Foreign country

$$\Delta c_t^f = \mu_c + x_{t-1}^f + \sigma \varepsilon_{c,t}^f$$
$$x_t^f = \rho x_{t-1}^f + \sigma \varphi_e \varepsilon_{x,t}^f$$

- Shocks are $i.i.d.$ within each country
- Shocks are correlated across countries
  - $\rho_c = corr(\varepsilon_{c,t}^h, \varepsilon_{c,t}^f)$
  - $\rho_x = corr(\varepsilon_{x,t}^h, \varepsilon_{x,t}^f)$

Calibration

δ	γ	ψ	θ	μ_c	σ	ρ	φ_e	ρ_x	ρ_c
.998	4.25	2	-6.5	.0015	.0068	.987	.048	1	.3

$$
\begin{aligned}
m_{t+1}^i &= \theta \log \delta - \frac{\theta}{\psi} \Delta c_t^i + (\theta - 1) \log R_{c,t+1}^i \\
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Preferences:

- Low risk aversion (γ)
- IES from Bansal, Gallant and Tauchen (2004)
- Monthly model: high discounting

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Consumption process:

- Average consumption growth $\approx 2\%$
- Standard deviation of consumption growth $\approx 2.5\%$
- Variance explained by long run risk $\approx 7 - 8\%$

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$$\begin{aligned} m^i_{t+1} &= \theta \log \delta - \frac{\theta}{\psi}\Delta c^i_t + (\theta - 1) \log R^i_{c,t+1} \\ \Delta c^i_t &= \mu_c + x^i_{t-1} + \sigma \varepsilon^i_{c,t} \\ x^i_t &= \rho x^i_{t-1} + \sigma \varphi_e \varepsilon^i_{x,t} \end{aligned}$$

Cross correlations of shocks:

- Correlation of consumption growths ≈ 0.3

Stochastic discount factors

$$\boxed{m^i_{t+1} = \theta \log \delta - \frac{1}{\psi} x^i_t - \gamma \sigma \varepsilon^i_{c,t+1} + \frac{\delta(1-\gamma\psi)}{\psi(1-\rho\delta)} \sigma \varphi_e \varepsilon^i_{x,t+1}}$$

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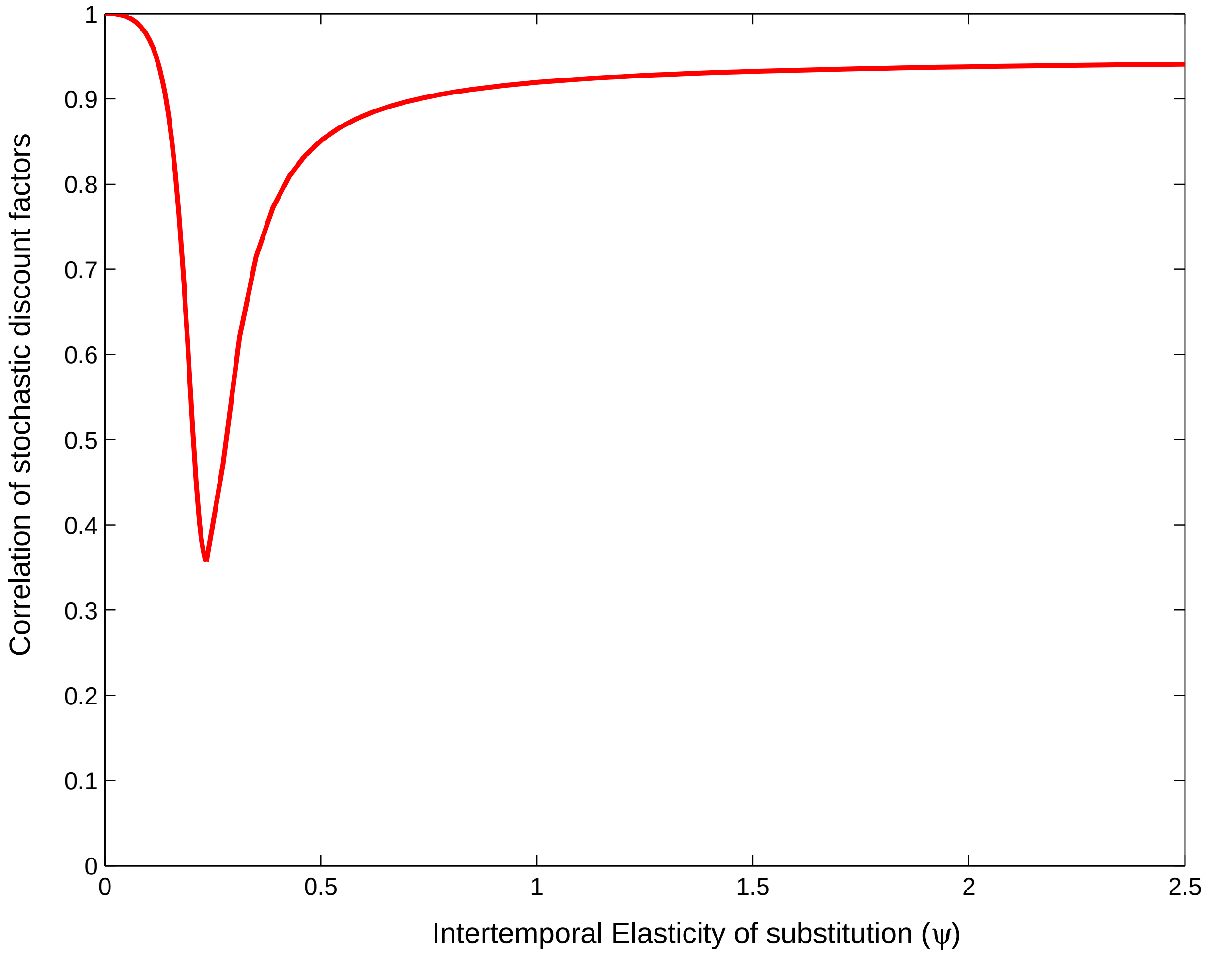

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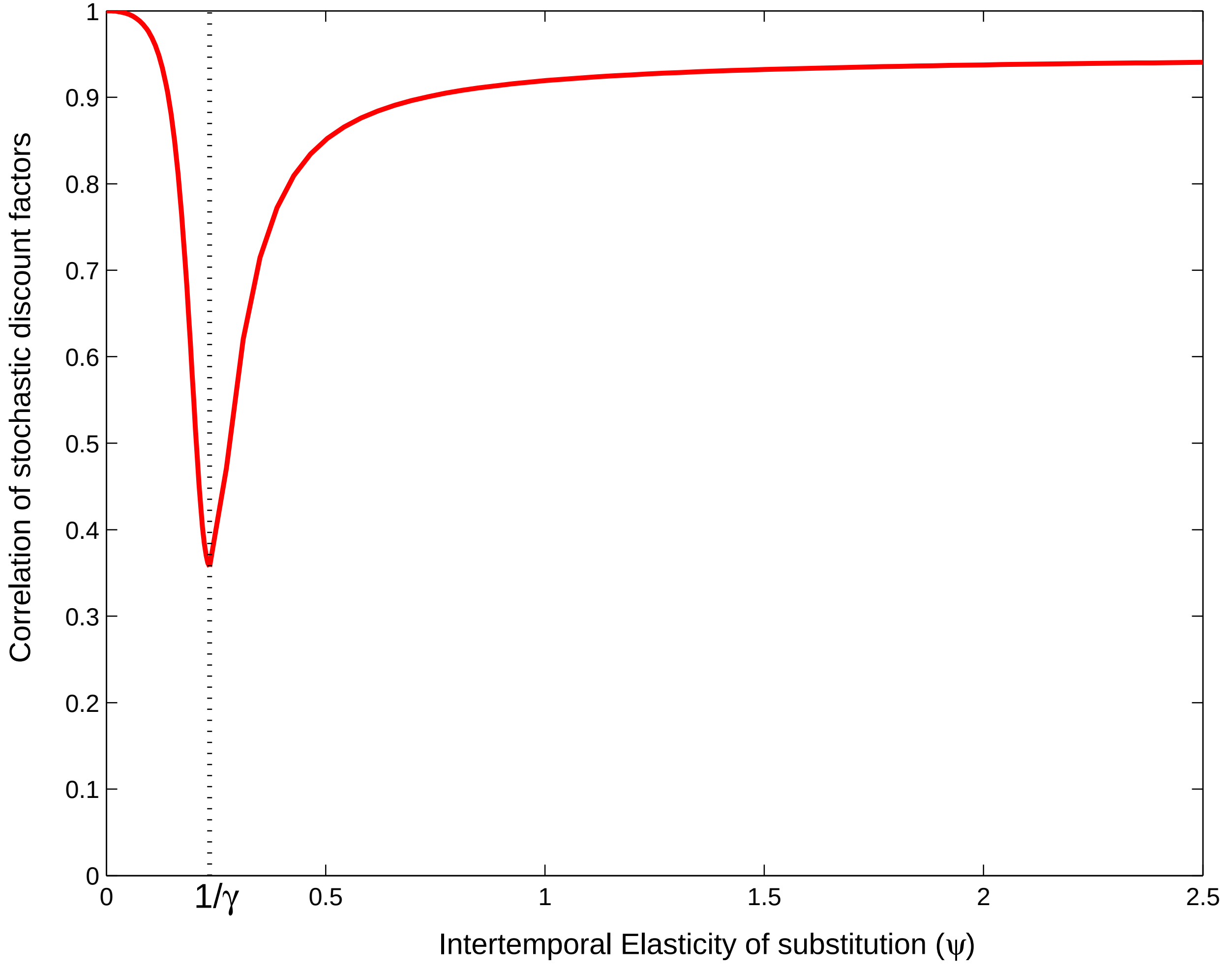

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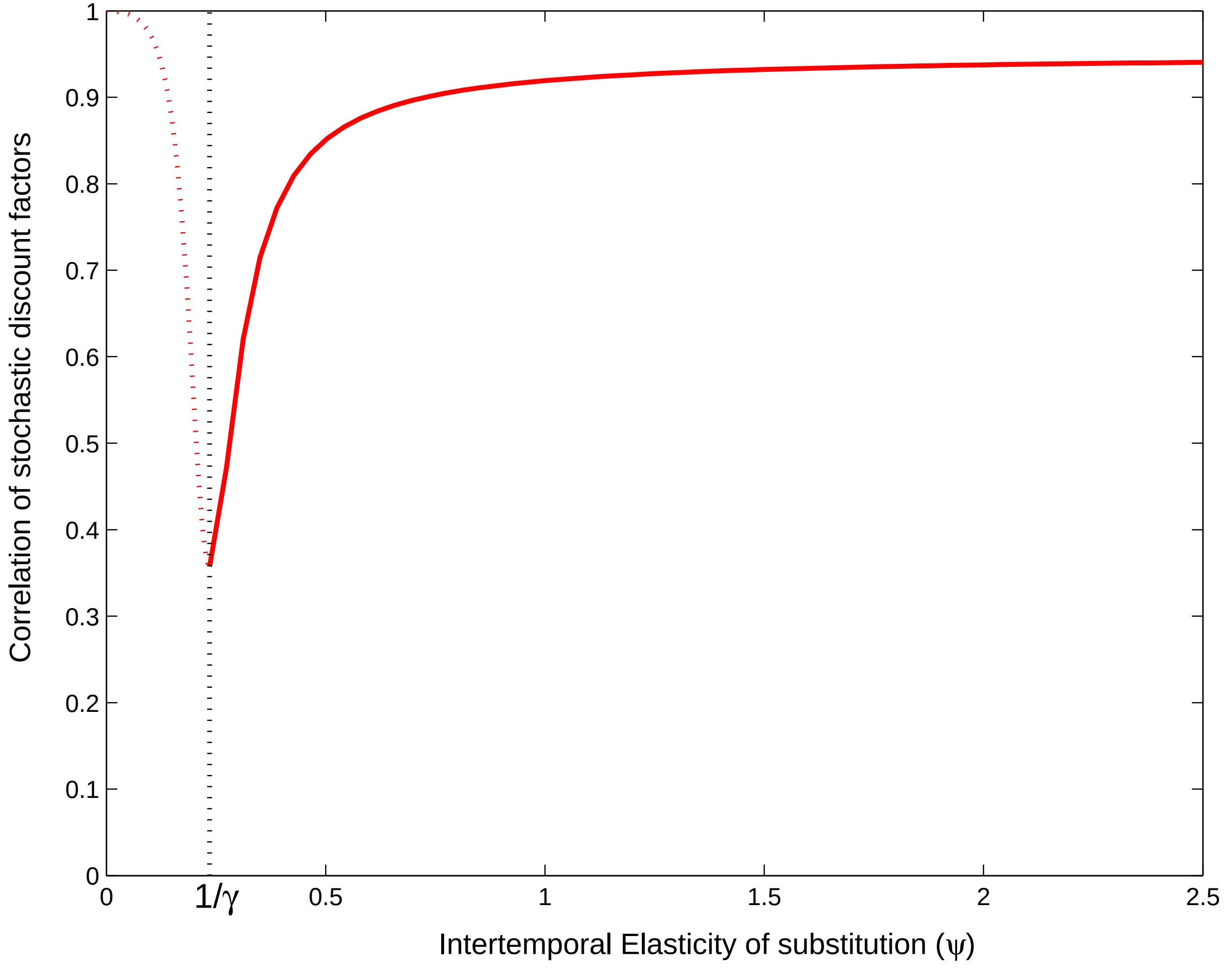

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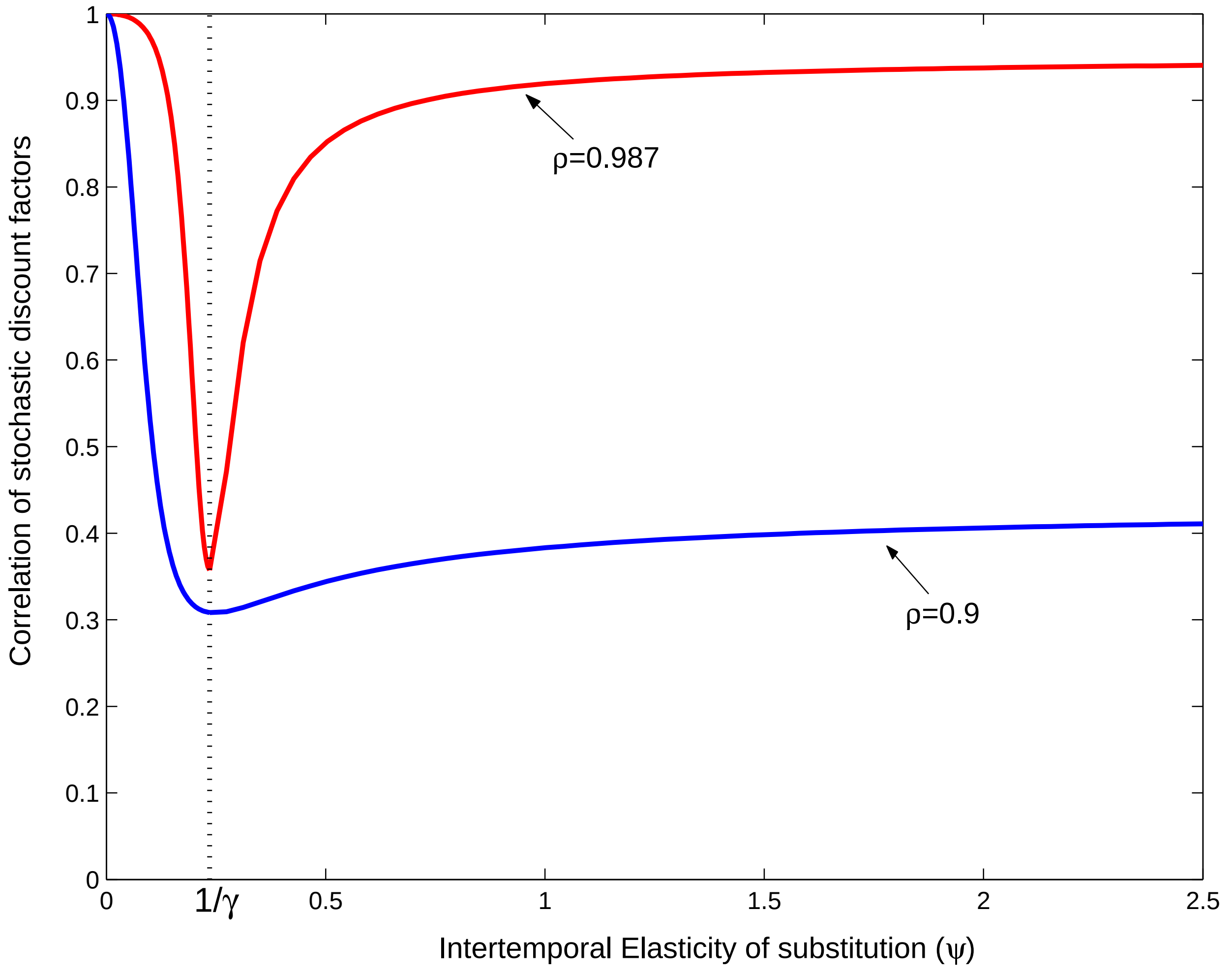

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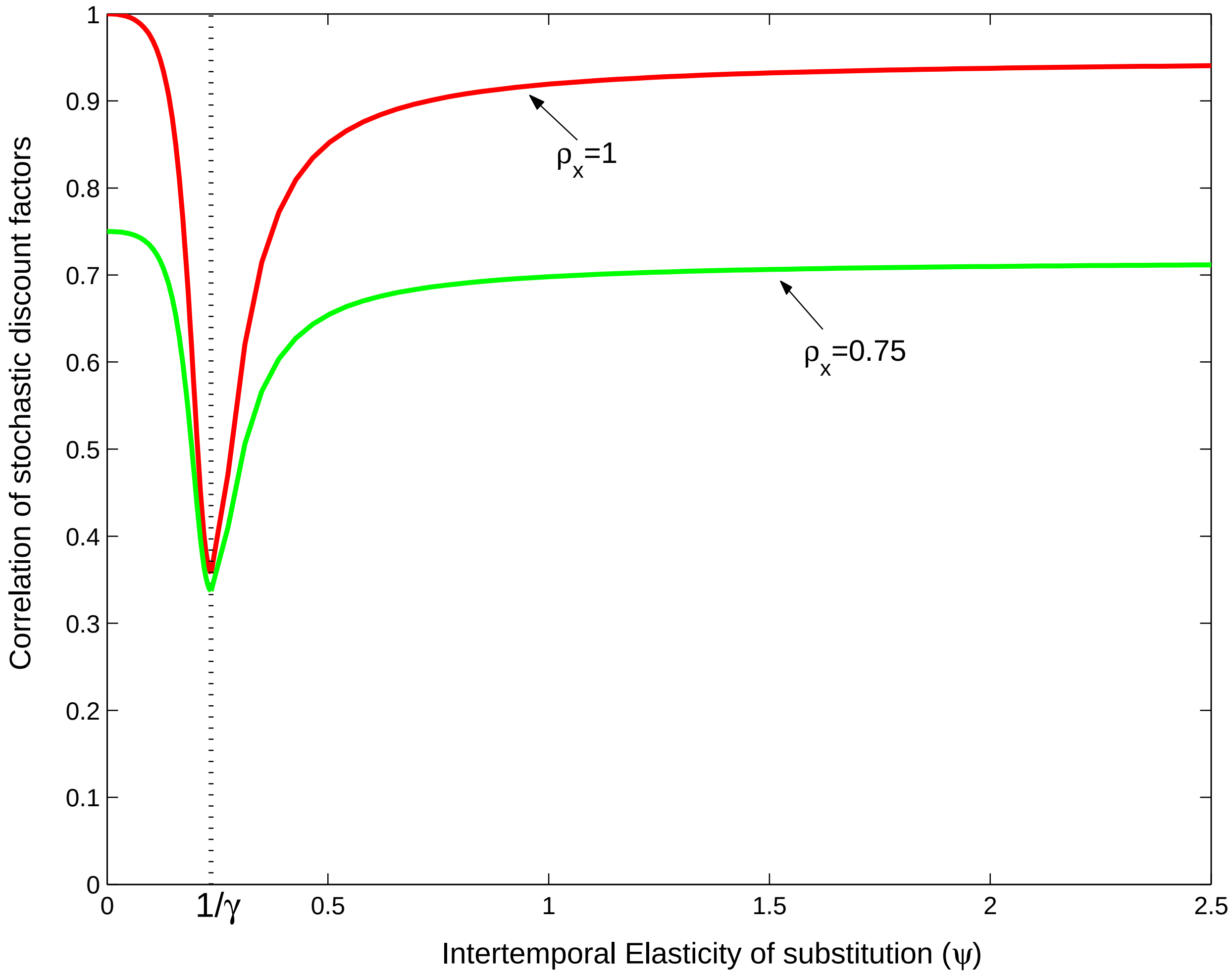

Exchange rate depreciation

$$Var\left(\frac{e_{t+1}}{e_t}\right) = \frac{2(1-\rho_x)}{\psi^2}\left\{\frac{1}{1-\rho^2} + \left[\frac{\delta(1-\gamma\psi)}{(1-\rho\delta)}\right]^2\right\}\varphi_e^2\sigma^2 + 2\gamma^2(1-\rho_c)\sigma^2$$

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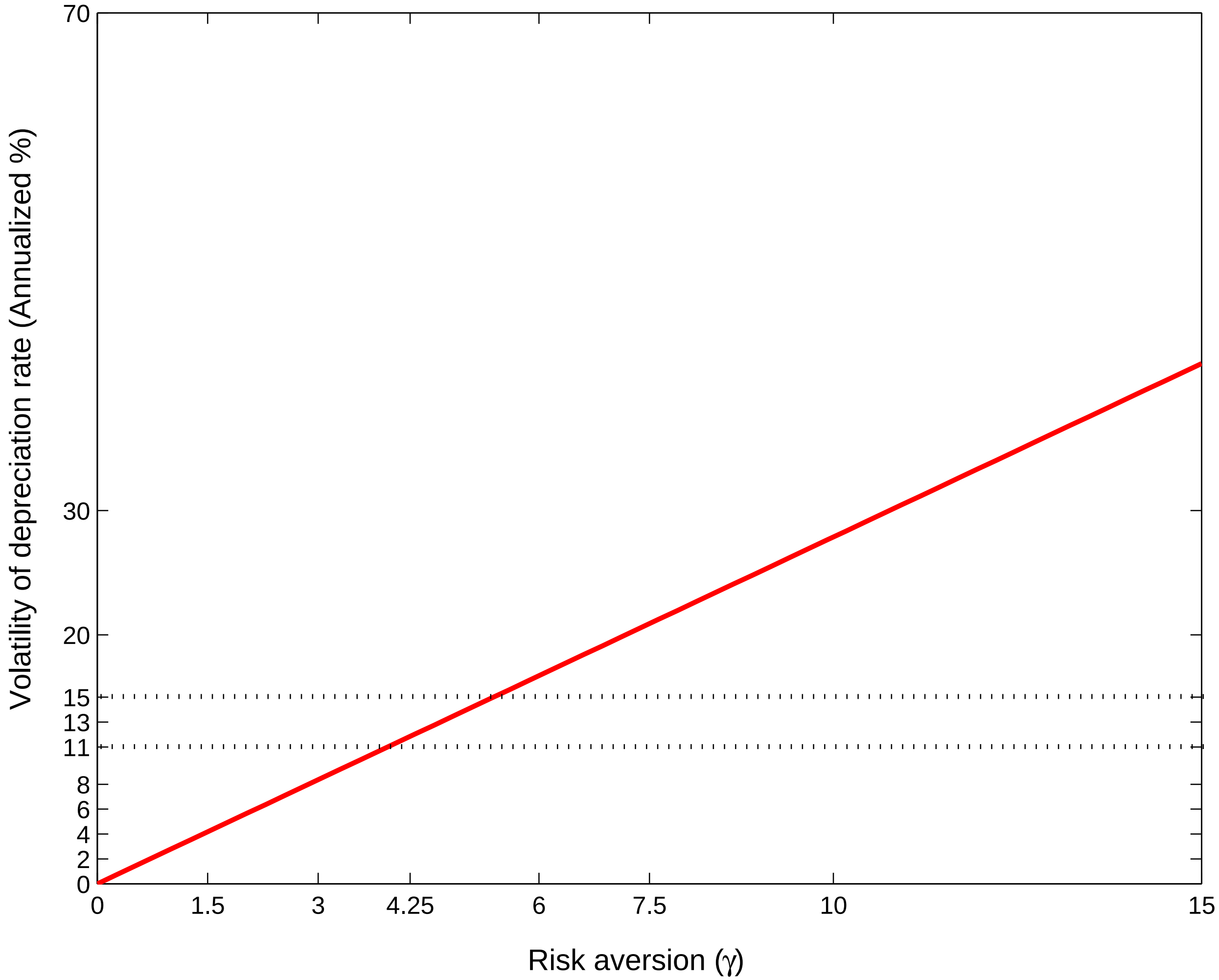

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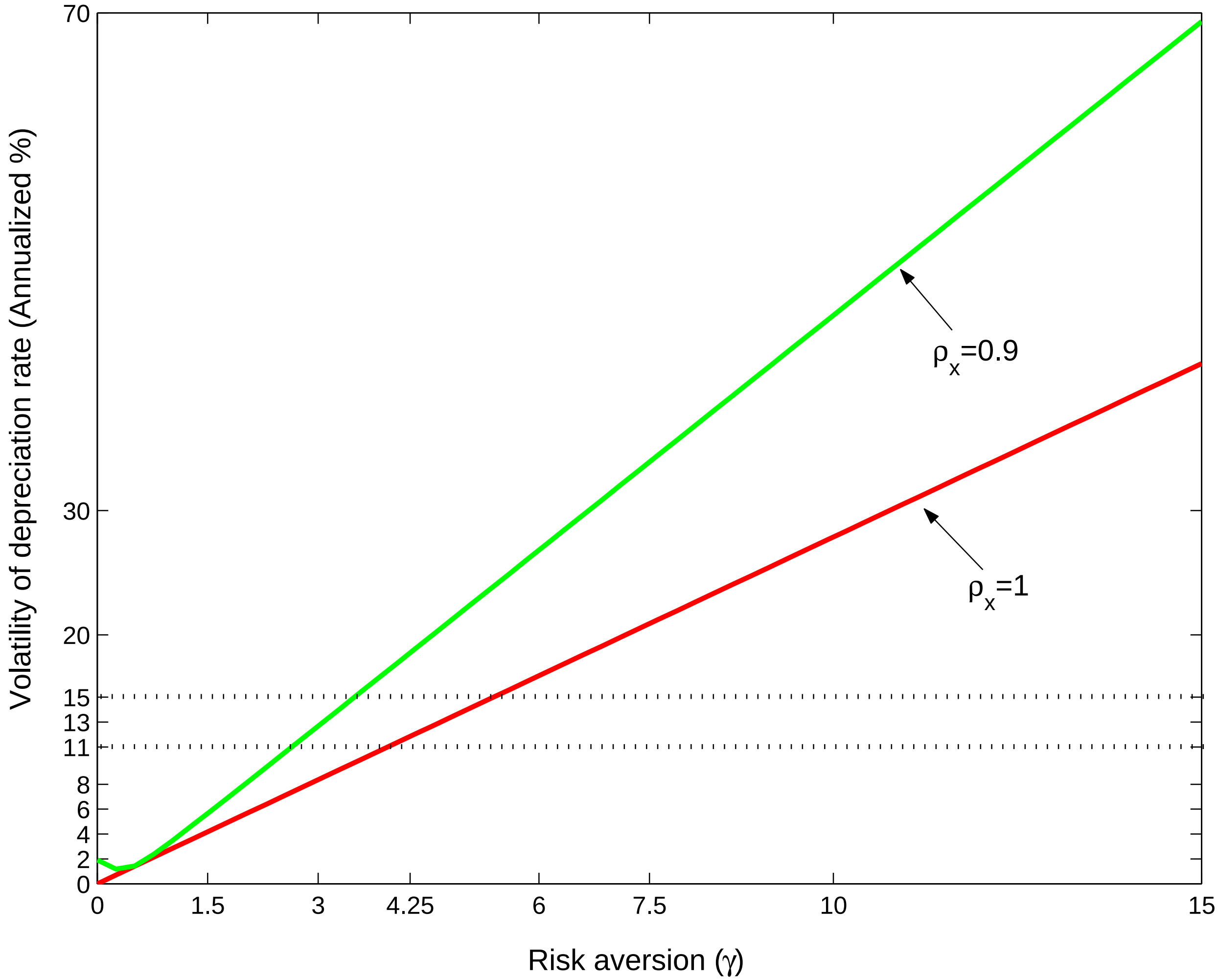

Every assumption counts

- Ingredients needed to solve the puzzle:
  1. Disentangle elasticity of substitution from risk aversion
  2. Highly persistent predictable component
  3. Highly correlated predictable components

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- Can we match key moments of international financial markets?

Dividends and Stochastic Volatility

- The system becomes $\forall i \in \{h, f\}$:

$$\begin{aligned}
\Delta c_t^i &= \mu_c + x_{t-1}^i + \sigma_t^i \varepsilon_{c,t}^i \\
\Delta d_t^i &= .0007 + 3 \cdot x_{t-1}^i + \sigma_t^i \cdot 5 \cdot \varepsilon_{d,t}^i \\
x_t^i &= \rho x_{t-1}^i + \sigma_t^i \varphi_e \varepsilon_{x,t}^i \\
\left(\sigma_t^2\right)^i &= .0068^2 + \nu_1 \left[\left(\sigma_{t-1}^2\right)^i - .0068^2 \right] + \sigma_w \varepsilon_{\sigma,t}^i
\end{aligned}$$

- Calibrate coefficients of dividend growth to match:
  - Average dividend growth $\approx 1\%$
  - Standard deviation of dividend growth $\approx 12\%$
  - Leverage is 5
- Guidelines to calibrate stochastic volatility given by

$$Var_t\left(r_{t+1}^d\right) = (1 - \nu_1)k_0 + \underbrace{\nu_1}_{.96} Var_{t-1}\left(r_t^d\right) + k_1 \underbrace{\sigma_w}_{.23e^{-5}} \varepsilon_{\sigma,t}$$

Results

	US	UK	No stoch vol	W/Stoch vol
$\rho\left(m^h, m^f\right)$	-	-	0.93	0.92
$\sigma\left(\frac{e_{t+1}}{e_t}\right)$	11.02		11.83	12.67
$E\left(r_d - r_f\right)$	7.02	9.17	7.01	7.03
$\sigma\left(r_d - r_f\right)$	17.13	22.83	19.60	19.41
$\rho\left(r_d^h - r_f^h, r_d^f - r_f^f\right)$	0.60		0.58	0.57
$E\left(r_f\right)$	1.47	1.62	1.33	1.33
$\sigma\left(r_f\right)$	1.53	2.92	1.19	1.22
$\rho\left(r_f^h, r_f^f\right)$	0.65		1.00	0.98
$\sigma\left(r_c\right)$	-	-	4.74	4.75
$\rho\left(r_c^h, r_c^f\right)$	-	-	0.85	0.85

Estimating long run risks

- Focus on consumption processes

$$\begin{bmatrix} \Delta c_t^h \\ \Delta c_t^f \end{bmatrix} = \begin{bmatrix} \mu_c \\ \mu_c \end{bmatrix} + \begin{bmatrix} x_{t-1}^h \\ x_{t-1}^f \end{bmatrix} + \sigma \begin{bmatrix} 1 & 0 \\ \rho_c & \sqrt{1-\rho_c^2} \end{bmatrix} \begin{bmatrix} \varepsilon_{c,t}^h \\ \varepsilon_{c,t}^f \end{bmatrix}$$

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- Can we justify high persistence and high correlation?
- Quarterly data from 1970 to 1998.
- Results from simulations and from real data.

Results

	ρ^h	ρ^f	ρ_x	ρ_c
Calibrated	0.987	0.987	1.000	0.300
Real Data	0.910	0.940	0.897	0.207
(T=120)	[0.547,0.995]	[0.308,0.995]	[0.696,1.000]	[0.003, 0.406]

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Simulations	0.987	0.987	0.986	0.302
(T=10000)	[0.983,0.990]	[0.983,0.991]	[0.953,1.0]	[0.285,0.318]

Future Research

What's next?

- Improve the estimation
- Relax assumption of complete home bias in consumption
- Where does x_t come from?