General equilibrium analysis of the Eaton-Kortum model of international trade

Fernando Alvarez, Robert E. Lucas, Jr.

Peter Karadi

12/04/2007

Outline of the paper

- Take the Eaton-Kortum (2002) model
  - a static, multi-country general equilibrium model of
  - technology, geography and international trade,
  - with perfect competition, and constant-returns-to-scale technology,
  - estimated for quantitative analysis.

Outline of the paper

- Take the Eaton-Kortum (2002) model
  - a static, multi-country general equilibrium model of
  - technology, geography and international trade,
  - with perfect competition, and constant-returns-to-scale technology,
  - estimated for quantitative analysis.
- Restate it to show that it is fairly standard
  - Standard existence proof
  - Uniqueness proof using the gross complementarity property

Outline of the paper

- Take the Eaton-Kortum (2002) model
  - a static, multi-country general equilibrium model of
  - technology, geography and international trade,
  - with perfect competition, and constant-returns-to-scale technology,
  - estimated for quantitative analysis.
- Restate it to show that it is fairly standard
  - Standard existence proof
  - Uniqueness proof using the gross complementarity property
- Some improvements
  - Add a final good producing sector (services): using relative tradable/non-tradable prices for calibration
  - Close it by assuming balanced trade (closer to an equilibrium model)
  - Solution for optimal tariff

Motivation

- Two competing trade models
  - Dornbusch, Fischer, Samuelson, 1977; Eaton-Kortum, 2002
  - Krugman, 1980; Melitz, 2003

Motivation

- Two competing trade models
  - Dornbusch, Fischer, Samuelson, 1977; Eaton-Kortum, 2002
  - Krugman, 1980; Melitz, 2003
- Similarities
  - Both with heterogenous firms and nice aggregation results
  - Both explain intra-industry trade across countries
  - Both explain effects of geography (EK stronger)
  - Lower trade barriers (special macro shock)
    - higher welfare through increased competition
    - increased TFP through reallocation towards more productive firms

Motivation

- Two competing trade models
  - Dornbusch, Fischer, Samuelson, 1977; Eaton-Kortum, 2002
  - Krugman, 1980; Melitz, 2003
- Similarities
  - Both with heterogenous firms and nice aggregation results
  - Both explain intra-industry trade across countries
  - Both explain effects of geography (EK stronger)
  - Lower trade barriers (special macro shock)
    - higher welfare through increased competition
    - increased TFP through reallocation towards more productive firms
- Main differences
  - KM model: monopolistic competition and fixed costs
  - EK model: perfect competition and iceberg costs

Motivation

- Two competing trade models
  - Dornbusch, Fischer, Samuelson, 1977; Eaton-Kortum, 2002
  - Krugman, 1980; Melitz, 2003
- Similarities
  - Both with heterogenous firms and nice aggregation results
  - Both explain intra-industry trade across countries
  - Both explain effects of geography (EK stronger)
  - Lower trade barriers (special macro shock)
    - higher welfare through increased competition
    - increased TFP through reallocation towards more productive firms
- Main differences
  - KM model: monopolistic competition and fixed costs
  - EK model: perfect competition and iceberg costs
- The paper seems to take sides.

The production

- Non-traded final good, using labor s, and intermediates q

$$c = s_f^{\alpha} q_f^{1-\alpha} \tag{1}$$

The production

- Non-traded final good, using labor s, and intermediates q

$$c = s_f^{\alpha} q_f^{1-\alpha} \tag{1}$$

- Continuum of traded intermediate goods with 'costs' (inverse TFP) $x \in [0, \infty)$, and cost distribution $\phi(x)$

The production

- Non-traded final good, using labor s, and intermediates q

$$c = s_f^{\alpha} q_f^{1-\alpha} \tag{1}$$

- Continuum of traded intermediate goods with 'costs' (inverse TFP) $x \in [0, \infty)$, and cost distribution $\phi(x)$
- Aggregated as CES

$$q = \left[\int_0^{\infty} q(x)^{(\eta-1)/\eta} \phi(x) dx \right]^{\eta/(\eta-1)} \tag{2}$$

The production

- Non-traded final good, using labor s, and intermediates q

$$c = s_f^{\alpha} q_f^{1-\alpha} \tag{1}$$

- Continuum of traded intermediate goods with 'costs' (inverse TFP) $x \in [0, \infty)$, and cost distribution $\phi(x)$
- Aggregated as CES

$$q = \left[\int_0^{\infty} q(x)^{(\eta-1)/\eta} \phi(x) dx \right]^{\eta/(\eta-1)} \tag{2}$$

- Intermediate goods production:

$$q(x) = x^{-\theta} s(x)^{\beta} q_m(x)^{1-\beta}, \tag{3}$$

The production

- Non-traded final good, using labor s, and intermediates q

$$c = s_f^{\alpha} q_f^{1-\alpha} \tag{1}$$

- Continuum of traded intermediate goods with 'costs' (inverse TFP) $x \in [0, \infty)$, and cost distribution $\phi(x)$
- Aggregated as CES

$$q = \left[\int_0^{\infty} q(x)^{(\eta-1)/\eta} \phi(x) dx \right]^{\eta/(\eta-1)} \tag{2}$$

- Intermediate goods production:

$$q(x) = x^{-\theta} s(x)^{\beta} q_m(x)^{1-\beta}, \tag{3}$$

- Endowment constraints

$$s_f + \int_0^{\infty} s(x)\phi(x)dx \leq 1; \quad q_f + \int_0^{\infty} q_m(x)\phi(x)dx \leq q \tag{4}$$

Probabilistic formulation

- Cost distribution: $\Phi^c(x) = \exp(-\lambda x)$

Probabilistic formulation

- Cost distribution: $\Phi^c(x) = \exp(-\lambda x)$
- Note: TFP parameter: $y = x^{-\theta}$ has a Fréchet distribution (type II extreme value distribution) $\Theta(y) = \exp(-\lambda y^{-1/\theta})$

Probabilistic formulation

- Cost distribution: $\Phi^c(x) = \exp(-\lambda x)$
- Note: TFP parameter: $y = x^{-\theta}$ has a Fréchet distribution (type II extreme value distribution) $\Theta(y) = \exp(-\lambda y^{-1/\theta})$
- Originally in EK: they start out by assuming Fréchet for the TFP terms
  - Advantage: some justification:
    - Innovations x arrive as a Poisson process, and
    - individual innovations have Pareto distribution $\Psi^c = (q/\bar{q})^{-1/\theta}$, then the
    - best ideas will have a Fréchet distribution with λ being the stock of knowledge
  - Disadvantage: distribution less known:
    - This paper uses an equivalent formulation
    - Makes it more accessible by using the exponential.

Multi-country setup

- n countries

Multi-country setup

- n countries
- Consumption: c_i, $i = 1 \dots n$ final goods, not traded

Multi-country setup

- n countries
- Consumption: c_i, $i = 1 \ldots n$ final goods, not traded
- Production:
  - Labor endowments: $L = (L_1, \ldots, L_n)$, not mobile
  - Technology parameters $\lambda = (\lambda_1, \ldots, \lambda_n)$
  - Same parameters: θ, β, α, η

Multi-country setup

- n countries
- Consumption: c_i, $i = 1 \dots n$ final goods, not traded
- Production:
  - Labor endowments: $L = (L_1, \dots, L_n)$, not mobile
  - Technology parameters $\lambda = (\lambda_1, \dots, \lambda_n)$
  - Same parameters: θ, β, α, η
- Variable trade costs:
  - Iceberg transportation costs: κ_{ij} amount arrives to i from j (lost)
  - Tariffs: ω_{ij} fraction of dollar gets to i from j (revenue generating)

Multi-country setup

- n countries
- Consumption: c_i, $i = 1 \ldots n$ final goods, not traded
- Production:
  - Labor endowments: $L = (L_1, \ldots, L_n)$, not mobile
  - Technology parameters $\lambda = (\lambda_1, \ldots, \lambda_n)$
  - Same parameters: θ, β, α, η
- Variable trade costs:
  - Iceberg transportation costs: κ_{ij} amount arrives to i from j (lost)
  - Tariffs: ω_{ij} fraction of dollar gets to i from j (revenue generating)
- Commodity space: $x = (x_1, \ldots, x_n)$, independent random draws: $\phi(x) \sim exp\left(\sum_{i=1}^{n} \lambda_i\right)$

Prices

- Take $w = (w_1, \ldots, w_n)$ as given (we are going to solve for this)

Prices

- Take $w = (w_1, \ldots, w_n)$ as given (we are going to solve for this)
- CRS technology, and perfect competition, so cost minimization can be used for prices:
    - Country aggregate c_i

$$p_i = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} w_i^{\alpha} p_{mi}^{(1-\alpha)} \tag{5}$$

Prices

- Take $w = (w_1, \ldots, w_n)$ as given (we are going to solve for this)
- CRS technology, and perfect competition, so cost minimization can be used for prices:
  - Country aggregate c_i

$$p_i = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} w_i^{\alpha} p_{mi}^{(1-\alpha)} \tag{5}$$

  - CES aggregate intermediate good q_i

$$p_{mi}^{1-\eta} = \int_{R_+^n} p_i(x)^{1-\eta} \phi(x) dx = E\left[p_i(x)^{1-\eta}\right] \tag{6}$$

Prices

- Take $w = (w_1, \dots, w_n)$ as given (we are going to solve for this)
- CRS technology, and perfect competition, so cost minimization can be used for prices:
  - Country aggregate c_i

$$p_i = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} w_i^{\alpha} p_{mi}^{(1-\alpha)} \tag{5}$$

  - CES aggregate intermediate good q_i

$$p_{mi}^{1-\eta} = \int_{R_+^n} p_i(x)^{1-\eta}\phi(x)dx = E\left[p_i(x)^{1-\eta}\right] \tag{6}$$

  - Each intermediate good x is bought in i from the lowest cost producer among countries $j = 1, \dots, n$

$$p_i(x) = B \min_j \left[\frac{w_j^{\beta} p_{mj}^{1-\beta}}{\kappa_{ij}\omega_{ij}} x_j^{\theta}\right] \tag{7}$$

where $B = \beta^{-\beta}(1-\beta)^{-(1-\beta)}$.

Prices II.

- Some amazingly useful properties of the exponential distribution:
  - $x \sim \exp(\lambda)$ and $k > 0$, then $kx \sim \exp(\lambda/k)$
  - $x \sim \exp(\lambda)$, $y \sim \exp(\mu)$, and independent, then $z = \min(x, y)$ is $z \sim \exp(\lambda + \mu)$
  - $\Pr\{x \leq y\} = \lambda/(\lambda + \mu)$

Prices II.

- Some amazingly useful properties of the exponential distribution:
  - $x \sim \exp(\lambda)$ and $k > 0$, then $kx \sim \exp(\lambda/k)$
  - $x \sim \exp(\lambda)$, $y \sim \exp(\mu)$, and independent, then $z = \min(x, y)$ is $z \sim \exp(\lambda + \mu)$
  - $\Pr\{x \leq y\} = \lambda/(\lambda + \mu)$
- Using the first two facts, after some algebra, we get that
  - individual intermediates: $p_i(x) \sim \exp(\mu(w))$, where

$$\mu(w) = B^{-1/\theta} \sum_{j=1}^{n} \psi_{ij}(w), \;\; \psi_{ij}(w) = \left(\frac{w_j^{\beta} p_{mj}(w)^{1-\beta}}{\kappa_{ij}\omega_{ij}} \right)^{-1/\theta} \lambda_j \tag{8}$$

Prices II.

- Some amazingly useful properties of the exponential distribution:
  - $x \sim \exp(\lambda)$ and $k > 0$, then $kx \sim \exp(\lambda/k)$
  - $x \sim \exp(\lambda)$, $y \sim \exp(\mu)$, and independent, then $z = \min(x, y)$ is $z \sim \exp(\lambda + \mu)$
  - $\Pr\{x \leq y\} = \lambda/(\lambda + \mu)$
- Using the first two facts, after some algebra, we get that
  - individual intermediates: $p_i(x) \sim \exp(\mu(w))$, where

$$\mu(w) = B^{-1/\theta} \sum_{j=1}^{n} \psi_{ij}(w), \ \ \psi_{ij}(w) = \left(\frac{w_j^{\beta} p_{mj}(w)^{1-\beta}}{\kappa_{ij}\omega_{ij}} \right)^{-1/\theta} \lambda_j \tag{8}$$

  - and the aggregate intermediates: n equations for $p_m = (p_{m1}, \ldots, p_{mn})$ as a function of w

$$p_{mi}(w) = AB \left(\sum_{j=1} n\psi_{ij}(w) \right)^{-\theta}, \quad A = \Gamma(1 + \theta(1 - \eta)) \tag{9}$$

Trade balance

- Let D_{ij} be the probability that a good x in i is the cheapest in j
  - By $LOLN$ it is also the fraction of goods from j
  - and, amazingly, it is the proportion of expenditure spent on goods from j, because
  - the price distribution $G_i(p)$ in country i is the same for all source countries j: higher productivity influences the range of products sold (D_{ij}) not their price distribution (differently from the Melitz model with fixed markups).

Trade balance

- Let D_{ij} be the probability that a good x in i is the cheapest in j
  - By $LOLN$ it is also the fraction of goods from j
  - and, amazingly, it is the proportion of expenditure spent on goods from j, because
  - the price distribution $G_i(p)$ in country i is the same for all source countries j: higher productivity influences the range of products sold (D_{ij}) not their price distribution (differently from the Melitz model with fixed markups).
- Using the 3. property of the exponential distribution, we can easily get that

$$D_{ij} = \frac{\psi_{ij}}{\sum_{k=1}^{n} \psi_{ik}} \tag{10}$$

Trade balance

- Let D_{ij} be the probability that a good x in i is the cheapest in j
  - By *LOLN* it is also the fraction of goods from j
  - and, amazingly, it is the proportion of expenditure spent on goods from j, because
  - the price distribution $G_i(p)$ in country i is the same for all source countries j: higher productivity influences the range of products sold (D_{ij}) not their price distribution (differently from the Melitz model with fixed markups).
- Using the 3. property of the exponential distribution, we can easily get that

$$D_{ij} = \frac{\psi_{ij}}{\sum_{k=1}^{n} \psi_{ik}} \tag{10}$$

- Trade balance

$$L_i p_{mi} q_i \sum_{j=1}^{n} D_{ij} \omega_{ij} = \sum_{j=1}^{n} L_j p_{mj} q_j D_{ji} \omega_{ji}. \tag{11}$$

Equilibrium wages

- Finding the equilibrium wages (w) reduces to finding the zeros of the following excess labor demand system:

$$Z_i(w) = \frac{1}{w_i}\left[\sum_{j=1}^{n} L_j \frac{w_j(1-s_{fj}(w))}{F_j(w)} D_{ji}(w)\omega_{ji} - L_i w_i (1-s_{fi}(w))\right] \tag{12}$$

where F_i is the country's spending on tradeables that reaches producers

$$F_i = \sum_{j=1}^{n} D_{ij}\omega_{ij}, \tag{13}$$

and s_{fi} is the labor's share in the production of the final goods

$$s_{fi} = \frac{\alpha\,[1-(1-\beta)F_i]}{(1-\alpha)\beta F_i + \alpha[1-(1-\beta)F_i]}. \tag{14}$$

Existence and Uniqueness

- Theorem 1: for any $w \in \mathbf{R}^n_{++}$ there is a unique $p_m(w)$, which is for each $p_{mi}(w)$ is
  - strictly increasing in w,
  - strictly decreasing in κ_{ij}, ω_{ij}

Existence and Uniqueness

- Theorem 1: for any $w \in \mathbf{R}^n_{++}$ there is a unique $p_m(w)$, which is for each $p_{mi}(w)$ is
  - strictly increasing in w,
  - strictly decreasing in κ_{ij}, ω_{ij}
- Theorem 2: There is a $w \in \mathbf{R}^n_{++}$ s.t. $Z(w) = 0$, because $Z(w)$ satisfies all the standard properties of an excess demand function

Existence and Uniqueness

- Theorem 1: for any $w \in \mathbf{R}^n_{++}$ there is a unique $p_m(w)$, which is for each $p_{mi}(w)$ is
  - strictly increasing in w,
  - strictly decreasing in κ_{ij}, ω_{ij}
- Theorem 2: There is a $w \in \mathbf{R}^n_{++}$ s.t. $Z(w) = 0$, because $Z(w)$ satisfies all the standard properties of an excess demand function
- Theorem 3: $\omega_{ij} = \omega_i$ and $\alpha \geq \beta$, and trade costs not too large, then there is exactly one solution to $Z(w) = 0$ with $\sum_{i=1}^{n} = 1$, because Z has the gross substitute property

$$\frac{\partial Z_i(w)}{\partial w_k} > 0 \tag{15}$$

Algorithm

- Discrete analogue of the tatonnement process

$$\Delta_w = \left\{ w \in \mathbf{R}_+^n : \sum_{i=1}^{n} w_i L_i = 1 \right\} \tag{16}$$

and mapping $T : \Delta_w \to \Delta_w$,

$$T(w)_i = w_i(1 + \nu Z_i(w)/L_i), i = 1, \ldots, n, \quad \nu \in (0, 1], \tag{17}$$

which is the percentage increase in country i's wage in proportion to a scaled version of its excess demand.

An easy example

- Zero gravity: $\kappa_{ij} = \omega_{ij} = 1$
  - GDP; higher dispersion ⇒ higher TFP

$$L_i w_i = \lambda_i^{\theta/(\beta+\theta)} L_i^{\beta/(\beta+\theta)} \tag{18}$$

  - Relative prices; higher productivity ⇒ higher relative price of non-tradables

$$\frac{p_i}{p_m} = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} \left(\frac{\lambda_i}{L_i}\right)^{\alpha\theta/(\beta+\theta)} p_m^{-\alpha} \tag{19}$$

An easy example

- Zero gravity: $\kappa_{ij} = \omega_{ij} = 1$
  - GDP; higher dispersion ⇒ higher TFP

$$L_i w_i = \lambda_i^{\theta/(\beta+\theta)} L_i^{\beta/(\beta+\theta)} \tag{18}$$

  - Relative prices; higher productivity ⇒ higher relative price of non-tradables

$$\frac{p_i}{p_m} = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} \left(\frac{\lambda_i}{L_i}\right)^{\alpha\theta/(\beta+\theta)} p_m^{-\alpha} \tag{19}$$

- Welfare gain to costless trade from autarky $(\kappa_{ij} = \omega_{ij} = 0)$

$$\log\left(\frac{c_{i0}}{c_i}\right) = (1-\alpha)\frac{\theta}{\beta}\log\left(\frac{\sum_{j=1}^{n} w_j L_j}{w_i L_i}\right). \tag{20}$$

the smaller the country, the more it gains, because size matters through cost advantage at home.

Some results

- Calibration
  - $\alpha = 0.75$
  - $\beta = 0.5$
  - $\theta = \{0.1, 0.15, 0.25\}$
  - $\kappa = 0.75$ (κ_{ij} using distance)
  - $[\omega_{ij}]$ from tariff data
  - $[L_i]$ and ($[\lambda_i]$) are estimated from using GDP and relative tradable/non-tradable prices

Some results

- Calibration
  - $\alpha = 0.75$
  - $\beta = 0.5$
  - $\theta = \{0.1, 0.15, 0.25\}$
  - $\kappa = 0.75$ (κ_{ij} using distance)
  - $[\omega_{ij}]$ from tariff data
  - $[L_i]$ and ($[\lambda_i]$) are estimated from using GDP and relative tradable/non-tradable prices
- Explaining trade volumes: 0.69 correlation

Some results

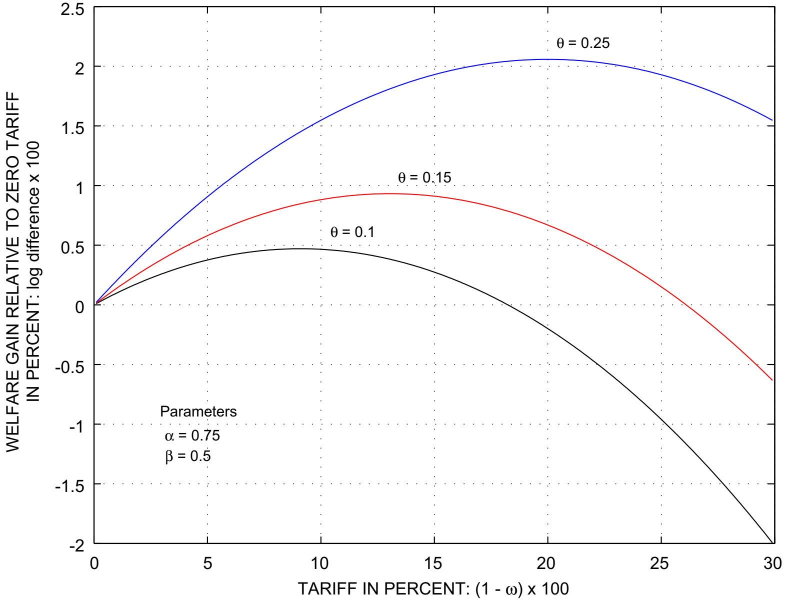
- Calibration
  - $\alpha = 0.75$
  - $\beta = 0.5$
  - $\theta = \{0.1, 0.15, 0.25\}$
  - $\kappa = 0.75$ (κ_{ij} using distance)
  - $[\omega_{ij}]$ from tariff data
  - $[L_i]$ and ($[\lambda_i]$) are estimated from using GDP and relative tradable/non-tradable prices
- Explaining trade volumes: 0.69 correlation
- Optimal tariff Tariff
  - Small, open limit economy: $L_1^r \to 0$ with $\lambda_1^r / L_1^r = k > 0$
  - Welfare maximizing best response ω to a given $\hat{\omega}$
  - Optimal $\omega^* > 0$, increasing in θ (elasticity of export demand)
  - Reason: monopoly pricing by the government

Conclusion

- Companion paper to Eaton-Kortum, 2002
  - While EK works hard with empirics using the bilateral international manufacturing trade data to explain effects of geography and technology on trade and prices
  - This paper works hard to make the theory stronger and more accessible

Conclusion

- Companion paper to Eaton-Kortum, 2002
  - While EK works hard with empirics using the bilateral international manufacturing trade data to explain effects of geography and technology on trade and prices
  - This paper works hard to make the theory stronger and more accessible
- Significant value-added
  - Presented the model in a standard form
  - Closed it more elegantly
  - Proved existence and uniqueness
  - Calculated optimal tariffs in a small economy



θ = 0.25
θ = 0.15
θ = 0.1
Parameters
α = 0.75
β = 0.5
WELFARE GAIN RELATIVE TO ZERO TARIFF
IN PERCENT: log difference x 100
TARIFF IN PERCENT: (1 - ω) x 100
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
5
10
15
20
25
30