The time-series properties of aggregate consumption: implications for the costs of fluctuations

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The Contribution of this Paper

- To measure the costs of *consumption fluctuations* by an accurate statistical model (endowment economy).
- To measure the costs of *income fluctuations* by an optimal consumption model (production economy).
- To adopt *analytical approximations* to recover the costs in both cases.

The main results

- In an endowment economy in which *consumption is highly persistent* the welfare costs of fluctuations are between $.5\%$ and 5% (even if RRA$= 5$).
- In an economy in which storage is possible, *precautionary saving* plays a very important role.
- When precautionary saving is possible, the impact of fluctuations on the welfare costs depends on *technology* .

Preferences and Welfare Costs λ

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$$C_t : \mu = E[\Delta c_t]$$

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Using $r = \delta + \frac{1}{eis}\mu = \delta + \gamma\mu$:

$$\log(1+\lambda) = \begin{cases} .5(1-e^{\mu-r})\sum_{t=0}^{\infty} e^{(\mu-r)t} V_0(c_t) & \text{if } \gamma = 1 \\ \frac{1}{\gamma-1}\log\left[(1-e^{\mu-r})\sum_{t=0}^{\infty} e^{(\mu-r)t} e^{.5\gamma(\gamma-1)V_0(c_t)}\right] & \text{if } \gamma \neq 1 \end{cases}$$

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1) Calibrate γ, μ, r

2) Model Δc_t and so $V_0(c_t)$.

Calibration

- $\gamma \in \{1, 3, 5\}$: focusing on consumption data!
- $r \in [3.2\ 5.2]$ (annual, after tax);
- $\mu = 2.2$ (annual);

Trend Stationary Log-Consumption

- Assume:

$$c_t = c_0 + \mu t + \eta c_{t-1} + \sigma \epsilon_t$$

- Then:

$$\begin{aligned} V_o(c_t) &= \sigma^2 \sum_{j=1}^{t} \eta^j \\ \log(1+\lambda) &= .5\gamma \frac{\sigma^2}{(r-\mu)+(1-\eta^2)} \end{aligned}$$

Three different points of view

- B. Lucas: $\eta = 0$
- OLS: $\eta = .92$
- B. Hall: $\eta = 1$

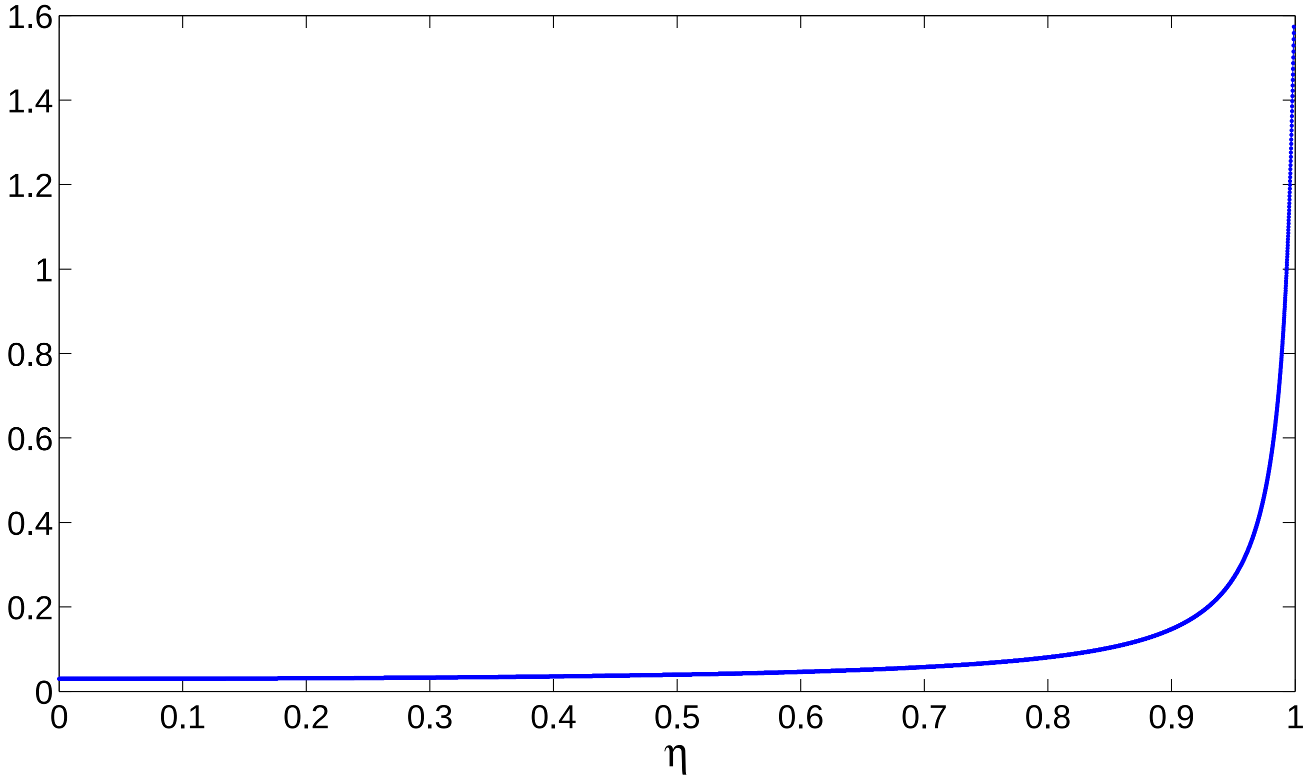
Lucas and OLS

Table 1 – Estimates of the costs of fluctuations in three simple models

Panel A: The Lucas statistical model			
	$\gamma = 1$	$\gamma = 3$	$\gamma = 5$
	0.04%	0.12%	0.20%
	(\$9)	(\$28)	(\$46)
Panel B: The AR(1) statistical model estimated by least squares			
	$\gamma = 1$	$\gamma = 3$	$\gamma = 5$
$r - g = 0.03$	0.03%	0.10%	0.17%
	(\$8)	(\$24)	(\$40)
$r - g = 0.02$	0.04%	0.11%	0.18%
	(\$8)	(\$25)	(\$43)
$r - g = 0.01$	0.04%	0.12%	0.19%
	(\$9)	(\$27)	(\$45)

Lucas and OLS: why so similar?

$$100\log(1+\lambda) = 50\gamma \frac{\sigma^2}{(r-\mu)+(1-\eta^2)}$$



$$\gamma = 5; r-\mu = 3\%; \sigma = .011$$

Close to B. Hall: local-to-unity model

- When $\eta \approx 1$:
  - the OLS estimator of η is downward biased;
  - the OLS estimator of the cost is not consistent;
  - the OLS estimator of the costs converges asymptotically to a random variable.

In the spirit of B. Hall and RLR

Table 4 – Estimates of the costs of fluctuations from ARMA models

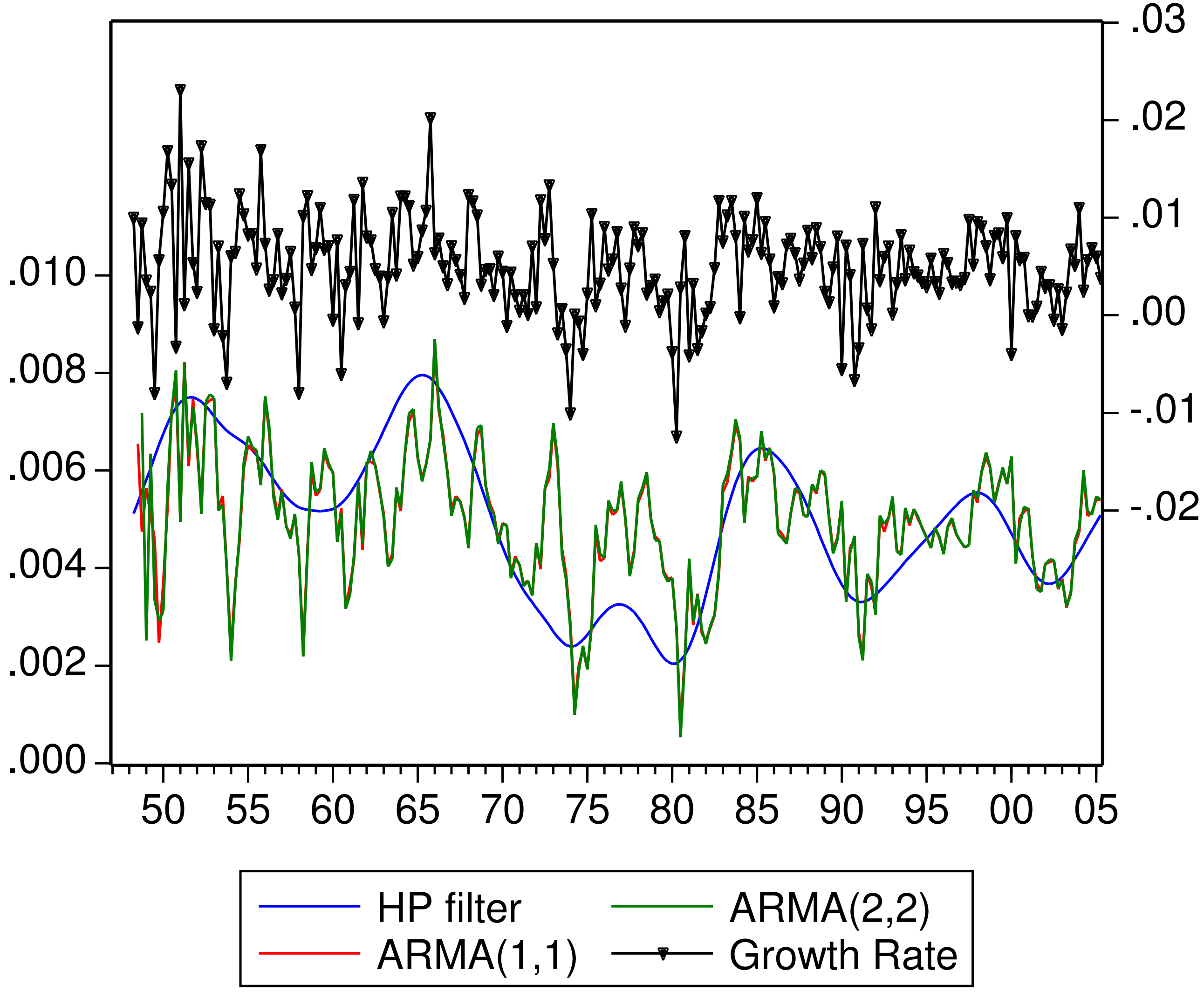
Panel A: Estimated ARMA (2,2) model

$$(1 - 0.66L - 0.32L^2)\Delta c_t = (1 + 1.03L + 0.56L^2)u_t, \quad \sigma_u = 0.011$$

	$\gamma = 1$	$\gamma = 3$	$\gamma = 5$
$r - g = 0.03$	0.31%	0.94%	1.60%
	(\$72)	(\$219)	(\$375)
$r - g = 0.02$	0.47%	1.43%	2.47%
	(\$109)	(\$334)	(\$579)
$r - g = 0.01$	0.94%	2.93%	5.33%
	(\$220)	(\$687)	(\$1248)

Panel B: Estimated ARMA (1,0) model

About RLR in Consumption Growth



Income fluctuations in "AK" models

$$\max_{\{C_t\}} \left\{ E_0 \left[\sum_{t=0}^{\infty} e^{-\delta t} \left(\frac{(C_t)^{1-\gamma} - 1}{1-\gamma} \right) \right] \right\}$$

s.t.: $$\begin{aligned} R_t K_t &= K_{t+1} + C_t \\ r_t &\sim N(r - .5\sigma^2, \sigma^2) \end{aligned}$$

The optimal consumption is $C_t^* = \pi R_t K_t$, so one can derive:

$$\begin{aligned} \Delta c_t &= \mu + \epsilon_t \\ \mu &= (r-\delta)/\gamma + .5\gamma\sigma^2 - \sigma^2 \\ C_0 &= (1 - e^{\mu - r}) R_0 K_0 \end{aligned}$$

Income fluctuations in "AK" models

Panel C: The random walk economic model

	$\gamma = 1$	$\gamma = 3$	$\gamma = 5$
$r - g = 0.03$	0.21%	0.62%	1.03%
	(\$48)	(\$145)	(\$242)
$r - g = 0.02$	0.31%	0.94%	1.56%
	(\$73)	(\$219)	(\$365)
$r - g = 0.01$	0.63%	1.88%	3.14%
	(\$147)	(\$441)	(\$735)

Each cell shows the per capita costs of fluctuations as a fraction of consumption and, in brackets, in 2003 dollars. The standard deviation of shocks is 0.028, 0.011, and 0.011, for panels A to C respectively.

Income fluctuations with D.M.R.C.

$$\max_{\{C_t\}} \left\{ E_0 \left[\sum_{t=0}^{\infty} e^{-\delta t} \left(\frac{(C_t)^{1-\gamma} - 1}{1-\gamma} \right) \right] \right\}$$

$$\begin{aligned} \text{s.t.: } K_{t+1} + C_t &= A_t^{1-\alpha} K_t^{\alpha} + (1-\delta) K_t \\ a_{t+1} &= a_t + \mu_a + \sigma_a \epsilon_{a,t+1} \end{aligned}$$

An analytical approximation of the problem of the agent when $\gamma = 1$ implies:

$$\log(1+\lambda) \approx .5\sigma_a^2 \left[\frac{1}{1 - e^{\mu_c - r}} - (1 - e^{\mu_c - r})(V_{aa} - V_a) \right]$$

Income fluctuations with D.M.R.C.

Panel C: Non-stationary productivity

	$\alpha = 0.36$	$\alpha = 0.75$
$r - g = 0.03$	0.55% (\$129)	0.26% (\$60)
$r - g = 0.02$	0.85% (\$199)	0.39% (\$92)
$r - g = 0.01$	1.76% (\$412)	0.81% (\$190)

Each cell shows the per capita costs of fluctuations as a fraction of consumption and, in brackets, in 2003 dollars.

Something "disturbing"

- The approximation $r \approx \delta + \frac{1}{eis}\mu$ neglects the risk-premium;
- A natural return to look at would be the r^f.
- His computations neglect the interest rate puzzle: $\delta = r^f - \frac{1}{eis}\mu > 0$.
- In Tallarini, the cost of uncertainty is $.5\gamma\sigma^2\frac{\delta}{1-\delta}$!!!!!