Escaping Nash Inflation / Shocks and Government Beliefs

Sargent, Williams, Cho and Zha

Motivation

- Imagine that the monetary authority doesn't know the true data generating process in the economy
- Forms expectations about the evolution of the economy according to a backward looking Phillips curve
- Estimates the parameters of the Phillips curve from observed data
- Can such a system explain the evolution of inflation over the recent past?

Setup [Cho, Williams and Sargent 2002]

- Non-Monetary Authority part of the economy evolves along the lines of Kidland and Prescott [1977]:
  - Natural Rate Phillips Curve

$$U_n = u - \theta(\pi_n - \bar{x}_n) + \sigma_1 W_{1n}$$

  - MA sets inflation up to a random component

$$\pi_n = x_n + \sigma_2 W_{2n}$$

  - Agents have rational expectations
  - Monetary authorities care about inflation and unemployment

$$V(\{U_n, \pi_n\}_{n=0}^{\infty}) = \sum_{n=0}^{\infty} \delta^n (U_n^2 + \pi_n^2)$$

Monetary Authority's View of the World

- Believe that the world evolves according to a backward looking Phillips Curve

$$U_n = \gamma_1 \pi_n + \gamma'_{-1} X_{n-1} + \eta_n$$
$$X_{n-1} = [U_{n-1}\ U_{n-2}\ \pi_{n-1}\ \pi_{n-2}\ 1]$$

- For any set of parameters γ, the MA will maximise objective function subject to this belief about the evolution of the economy. This best response is characterised by the linear equation $h(\gamma)' X_{t-1}$
- For beliefs $\gamma_1 = -\theta$, $\gamma_{-1} = [0\ 0\ 0\ 0\ u]$, MA will set inflation $x_n = \theta u$. This is the outcome one gets in KP if MA cannot commit (Nash Inflation)
- For beliefs $\gamma_1 = 0$, $\gamma_{-1} = [0\ 0\ 0\ 0\ u^*]$, MA will set inflation $x_n = 0$. The is outcome one gets in KP if MA can commit (Ramsay Inflation)

Self Confirming Equilibrium

- Equilibrium concept in which beliefs of MA are consistent with the DGP process generated by those beliefs.
- Definition: A *Self Confirming Equilibrium* is a set of parameters γ such that

$$E[g(\gamma,\xi_n)] = E[\eta_n \begin{bmatrix} \pi_n \\ X_{n-1} \end{bmatrix}] = 0$$

- Let $\xi_n = [W_{1n}\ W_{2n}\ X'_{n-1}]$, which is dependant on γ and the history of shocks W_1^n and W_2^n
- The equilibrium condition is the orthogonality condition which makes γ the least squares regressors of the equation which describes the beliefs of the MA.
- Unique self confirming equilibrium of our system is the Nash Inflation outcome

A Rule for Generating Beliefs

- Up until now, we have not discussed how γ are calculated by the MA. We now assume that they are estimated in real time using the following recursive algorithm

$$\begin{aligned} \gamma_{n+1} &= \gamma_n + \varepsilon R_n^{-1} g(\gamma_n, \xi_n) \\ R_{n+1} &= R_n + \varepsilon (M_n - R_n) \end{aligned}$$

where $M_n = \begin{bmatrix} \pi_n \\ X_{n-1} \end{bmatrix} \begin{bmatrix} \pi_n \\ X_{n-1} \end{bmatrix}'$

- Known as a 'fixed gain' algorithm. Doesn't quite approximate least squares estimation, as puts more weight on more recent events. In scalar case, estimate would be

$$\bar{\beta} = \frac{\sum_{i=0}^{n} (1-\varepsilon)^i x_{n-i} y_{n-i}}{\sum_{i=0}^{n} (1-\varepsilon)^i x_{n-i}^2}$$

Mean Dynamics

- The above process converges to the following Ordinary Differential Equation as $\varepsilon \to 0$

$$\begin{aligned} \dot{\gamma} &= R^{-1} E g\left(\gamma, \xi\right) \\ \dot{R} &= E(M_n) - R \end{aligned}$$

- These equations are defined in terms of continuous time $t = n\varepsilon$
- This means that, as $\varepsilon \to 0$ and $n \to \infty$, the trajectory of the parameter estimates $\gamma_n,\ R_n$ converges weakly to the trajectory of the above ODEs
- The unique stable point of the ODE is the self confirming equilibrium of the system
- Implies that, as $\varepsilon \to 0$ and $n \to \infty$ with high probability the economy will be near the Nash inflation outcome

Escape Dynamics

- For a given ε, the economy will experience repeated 'escapes' from the neighborhood of the Nash inflation outcome.
- For some neighborhood G of $\bar{\gamma}$, an escape path is a sequence of parameter estimates $\{\gamma_n\}_{n=0}^{\infty}$ such that

$$\begin{aligned} \gamma_0 &\in G \\ \gamma_n &\notin G \quad \text{some } 0 < n < \infty \end{aligned}$$

- A *Dominant Escape Path* ϕ is one which is minimal across exit paths in exit time from G, and for which, as $\varepsilon \to 0$, all exit paths converge to a neighborhood of ϕ.
- It turns out that, for the above system the dominant escape path takes the economy through the region Ramsay inflation

An Estimated Version of the Model

[Sargent, Williams and Zha 2005]

- Economy similar to that of Cho, Williams, Sargent [2002]
- The MA assumes that parameters 'drift' over time. Estimates of $\bar{\alpha}_{t|t-1}$ based on the following model

$$\begin{aligned} u_t &= \alpha_t' \Phi_t + \sigma w_t \\ \alpha_t &= \alpha_{t-1} + \Lambda_t \end{aligned}$$

- Offers an explanation as to why MA continues to put weight on new information - parameter drift means that old observations carry less and less information about current state. Given MA's beliefs, optimal estimation of $\bar{\alpha}'_{t|t-1}$ is by Kalman Filter
- Estimate using MCMC method

Results

- Model fits inflation data extremely well (better than BVAR models) if V is estimated in sample, and jointly as part of the above problem
- MA beliefs show that inflation cutting is believed to be very costly in the 1970's and much less so in the 1980's
- Nash inflation close to π^*, implying that escape dynamics will have little effect on inflation
- Driving force of inflation history is not oscillation between mean dynamics pushing towards Nash inflation and escape dynamics pushing toward Ramsay inflation