Stationary Equilibria in Asset-Pricing Models With Incomplete Markets and Collateral

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Setup

- Exo. shock $y \in \{1, ..., Y\}$ with transition matrix π
- History nodes: $\sigma = (\sigma^* y)$
- H agents and single perishable consumption good

Endowments, Tech and Pref

- Endowment process $e^h(\sigma^* y) = e^h(y)$
- Set of Lucas trees, A. Shares of each tree, $\theta_a^h(\sigma) \geq 0$, are held by each agent.
- A tree gives end-of-period dividend $\theta_a^h(\sigma) \cdot d_a^h(y)$
- $U_h(c) = E[\sum_{t=0}^{\infty} \beta_h^t u_h(c_t, y_t)]$ and are never satiated at after any shock, y.

Markets

- Trading in each period at beginning of period.
- $q_a(\sigma) =$ price of tree a at node σ.
- J financial assets: one period securities with payoff $b_j(\sigma y) = b_j(y) > 0$.
- agent h's portfolio: ϕ^h.
- $p_j(\sigma) =$ price of security j.

Collateral

- h can short sell only with collateral.
- Def: $k^j(\sigma) > 0$ is A-dimensional vector of tree holdings necessary to sell one unit of j short.
- Cannot double count collateral.
- $k^j_a(\sigma) = \overline{k}^j_a(p(\sigma), q(\sigma), \{\theta^h(\sigma), c^h(\sigma)\}_{h\in H}, y) =$ space of fn cont on all var but y such that for all $j, y, q, c, \theta, \quad \inf_{p\geq 0} \overline{k}^j_a > 0$ for some $a \in A$.

Default

- Payoff at node $\sigma = (\sigma^* y)$:

$$f_j(\sigma) = \min\{b_j(y), \sum_{a \in A} k_a^j(\sigma^*) \frac{q_a(\sigma)}{q_a(\sigma^*)}\}$$

- can default on just one security
- no reputation effects
- can allow for endogenous margin requirements by introducing securities with same payoff but with different k^j. In eq., only some securities will be traded.

Financial Markets Equilibrium

Def: Financial Markets Equilibrium for an economy with initial shock y_0 is a collection $\{\{\theta^h(\sigma), \phi^h(\sigma), c^h(\sigma)\}_{h \in H}, \{q_a(\sigma)\}_{a \in A}, \{p_j(\sigma)\}_{j \in J}\}_{\sigma \in \Sigma}$ such that:

$$\sum_{h \in H} \theta^h(\sigma) = 1 \quad \text{and} \quad \sum_{h \in H} \phi^h(\sigma) = 0 \quad \text{for all} \quad \sigma \in \Sigma$$

Financial Markets Equilibrium

Def: Financial Markets Equilibrium for an economy with initial shock y_0 is a collection $\{\{\theta^h(\sigma), \phi^h(\sigma), c^h(\sigma)\}_{h\in H}, \{q_a(\sigma)\}_{a\in A}, \{p_j(\sigma)\}_{j\in J}\}_{\sigma\in\Sigma}$ such that:

And for each agent h:

$$\{\theta^h(\sigma), \phi^h(\sigma), c^h(\sigma)\} \in \arg\max \theta \geq 0, \phi, c \geq 0 U_h(c) \quad \text{s.t.}$$

$$c^h(\sigma) = e^h(y) + \phi^h(\sigma^*) \cdot f(\sigma) +$$

$$\theta^h(\sigma^*) \cdot q(\sigma) + \theta^h(\sigma) \cdot (d^h(y) - q(\sigma)) - \phi^h(\sigma) \cdot p(\sigma),$$

$$q_a \theta_a^h(\sigma) + \sum_{j\in J:\phi_j^h(\sigma)<0} k_a^j \phi_j^h(\sigma) \geq 0 \quad \text{for all } a \in A$$

Markov Characterization

- State space $S = Y \times Z$. $Z = \hat{Z} \times \Delta^{H-1}$. Z is all endo. var consistent with eq.
- Def: Expectations Correspondence: $g : S \Rightarrow Z^Y$. $\{z^y\}_{y \in Y} \in g(s)$ if it solves the Lagrangian for all households.
- Lemma 1: Graph of g is a closed subset of $S \times Z^Y$. Def: Markov Eq. is a policy corr P and a trans. fn. F:

$$P : Y \times \Delta^{H-1} \Rightarrow \hat{Z} \quad \& \quad F : graph(P) \to Z^Y$$

s.t. $\forall x \in graph(P)$ and $\forall y$:

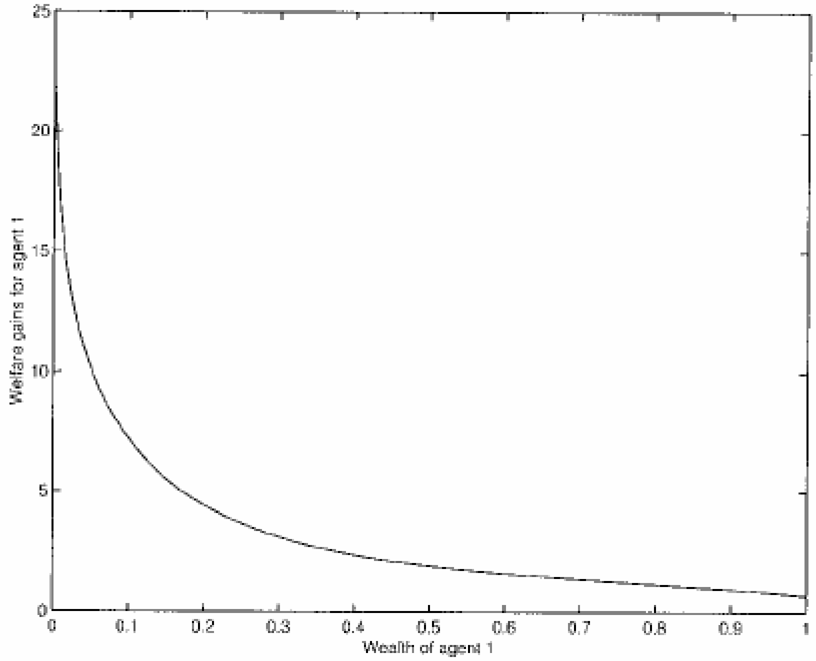
$$F(s) \in g(s) \quad \text{and} \quad (y, F_y(s)) \in graph(P)$$

How to build a Markov Eq.

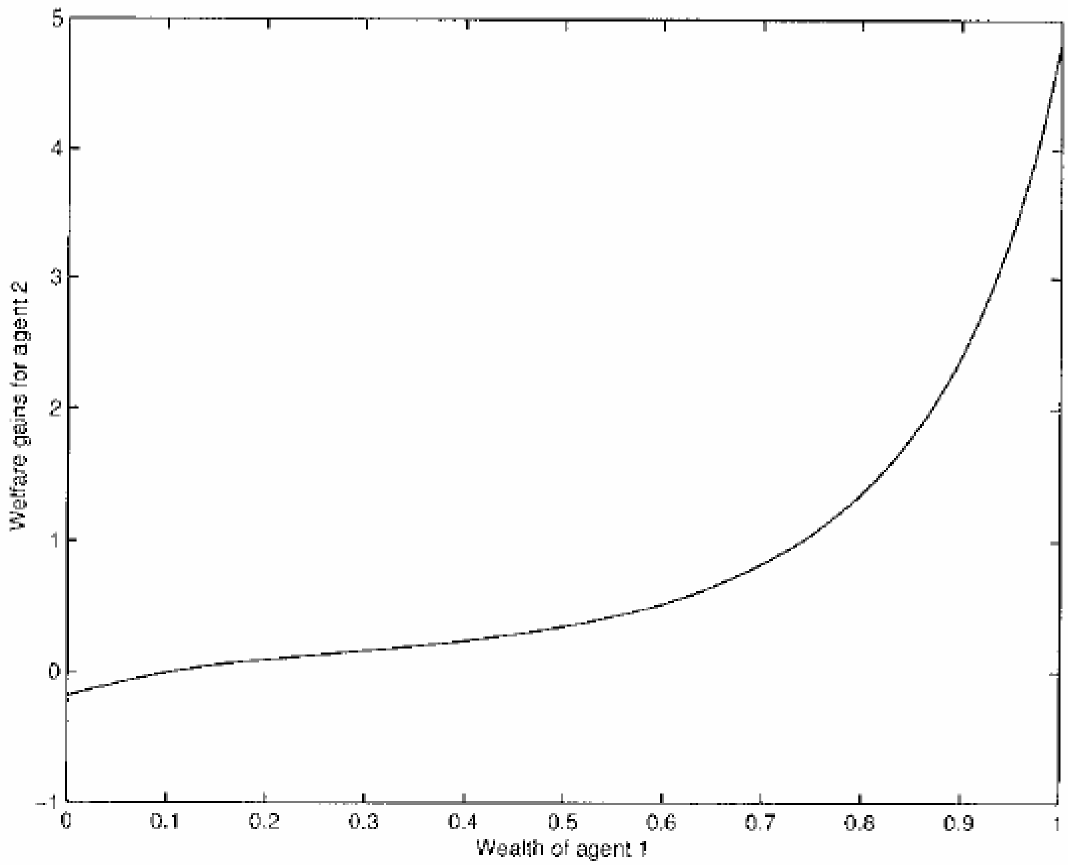
- Take compact set $\tau \subset \hat{Z}$ sufficiently large to contain all possible endogenous eq. values in an eq of a finitely truncated economy.
- Existence of τ is an extension of Geanakopolos and Zame '02.
- Get policy corr by usual iteration
- Problem: don't know general conditions for uniqueness.

Markov Eq. Computation

- Never exact - can be viewed as an eq. to an economy with perturbed endowments and preferences.
- Def: ϵ-eq. as one where all agents' euler-eq hold within ϵ and goods and securities markets clear.
- A simple (P is fn) ϵ-eq always exists, even if simple exact eq. doesn't.
- Problem: don't really know how smooth our transition fn is: don't know how approximate our errors are.



Welfare gains for agent 1
Wealth of agent 1
25
20
15
10
5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



Welfare gains for agent 2
Wealth of agent 1