Speculative Investor Behavior and Learning

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Introduction and Motivation

- Harrison and Kreps (1978) framework without persistently “naive” agents
- Show that small differences in priors can lead to large initial premiums.
- Framework consistent with stylized facts of IPOs.
- Concludes with defense of heterogeneous priors.

Setup

- Risky asset pays a dividend of $1 in each period or not.
- Probability in each period is θ.
- Agents have prior beliefs about θ
- Riskless bond with interest rate r
- Can short bond but not risky asset
- All common knowledge.

Agents

- Finite collection of risk-neutral traders (or types) $J = 1, ..., I$
- Type i's prior beliefs are represented by the density π_i over possible values of $\theta \in [0, 1]$. π_i twice differentiable and uniformly bounded below.
- Agent type i, after observing s dividends paid in t periods, has posterior density over θ:

$$\eta_i(\theta|s,t) = \frac{\theta^s(1-\theta)^{t-s}\pi_i(\theta)}{\int_{\rho=0}^{1} \rho^s(1-\rho)^{t-s}\pi_i(\rho)d\rho}$$

Fundamental Values

- The probability agent i attaches to a dividend next period is:

$$\mu_i(s,t) = \int_{\theta=0}^{1} \theta \eta_i(\theta | s,t) d\theta = \frac{\int_{\theta=0}^{1} \theta^{s+1}(1-\theta)^{t-s}\pi_i(\theta)d\theta}{\int_{\theta=0}^{1} \theta^{s}(1-\theta)^{t-s}\pi_i(\theta)d\theta}$$

- Value to agent with estimate $\mu_i(s,t)$ of buying after t and holding forever the risky asset is $\frac{1}{1+r}\mu_i(s,t) + \frac{1}{(1+r)^2}\mu_i(s,t) + ... = \mu_i(s,t)/r$.
- Ignoring $1/r$, call $\mu_i(s,t)$ her Fundamental Value
- Lemma: For all $\theta_0 \in [0,1]$ and $i \in J$, $\mu_i(\theta_0 t, t) \rightarrow \theta_0$ as $t \rightarrow \infty$

Defintions

- Def: Agent k is a Global Optimist if $\mu_k(s,t) \geq \mu_i(s,t)$ for all $i \in J$ and histories (s,t).
- Def: Agent k is a Local Optimist if there exists a history (s,t) such that $\mu_k(s',t') \geq \mu_i(s',t')$ for all $i \in J$ and histories (s',t') containing (following) (s,t).
- Def: Beliefs $\{\pi_i\}_{i \in J}$ satisfy Perpetual Switching if, for every i and history (s,t), there exists $j \neq i$ and history (s',t') such that $\mu_j(s',t') > \mu_i(s',t')$.
- Trivially, beliefs satisfy perpetual switching iff there is no local optimist.

- Def: Agent k is Rate Dominant if $d/d\theta(ln(\pi_k(\theta))) \geq d/d\theta(ln(\pi_i(\theta)))$ for all $i \in J, \theta \in [0,1]$.

  Same as a Miilgrom's (1981) monotonic likelihood ratio property btw. π_k and π_i, ensuring that agent k has the density that is always increasing at the fastest rate.

- Theorem: The following claims are equivalent: k is rate dominant, k is a global optimist, k is a local optimist.

Stochastic Dominance Example

- Stochastic dominance in priors does not ensure "optimism"
- Two traders: $\pi_1(1/5) = 1/2, \pi_1(4/5) = 1/2$ and $\pi_2(1/5) = 1/2, \pi_2(3/5) = 1/2$.
- history: $(3, 6)$.
- So $\eta_1(4/5|(3,6)) = 1/2$, $\eta_2(3/5|(3,6)) = 27/35$.
  So $\mu_1(3,6) = .5 * 1/5 + .5 * 4/5 = 1/2$ and $\mu_2(3,6) = (27/35)(3/5) + (8/35)(1/5) = 89/175 > 1/2$

Market Prices

- Let $\mu_*(s,t) = \max_{i \in J} \mu_i(s,t)$.
- Equilibrium prices after (s,t) and with interest rate r satisfy:

$$\begin{aligned} P(s,t,r) &= \frac{1}{1+r}[\mu_*(s,t)(1+P(s+1,t+1,r)) \\ &+ (1-\mu_*(s,t))P(s,t+1,r)] \end{aligned}$$

- Normalize price by riskless asset: $p(s,t,r) = rP(s,t,r)$

$$\begin{aligned} p(s,t,r) &= \frac{1}{1+r}[\mu_*(s,t)(r+p(s+1,t+1,r)) \\ &+ (1-\mu_*(s,t))p(s,t+1,r)] \end{aligned}$$

Market Price Iteration

- Can follow Harrison and Kreps iterative calculation method:
- Set $p^0(s,t,r) = 0$ and for all $s \leq t$ and $r > 0$:

$$\begin{aligned} p^{n+1}(s,t,r) &= \frac{1}{1+r}[\mu_*(s,t)(r + p^n(s+1,t+1,r)) \\ &+ (1-\mu_*(s,t))p^n(s,t+1,r)] \end{aligned}$$

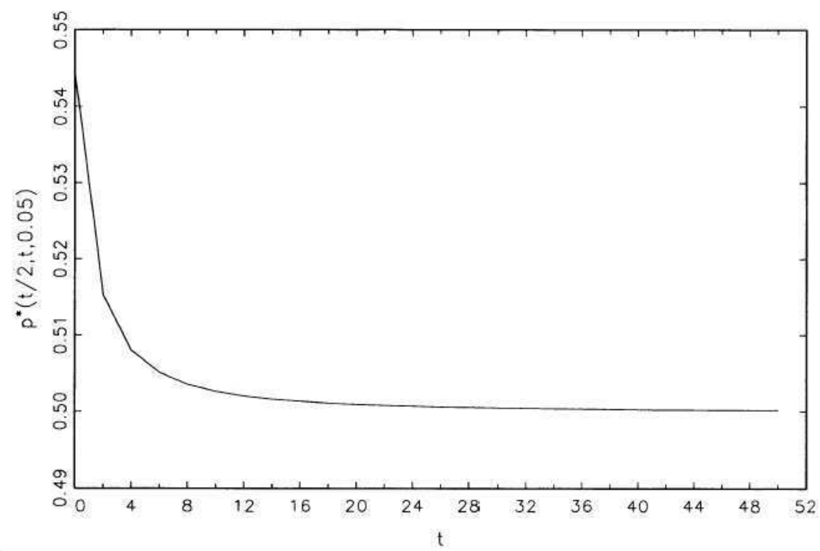
- Note: $p^n(s,t,r)$ bounded above by 1 and non-decreasing in n.
- So: def: $p^*(s,t,r) = \lim_{n\to\infty} p^n(s,t,r)$.

Speculative Premiums

- If k is an optimist, then $p^*(s,t,r) = \mu_k(s,t)$ for all histories and r.
- If there is no optimist, then $p^*(s,t,r) > \mu_i(s,t)$ for all histories, r and i.
  As $t \to \infty$, $p^*(\theta_0 t, t, r) \to \mu_i(\theta_0 t, t) \to \theta_0$
  for all $\theta \in [0,1]$, r and i.
- As $r \to \infty, p^*(s,t,r) \to \mu_*(s,t)$ for all (s,t).
- There are also Ponzi scheme pricing equilibria.

Example

- $\pi_1(\theta) = 1$, $\pi_2(\theta) = 1/\sqrt{\theta(1-\theta)}$.
- So, $\mu_1(s,t) = (s+1)/(t+2)$, $\mu_2(s,t) = (s+1/2)/(t+1)$. Note: by rate comparison, there is no optimist.



Towards an IPO theory

- Learning gives some appealing price dynamics for IPOs without as many of the “winner’s curse” problems as the over-confidence literature (e.g. Hong, Scheinkman and Xiong (2005)).
- Heterogeneity with learning is not as much of an affront to rational expectations.
- Further research: robust control, risk aversion (with wealth distribution).