Idiosyncratic and Aggregate Risk in the Presence of Government's Moral Hazard

Veronica Rappoport

November 27, 2007

Introduction

- Government's fiscal policy plays an important role in insuring Home country agents against idiosyncratic risk as well as in international risk sharing.
- International risk sharing involves foreign agents holding domestic government debt. At the same time taxes are levied exclusively on resident agents. This results in domestic government's moral hazard problem.
- Lack of commitment technology affects the capacity of resident agents to diversify risk optimally.
- How should the institutions be designed so as to overcome the moral hazard problem?

Outline

- Model setup.
- Financial market equilibrium.
- Ex-ante optimal policy.
- Markov perfect equilibrium.
- Equilibrium with reputation.

Model

- Two generations in Home economy: unit mass of young agents $(w_1; c_{1,t})$, and unit mass of old agents $(w_{is,t}; c_{is,t})$.
- Two generations in Foreign economy: 1 young agent $(w_1^*; c_{1,t}^*)$ and 1 old agent $(w_{s,t}^*; c_{s,t}^*)$.
- Endowment of old generations: $w_{is} = w_s + \epsilon_i$ *vs.* w_s^* where $(w_s, w_s^*)' \sim \mathcal{N}\left((\overline{w}, \overline{w}^*)', \begin{bmatrix} \sigma^2 & \sigma_{fd} \\ \sigma_{fd} & \sigma^{*2} \end{bmatrix}\right)$, and $\epsilon_i \sim iid\mathcal{N}\left(0, \sigma_i^2\right)$.
- Agents have CARA preferences: $U_{it} = u\left(c_{1,t}\right) + \beta E u\left(c_{is,t+1}\right)$ where $u\left(c\right) = -\frac{1}{\gamma}e^{-\gamma c}$ where $\beta(1+r) = 1$.
- Budget constraints

$$c_{1,t} + B_{0,t} + B_{d,t}p_{d,t} + B_{f,t}p_{f,t} = \left(1 - \tau_{1,t}\right) w_1$$

$$c_{is,t} = \left(1 - \tau_{2,t}\right) w_{is,t} + B_{0,t-1}\left(1 + r\right) + B_{d,t-1}R_{s,t} + B_{f,t-1}w_{s,t}^*$$

Model Economy II

- The government's policy: $\mathcal{P}_t = \{\tau_{1,t}, \tau_{2,t}, R_{s,t}, a_t\}$.
- The government's budget constraints

$$\begin{aligned} R_{s,t} + a_t &\leq \tau_{2,t} w_{s,t} + \tau_{1,t} w_1 + p_{d,t} + a_{t-1}(1+r) \\ a_t &\geq 0 \end{aligned}$$

Financial Market Equilibrium

Definition For a given policy path $\{\mathcal{P}_t\}_{t=1}^{\infty}$, a Financial Market Equilibrium is an allocation $\{\mathcal{B}_t, \mathcal{B}_t^*\}$, and a price vector $\{p_{d,t}, p_{f,t}\}$ such that:

(i) given prices, $\mathcal{B}_t = \{B_{d,t}, B_{f,t}, B_{0,t}\}$ maximizes $U_{it} = u\left(c_{1,t}\right) + \beta Eu\left(c_{is,t+1}\right)$ subject to Home households' budget constraint;

(ii) given prices, $\mathcal{B}_t^* = \left\{B_{d,t}^*, B_{f,t}^*, B_{0,t}^*\right\}$ maximizes $U_{it} = u\left(c_{1,t}^*\right) + \beta Eu\left(c_{is,t+1}^*\right)$ subject to Foreign households' budget constraint;

(iii) for all t the government's budget constraint is satisfied;

(iiv) markets clear, that is, $B_{d,t} + B_{d,t}^* = 1$, $B_{f,t} + B_{f,t}^* = 1$.

Ex-ante Optimal Policy

The ex-ante optimal policy is a path $\{\mathcal{P}_t\}_{t=1}^{\infty}$ with $\mathcal{P}_t = \{\tau_{1,t}, \tau_{2,t}, R_{s,t}, a_t\}$ that maximizes

$$\max_{\{\mathcal{P}_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^{t-1} \left[u\left(c_{1,t}\right) + \beta E u\left(c_{is,t+1}\right)\right]$$

$$s.t. \qquad R_{s,t} + a_t = \tau_{1,t} w_1 + \tau_{2,t} w_{s,t} + p_{d,t} + a_{t-1}(1+r)$$

$$p_{d,t} = \beta \left[\overline{R}_{t+1} - \frac{\gamma}{2} \tau_{2,t+1} \left(\sigma^2 + \sigma_{fd}\right)\right]$$

$$c_{1,t} = \frac{1}{1+\beta} \left\{ w_1 + \beta \overline{w} - \beta \frac{\gamma}{4} \left(\sigma^2 - \sigma^{*2}\right) - \beta \frac{\gamma}{2} Var\left(c_{is,t+1}\right) + T_t \right\}$$

$$c_{1,t} = E\left(c_{is,t+1}\right)$$

where $Var\left(c_{is,t+1}\right) = \left(1 - \tau_{2,t+1}\right)^2 \sigma_i^2 + \frac{1}{4}\left(\sigma^2 + \sigma^{*2} + 2\sigma_{fd}\right)$

$$T_t = -\tau_{1,t} w_1 - \beta \tau_{2,t+1} \left[\overline{w} - \frac{\gamma}{2}\left(\sigma^2 + \sigma_{fd}\right)\right]$$

Ex-ante Optimal Policy: Results

- The ex-ante optimal policy results in the first-best risk diversification with $\tau_{2,t} = 1$ for all t.
- The utility is constant across generations:

$$u'(c_{1,t}) = Eu'(c_{is,t}) = u'(c_{1,t-1})$$

- Ex-ante optimal policy is time inconsistent.
- Time inconsistency $\rightarrow$ consider *Markov Perfect Equilibria (MPE)*

Markov Perfect Equilibria

Definition A stationary Markov Perfect Equilibrium is a government policy function $\mathcal{P}$, households' policy functions $\mathcal{B}, \mathcal{B}^*$ and price vectors p_d, p_f such that:

(i) $\mathcal{B}, \mathcal{B}^*, p_d, p_f$ is a Financial Market Equilibrium given $\mathcal{P}$;

(ii) $\mathcal{P}$ satisfies

$$W\left(\mathcal{B}_{-1}, a_{-1}\right) = \max_{\mathcal{P}} \left\{u\left(c_1\right) + Eu\left(c_{is}\right) + \beta EW\left(\mathcal{B}, a\right)\right\}$$

subject to government's budget constraint and the transversality condition;

(iii) the allocation of consumption is stationary.

Markov Perfect Equilibria: Results

- Sub-optimal tax on risky Home endowment: $\tau_2 \in \left(\frac{1}{2}, 1\right)$ with $\frac{\partial \tau_2}{\partial \sigma^2} > 0$.
- Excessive volatility of Home consumption:

$$
\begin{aligned}
c_{1,t} &= \frac{1}{1+\beta}\left\{w_1 + \beta \overline{w} - \beta\frac{\gamma}{4}\left(\sigma^2 - \sigma^{*2}\right) - \beta\frac{\gamma}{2} Var\left(c_{is}\right) + a_0 r\right\} \\
Var\left(c_{is}\right) &= \left(1-\tau_2\right)^2 \sigma_i^2 + \frac{1}{4}\left(\sigma^2 + \sigma^{*2} + 2\sigma_{fd}\right)
\end{aligned}
$$

- Suboptimality $\rightarrow$ consider *reputation mechanism*

Equilibrium with Reputation

$$\max_{\{\mathcal{P}_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[u\left(c_{1,t}\right) + \beta E u\left(c_{is,t+1}\right) \right]$$

$$s.t. \qquad R_{s,t} + a_t = \tau_{1,t} w_1 + \tau_{2,t} w_{s,t} + p_{d,t} + a_{t-1}\left(1+r\right)$$

$$p_{d,t} = \beta \left[\overline{R}_{t+1} - \frac{\gamma}{2} \tau_{2,t+1} \left(\sigma^2 + \sigma_{fd} \right) \right]$$

$$c_{1,t} = \frac{1}{1+\beta} \left\{ w_1 + \beta \overline{w} - \beta \frac{\gamma}{4} \left(\sigma^2 - \sigma^{*2} \right) - \beta \frac{\gamma}{2} Var\left(c_{is,t+1}\right) + T_t \right\}$$

$$c_{1,t} = E\left(c_{is,t+1}\right)$$

$$\forall t, W\left(\mathcal{B}_{t-1}, a_{t-1}, \{\mathcal{P}_{t'}\}_t^{\infty} \mid \{\mathcal{P}_{t'}\}_0^{t-1} \right) \geq W\left(\mathcal{B}_{t-1}, a_{t-1}, \{\mathcal{P}'_{t'}\}_t^{\infty} \mid \{\mathcal{P}_{t'}\}_0^{t-1} \right)$$

where $\mathcal{P}'_{t'}$ corresponds to MPE policy.

Equilibrium with Reputation: Results

- If the government follows the ex-ante optimal rule, then the welfare is given by

$$W\left(\mathcal{B}_{t-1}, a_{t-1}, \{\mathcal{P}_{t'}\}_t^{\infty} \,|\, \{\mathcal{P}_{t'}\}_0^{t-1}\right) = u\left(c_1^F\right)\frac{2}{1-\beta}$$

- If the government abandons the rule at some t_D, it succeeds in surprising the market for one period only (recall reversion to MPE). Then
  - for $t = t_D$, $\tau_{2,t} = \tau_2^D$ and for $t > t_D$, $\tau_{2,t} = \tau_2 < \tau_2^D$;
  - the welfare is

$$W\left(\mathcal{B}_{t-1}, a_{t-1}, \{\mathcal{P}'_{t'}\}_{t_D}^{\infty} \,|\, \{\mathcal{P}_{t'}\}_0^{t_D-1}\right) = u\left(c_1^D\right)\left[\frac{2}{1-\beta} - \frac{1-\tau_2}{\tau_2}\right]$$

Equilibrium with Reputation: Results II