Competitive Pooling: Rothschild-Stiglitz Reconsidered

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Introduction

- **Background:** In traditional ge theory, insurance is a special case of securities with contingent payoffs. A household with low endowment in some state could insure himself by buying a security which delivered when he most needed the money.
- **Pooling** missing from the traditional analysis. Insurance company: clients sign the same insurance contract but "deliver" differently; shareholder of the company holds a pool of different liabilities.
- **Adverse selection** stems from pooling: riskier households tend to take out more insurance and thus to be more than proportionately represented in the pool.

Framework

- **In this paper**, a theory of competitive pooling is introduced.
- **How does a pool work?** Some agents sell their promises into the pool and are obliged to deliver an exogenously prescribed (state-contingent) amount per promise. The buyers (shareholders) of the pool receive a pro rata share of all its different sellers' deliveries. Trading take place through the anonymous pool.
- **Examples of pools:**
  - mortgage pools traded on Wall Street;
  - land pools or cooperatives.

Setup I

- **Sellers:** Agent h chooses to sell φ_j^h promises into pool j; must deliver an exogenously prescribed d_j^h. Promises of different sellers cannot be distinguished.
- **Buyers (shareholders)** of pool j receive a pro rata share of all its different sellers' deliveries. Each share of pool j delivers K_j:

$$K_j = \frac{\sum_h \varphi_j^h d_j^h}{\sum_h \varphi_j^h}$$

All that matters to the buyer is K_j and price π_j.

Setup II

- **Adverse selection** stems from pooling: a buyer must worry that unreliable sellers with a proclivity for lower deliveries will tend to sell more promises into the pool, worsening the anticipated rate K_j.
- **Signaling** is incorporated into the analysis by assuming that there are many pools j, each with its own quantity limit Q_j imposed on sales into the pool. Moreover, no one is permitted to sell into more than one pool (exclusivity and full observability of trades). Those who sell into a pool with low Q_j signal their good quality.

Setup III

- **Perfect competition:** all agents view (Q_j, π_j, K_j) at each pool j as given. The terms Q_j of pool j are set exogenously. The prices π_j, the anticipated delivery rates K_j, and the trades at each pool j are all determined endogenously at equilibrium by the market forces of supply and demand.
- **At equilibrium,** given the exogenously set Q_j's of pool j's, prices π_j, anticipated delivery rates K_j, and trades at each pool j are determined.
- **Many inactive pools** result from the equilibrium.

DG (2002) vs. RS (1976)

- **RS (1976)**:
  - continuum of agents;
  - oligopolistic, risk-neutral insurance companies;
  - insurance contracts specify both Q_j and π_j;
  - if an equilibrium exists, it is separating and only two contracts are offered.
- **DG (2002):**
  - continuum of agents; every agent is a price taker;
  - households insure each other through the pool;
  - in equilibrium, if $Q_i < Q_j$, then $\pi_i > \pi_j$.
  - which contracts will emerge is determined by the invisible hand, not the "visible" company: equilibrium always exists (and always unique).

One pool (Cooperative) I

- **Households:** $h \in H = \{1, \ldots, H\}$, each with risky endowment $e_h^s \in \mathbb{R}_+$, where $s \in \Sigma = \{1, \ldots, S\}$. $(\Sigma_{h \in H} e^h \gg 0)$.
- **Ex-ante utility:** $u^h : \mathbb{R}_+^\Sigma \to \mathbb{R}$, where u is "nice".
- Each $h \in H$ contributes $0 \leq \varphi^h \leq Q$ promises and delivers $d_s^h \in \mathbb{R}_+^\Sigma$ per unit of promise.
- **Average delivery of the pool in state s:**

$$K_s(\varphi) = \frac{\sum_h \varphi^h d_s^h}{\sum_h \varphi^h}$$

where $\varphi \equiv (\varphi^1, \ldots, \varphi^H)$; $K_s(\varphi) =$? if $\sum_h \varphi^h = 0$.

One pool (Cooperative) II

- **Final bundle for h:** $\chi(\varphi^h, K(\varphi)) \in \mathbb{R}^\Sigma$ where,

$$\chi_s(\varphi^h, K(\varphi)) = e_s^h + \varphi^h(K_s(\varphi) - d_s^h)$$

  for $s \in \Sigma$.
- **Notice:** Sellers become shareholders.
- **Feasible set of contributions:**

$$\{\theta^j \in [0, Q] : \chi^h(\theta^h, K(\varphi|\theta^h)) \in \mathbb{R}_+^\Sigma\}$$

  where $(\varphi|\theta^h) \equiv (\varphi^1, \ldots, \varphi^{h-1}, \theta^h, \varphi^{h+1}, \ldots, \varphi^H)$.
- **Noncooperative** game with payoffs $u^h(\chi(\varphi^h, K(\varphi)))$ for $h \in H$ results from the rules of the cooperative.

Perfectly competitive cooperative I

- **Large number of households**: each can ignore her effect on $K(\varphi)$ and concentrate on how much net trade $(K - d^h)$ implement.
- **Continuum of households:** $t \in (0, H]$, where all $t \in (h-1, h]$ are of type h and are identical: $d^t = d^h$, $e^t = e^h$ and $u^t = u^h$.
- **Units of aggregate promise**: $\overline{\varphi} \equiv \int_0^H \varphi^t dt$.
- **Delivery per unit in state** s: $K_s \equiv (1/\overline{\varphi}) \int_0^H \varphi^t d_s^t dt$.
- **Consumption** of household $t \in (h-1, h]$ in $s \in S$: $x_t^s = \chi_s^t(\varphi^t, K) \equiv e_s^h + \varphi^t(K_s - d_s^h)$;

Perfectly competitive cooperative II

- **B. set:** $\Sigma^t(K) = \{(\theta, y) \in [0, Q] \times \mathbb{R}_+^\Sigma : y = \chi^t(\theta, K)\}$.
- $(K, \varphi, x) \in \mathbb{R}_+^\Sigma \to [0, Q]^{(0,H]} \to [\mathbb{R}_+^\Sigma]^{(0,H]}$ is an **equilibrium** for the one-pool economy $((u^h, d^h, e^h)_{h\in H}, Q)$ iff φ and x are measurable and
  - ▷ $K = (1/\overline{\varphi}) \int_0^H \varphi^t d^t dt$, if $\overline{\varphi} > 0$;
  - ▷ $(\varphi^t, x^t) \in \arg\max_{(\theta,y)\in\Sigma^t(K)} u^t(y)$ for almost all t.
- **With one pool only,** can ignore the case where the pool is inactive. **With multiple competing pools,** choice of K when $\overline{\varphi} = 0$ is crucial and so is the refinement of the equilibrium.

Competing cooperatives I

- **Competition** in Q's to lure away dissatisfied households at other pools.
- **Collection** of cooperatives $j \in \vartheta = \{1, \ldots, J\}$ with same deliveries d_s^t but different quantity restrictions $\varphi_j^t \leq Q_j$.
- **The economy**: $((u^h, d^h, e^h)_{h \in H}, (Q_j)_j \in \vartheta)$.
- **Now**, household t chooses $\theta = (\theta_1, \ldots, \theta_J) \in \mathbb{R}_+^J$. Denote $K_j = (K_{1j}, \ldots, K_{Sj})$ and $K = (K_1, \ldots, K_S)$.
- **Consumption** for t: $\chi^t(\theta, K) \equiv e^t + \sum_{j \in \vartheta} \theta_j (K_j - d^t)$.

Competing cooperatives II

- **Budget set with exclusivity:**

$$\Sigma^t(K) = \{(\theta, y) \in \underline{\Sigma}^t(K) : \theta_j > 0 \Rightarrow \theta_k = 0, \forall k \in \vartheta \backslash \{j\}\}$$

- $(K, \varphi, x) \in \mathbb{R}_+^{\Sigma \to \vartheta} \to \mathbb{R}^{\vartheta \to (0,H]} \to \mathbb{R}_+^{\Sigma \to (0,H]}$ is an **equilibrium** iff φ and x are measurable and
  - $K_{sj} = (1/\overline{\varphi}_j) \int_0^H \varphi_j^t d_j^t dt$, if $\overline{\varphi}_j > 0, \forall j \in \vartheta$;
  - $(\varphi^t, x^t) \in \arg\max_{(\theta,y) \in \Sigma^t(K)} u^t(y)$ f.a.a. $t \in (0, H]$.
- **Problem:** if people anticipate low K_j at pool j, then they won't contribute to it. But then the ex-post check on delivery K_j is impossible. Need a way to formulate expectation on K_j when j inactive.

Refinement

- **The spirit**: "$\dots$it is interesting to study equilibrium in which anticipations are always reasonably optimistic. It is of central importance for us to understand which markets are open and which are not, and we do not want our answer to depend on the agents' whimsical pessimism$\dots$"
- **Anticipated deliveries from inactive pools** are analogous to beliefs in game theory "off the equilibrium path". The refinement is a tremble "on the market".
- **Introduce** external $d-$agent who contributes $\epsilon(n) = (\epsilon_j(n))_{j\in\vartheta} \geq 0$ and delivers $d = (d, \dots, d)$ per unit contributed. $d \geq \max_{h\in H} d_h^s$ for all $s \in S$ (d optimistic). $\epsilon(n) \to 0$ as $n \to \infty$.

Refined Equilibrium I

- $E = (K, \varphi, x) \in \mathbb{R}_+^{SJ} \to \mathbb{R}^{J \to (0,H]} \to \mathbb{R}_+^{S \to (0,H]}$ is a **refined equilibrium** if there is a sequence $E_d(n) = (K(n), \varphi(n), x(n), \epsilon(n)) \in \mathbb{R}_+^{SJ} \to \mathbb{R}^{J \to (0,H]} \to \mathbb{R}_+^{S \to (0,H]} \to \mathbb{R}_+^J$ such that d is optimistic, $\varphi(n)$ and $x(n)$ are measurable for $n = 1, 2, \ldots$ and
  - $\epsilon(n) \to 0, K(n) \to K$ and $\phi^t(n) \to \phi^t, x^t(n) \to x^t$ a.a. t.
  - $(\varphi^t(n), x^t(n)) \in \arg\max_{(\theta,y) \in \Sigma^t(K(n))} u^t(y)$ a.a. t and all n.
  - $\epsilon_j(n) > 0$ if $\overline{\varphi}_j(n) = 0$, for all $j \in \vartheta$ and all n;

Refined Equilibrium II

- for all n and for all $j \in J^* = \{j \in J : \overline{\phi}_j(n) = 0\}$,

$$K_{sj} = \frac{1}{\epsilon_j(n) + \overline{\varphi}_j(n)} \left[\epsilon_j(n) d + \int_0^H \varphi_j^t(n) d_s^t dt \right]$$

- **Theorem:** Consider the finite type continuum model with competing cooperatives and the exclusivity constraint. Then a refined equilibrium always exists.