Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard

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Introduction

- **Purpose:** General equilibrium analysis of Pareto optima and competitive equilibria with moral hazard and adverse selection problems.
- **Examples:**
  - ▷ adverse selection insurance economy *a la* Rothschild-Stiglitz ('76);
  - ▷ signalling economy *a la* Spence ('74);
  - ▷ moral hazard insurance economy;
  - ▷ private-information labor market economy.

Structure of the paper

- **Unification** of these seemingly different economies under a common structure with convex constraints and preferences (Introduction of consumption lotteries).
- **Pareto optimal allocations** determined as solutions to the problem of maximizing weighted averages of the agent-type utilities subject to incentive-compatibility conditions and resource constraints.
- **Competitive equilibria:** existence and optimality depend only on the way (distinct) agents enter the resource constraints (homogeneously or not).

Structure of the presentation

- **Concentrate on:**
  - adverse selection insurance economy *a la* Rothschild-Stiglitz ('76);
  - signalling economy *a la* Spence ('74);
  - moral hazard insurance economy;
  - private-information labor market economy.
- **Study:**
  - the mapping of this economy into the general structure;
  - the Pareto optimal allocation;
  - and its decentralization.

General mathematical structure

- **Agent types:** $i = 1, \ldots, I$; λ_i fraction of each type.
- **Commodity space:** L, linear space.
- **Cons. poss. set:** $\bar{X} \subset L$, closed and convex.
- **Utility for type *i*:** $u_i : \bar{X} \to R$, concave (linear).
- **Common endowment:** $\xi \in L$.
- $x = (x_i)$ is **implementable** if:
  - $x_i \in \bar{X}$, $\forall i$;
  - $u_i(x_i) \geq u_i(x_j)$, $\forall i, j$;
  - $\sum_i \lambda_i r_{ik}(x_i - \xi) \leq 0$, where r_{ik} is a real-valued linear function on L and $k = 1, \ldots, K$.

Adverse selection *a la* RS

- **Agent types:** $i = 1, 2$; private information.
- **Endowment:** $z = \begin{cases} z_0 & \text{with prob. } \theta_i \\ z_1 & \text{with prob. } 1 - \theta_i \end{cases}$
  common information; $\theta_1 < \theta_2$.
- **No aggr. uncertainty:** λ_i known; by LLN, θ_i fraction of those who suffer a loss in group i.
- **Preferences:** $U : C \to R$, where $C \subset R_+$, U "nice" and $z_0, z_1 \in C$.
- Upon realization of z_0 (z_1), agent i gets c_0 (c_1) where $c_0, c_1 \in C$; **expected utility**: $\theta_i U(c_0) + (1 - \theta_i) U(c_1)$.

Adverse selection *a la* PT (I)

- $L = R^2$.
- $\xi = (z_0, z_1)$.
- $x_i = (c_{i0}, c_{i1})$.
- **Then**, $\forall i, j$:
  - $x_i \in \bar{X} \equiv C \times C$; closed and convex if C is;
  - $\sum_i \lambda_i[\theta_i(c_{i0} - z_0) + (1 - \theta_i)(c_{i1} - z_1)] \leq 0$;
  - $\theta_i U(c_{i0}) + (1 - \theta_i)U(c_{i1}) \geq \theta_i U(c_{j0}) + (1 - \theta_i)U(c_{j1})$.
- **But**, given the strict concavity of U, the space of consumption allocations restricted by these constraints is not convex.

Adverse selection *a la* PT (II)

- **Introduce lotteries** over C (finite with n elements): $\mu = ((\mu(c))_{c \in C}$, where $\sum_c \mu(c) = 1$ and $\mu(c) \geq 0$ $\forall c \in C$;
- If a loss is suffered, each agent subject to μ_0; otherwise, μ_1 is "played"; notice that LLN extends to $\mu_0(c)$ and $\mu_1(c)$.
- $L = R^{2n}$;
- $x_i = (x_{i0}, x_{i1})$, where $x_{i0} = \mu_0$ and $x_{i1} = \mu_1$.
- $\xi = (\xi_0, \xi_1)$: a pair of degenerate probability distributions which assign probability 1 to the event z_0 and z_1 respectively.

Adverse selection *a la* PT (II)

- **Consumption possibility set:**
  $\bar{X} = \{x \in L : \sum_c x_{0c} = 1, \sum_c x_{1c} = 1, x \geq 0\}$.
- **Expected utility:**
  $W_i(x_i) = \theta_i \sum_c x_{i0c} U(c) + (1-\theta_i) \sum_c x_{i1c} U(c)$.
- **Resource constraint:**
  $$\sum_i \lambda_i \left[\theta_i \sum_c x_{i0c} c + (1-\theta_i) \sum_c x_{i1c} c\right] \leq \sum_i \lambda_i \left[\theta_i \sum_c \xi_{0c} c + (1-\theta_i) \sum_c \xi_{1c} c\right].$$
- **So, we have:**
  - $x_i \in \bar{X}$, $\forall i$ with $\bar{X}$ closed and convex;
  - $W_i(x_i) \geq W_i(x_j)$, $\forall i, j$;
  - $\sum_i \lambda_i r_i(x_i - \xi) \leq 0$.

Progress

- **Concentrate on:**
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  - signalling economy *a la* Spence ('74);
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Pareto optima (general structure)

- **Set of weights:** $\Gamma = \left\{\gamma \in R^I : \gamma_i \geq 0, \sum_i \gamma_i = 1\right\}$.
- For $\gamma \in \Gamma$, $\phi(\gamma)$ is the set of consumption allocations that solve the **program**:

$$\max_{x=(x_i)} \sum_i \gamma_i u_i \cdot x_i \quad \text{(e.g.,} u_i \cdot x_i = W_i(x_i))$$

  subject to

  - $x_i \in \bar{X}$, $\forall i$;
  - $u_i \cdot x_i \geq u_i \cdot x_j$, $\forall i, j$;
  - $\sum_i \lambda_i r_{ik}(x_i - \xi) \leq 0$, where r_{ik} is a real-valued linear function on L and $k = 1, \ldots, K$.

Pareto optima (general structure)

- Define $\Phi = \bigcup_{\gamma \in \Gamma} \phi(\gamma)$.
- **Lemma:** The set Φ contains all the Pareto optima. If $\gamma > 0$, then all allocations belonging to $\phi(\gamma)$ are optima. Finally, if $x = (x_i)$ belongs to $\phi(\gamma)$ and if x is not Pareto dominated by another element belonging to $\phi(\gamma)$, then x is an optimum.
- **Theorem:** If the set $\bar{X}$ is compact and contains ξ, the set of Pareto optimal allocations is nonempty.

Pareto optima (Adverse selection)

- Let $\bar{z}$ be the **ex post per capita endowment:**

$$\bar{z} = \sum_i \lambda_i \left[\theta_i z_0 + (1-\theta_i) z_1\right]$$

- **Program**: for $\gamma \in \Gamma$,

$$\max \sum_i \gamma_i \sum_c U(c) \left[x_{i0c}\theta_i + x_{i1c}(1-\theta_i)\right],$$

$$x_1, x_2 \geq 0$$

Pareto optima (Adverse selection)

- **Subject to:**

(μ_1) $\sum_c U(c)\left[(x_{10c} - x_{20c})\theta_2 + (x_{11c} - x_{21c})(1-\theta_2)\right] \leq 0$: agents of type 2 weakly prefer x_2 to x_1;

(μ_2) $\sum_c U(c)\left[(x_{20c} - x_{10c})\theta_1 + (x_{21c} - x_{11c})(1-\theta_1)\right] \leq 0$: agents of type 1 weakly prefers x_1 to x_2;

(μ_3) $\sum_{i,c} \lambda_i c\left[x_{i0c}\theta_i + x_{i1c}(1-\theta_i)\right] \leq \bar{z}$: this is the single resource constraint;

(μ_4) $\sum_c x_{10c} = 1$;

(μ_5) $\sum_c x_{11c} = 1$;

(μ_6) $\sum_c x_{20c} = 1$;

(μ_7) $\sum_c x_{21c} = 1$;

Pareto optima (Adverse selection)

- **First order condition** for x_{10c}:

$$\gamma_1 U(c)\theta_1 - \mu_1 U(c)\theta_2 + \mu_2 U(c)\theta_1 - \mu_3 c\theta_1\lambda_1 + \mu_4 \leq 0$$

  Analogous for $x_{11c}, x_{20c}, x_{21c}$.

- It turns out that x_{10c} equals one for some $c \in C$ and zero otherwise: probability measure x_{10} puts all mass on some $c10$. Analogous conclusions for x_{11}, x_{20}, x_{21}.
- The consumptions points depend on the weights γ.

Pareto optima (Adverse selection)

- **Divide** the Pareto optima in three sets:
  - Everyone consumes $\bar{z}$ with certainty;
  - Expected utility for type 1 exceeds $U(\bar{z})$;
  - Expected utility for type 2 exceeds $U(\bar{z})$.
- **Separating equilibrium** *a la* Rothschild-Stiglitz with $c_{20} = c_{21} = c_2$ and $c_{10} < c_2 < c_{11}$: the allocation is Pareto optimal in the larger space of lotteries; randomness used to separate the agents; the low risk accept higher utility for higher expected consumption.

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Decentralization

- It turns out that the standard competitive equilibrium construct (price system, choices that maximize utilities and production given the price system and markets that clear) is successful in economies in which $r_{ik} = r_k, \forall i$.
- Adverse selection is **not** one of those economies. There, the resource constraints are of the form $\sum_i \lambda_i r_{ik}(x_i - \xi) \leq 0$ rather than the less general form $\sum_i \lambda_i r_i(x_i - \xi) \leq 0$.
- For example, in the Rothschild-Stiglitz economy, a competitive equilibrium does **not** exist if there is more than one type in the resource constraint.

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