A Theory of Financing Constraints and Firm Dynamics

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Introduction

- **Objective:** Study the effect of borrowing constraints on firms' growth and survival.
- **Framework:** Long-term contractual relationship between an entrepreneur (borrower) and an investor (lender) with asymmetric information.
- **Results:**
  - Borrowing constraints as endogenous "products" of the optimal contract; less stringent the higher the entrepreneur's share of total firm value (Equity);
  - Optimal contract generates firm dynamics that match stylized facts qualitatively.

Model I

- **Time:** Discrete; infinite horizon.
- **Players:**
  - A risk-neutral entrepreneur has a project "in mind" at $t = 0$. She owns M but needs $I_0 > M$ to start. If started, the project takes working capital k_t as input and returns output $\tilde{y}_t$, $\forall t \geq 1$.
  - A risk-neutral investor (with the same discount factor as the entrepreneur) covers the start-up cost and provides working capital. She cannot monitor the project, must rely on entrepreneur's reports.
- **Liquidation:** The project can be scrapped at any time, generating value S.

Model II

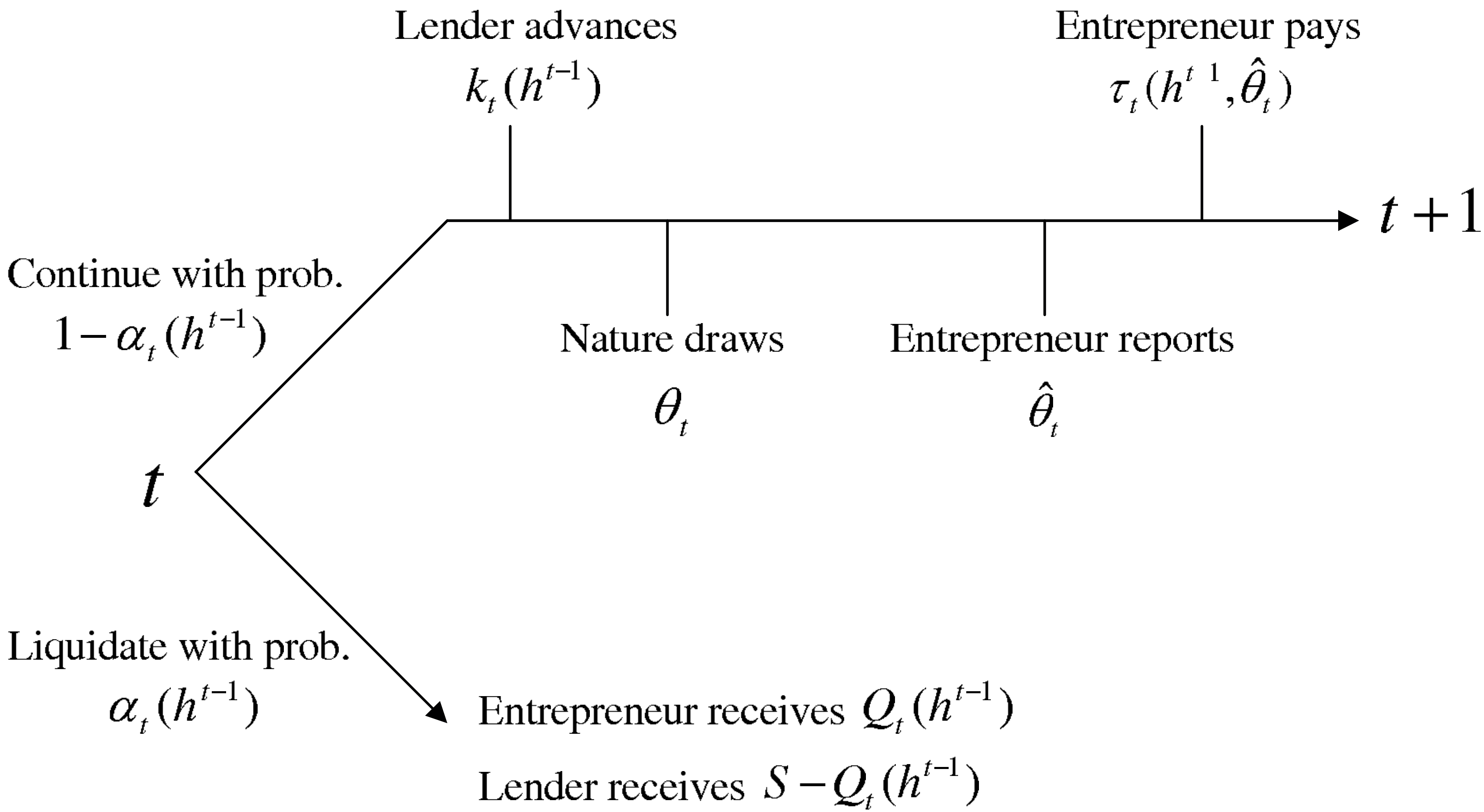
- **Productivity shocks:** $\theta \,\epsilon\, \Theta \equiv \{H, L\}$; θ takes value H with probability p. Accordingly,

$$\tilde{y}_t = \begin{cases} R(k_t) & \text{with prob. } p \\ 0 & \text{with prob. } 1-p \end{cases}$$

- **Reporting strategy:** $\hat{\theta} = \{\hat{\theta}_t(\theta^t)\}_{t=1}^{\infty}$, where $\theta^t = (\theta_1, \ldots, \theta_t)$; history of reports thus $h^t = (\hat{\theta}_1, \ldots, \hat{\theta}_t)$.
- **Contract:** At $t = 0$, the investor offers a contract to the entrepreneur. The terms specify liquidation policy, payments and capital advancements and are made contingent on common information:

$$\sigma = \{\alpha_t(h^{t-1}), Q_t(h^{t-1}), k_t(h^{t-1}), \tau_t(h^t)\}$$

Timing


Lender advances
$k_t(h^{t-1})$
Entrepreneur pays
$\tau_t(h^{t\ 1}, \hat{\theta}_t)$
$t+1$
Continue with prob.
$1-\alpha_t(h^{t-1})$
Nature draws
θ_t
Entrepreneur reports
$\hat{\theta}_t$
t
Liquidate with prob.
$\alpha_t(h^{t-1})$
Entrepreneur receives $Q_t(h^{t-1})$
Lender receives $S-Q_t(h^{t-1})$

Equity and Debt

- σ **feasible** if, $\forall t \geq 1$ and $\forall h^{t-1} \epsilon \Theta^{t-1}$
  - $\alpha_t(h^{t-1}) \epsilon [0,1]$;
  - $Q_t(h^{t-1}) \geq 0$;
  - $\tau_t(h^{t-1}, H) \leq R(k_t(h^{t-1}))$ and $\tau_t(h^{t-1}, L) \leq 0$.
- $\forall h^{t-1}$, the pair $(\sigma, \hat{\theta})$ implies expected discounted cash flows for:
  - Entrepreneur: $V_t(\sigma, \hat{\theta}, h^{t-1})$ or Equity;
  - Investor: $B_t(\sigma, \hat{\theta}, h^{t-1})$ or Debt.
- σ **incentive compatible** if, $\forall \hat{\theta}$

$$V_1(\theta, \sigma, h^0) \geq V_1(\hat{\theta}, \sigma, h^0)$$

Pareto Frontier

- **Set of equity values** that can be generated by feasible and incentive compatible contracts:

$$\mathcal{V} \equiv \{V | \exists\, \sigma \text{ s.t. feas., i.c. and } V_1(\theta, \sigma, h^0) = V\}$$

- Can define a **Frontier of values** thus: for any $V \epsilon \mathcal{V}$, the optimal contract such that

$$B(V) = sup\{B | \exists\, \sigma \text{ s.t. } V_1(\theta, \sigma, h^0) = V \text{ and } B_1(\theta, \sigma, h^0) = B\}$$

- Each point on the frontier corresponds to a **capital structure** $(V, B(V))$.
- In general, the **Modigliani-Miller Theorem** does not hold: $W(V) = V + B(V)$ not invariant to capital structure.

Symmetric Information

- **Benchmark:** With symmetric information, the contract simply maximizes the expected discounted surplus of the project:

$$k^* = \arg\max_k pR(k) - k$$

$$\pi^* = pR(k^*) - k^*$$

$$\tilde{W} = \pi^*[1 + \delta + \delta^2 + \ldots] = \frac{\pi^*}{1 - \delta}$$

The project is undertaken if $\tilde{W} > I_0$. Any division of $\tilde{W}$ between investor and entrepreneur is feasible as long as the latter gets a nonnegative value.

Asymmetric Information I

- Suppose **liquidation has not occurred** at the beginning of the period. The problem is then:

$$\hat{W}(V) = \max_{k,\tau,V^H,V^L} [pR(k) - k] + \delta[pW(V^H) + (1-p)W(V^L)]$$

subject to

PK: $V = [pR(k) - \tau] + \delta[pV^H + (1-p)V^L]$

IC: $R(k) - \tau + \delta V^H \geq R(k) + \delta V^L$

LL: $\tau \leq R(k)$

$V^H, V^L \geq 0$

Asymmetric Information II

- Step back at the beginning of the period. **Liquidation decision** to be made:

$$W(V) = \max_{\alpha, Q, V_c} \alpha S + (1-\alpha)\hat{W}(V_c)$$

subject to

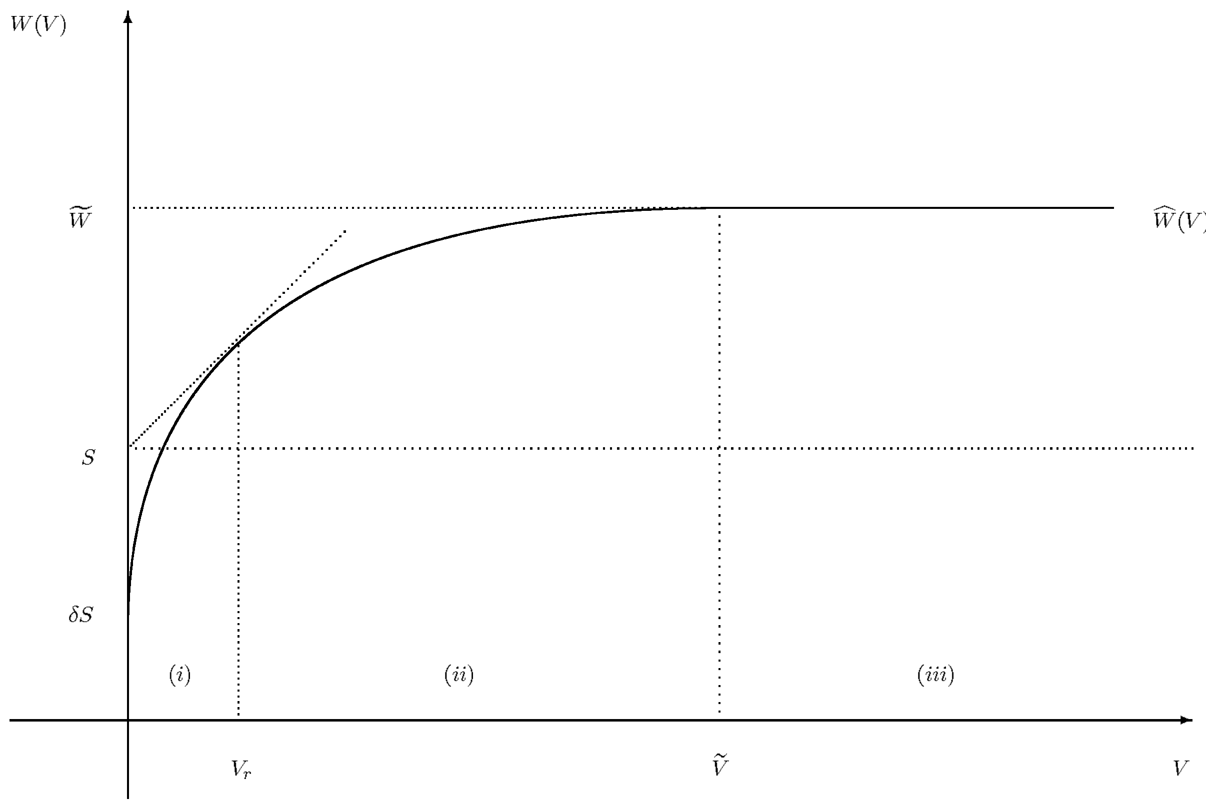
PK: $V = \alpha Q + (1-\alpha) V_c$

$Q, V_c \geq 0$

Solution:

- Set $Q = 0 \Rightarrow \alpha(V) = \frac{V_c(V) - V}{V_c(V)}$;
- Set $V_c(V) = V_r$ such that $0 \leq \alpha(V) \leq 1$, for $V \leq V_r$;
- Three regions emerge (see graph below).

Value Function



$W(V)$ linear for $V \leq V_r$, where random liquidation occurs; strictly incr. for $V < \tilde{V}$ and equal to $\tilde{W}$ for $V \geq \tilde{V} \equiv \frac{pR(k^*)}{(1-\delta)}$.

Optimal Contract I

- **Capital Advancement:**
  - $k(V)$ single valued and continuous;
  - for $V < \tilde{V}$, $k(V) < k^*$, i.e. borrowing constraint binds;
  - for $V \geq \tilde{V}$, $k(V) = k^*$;

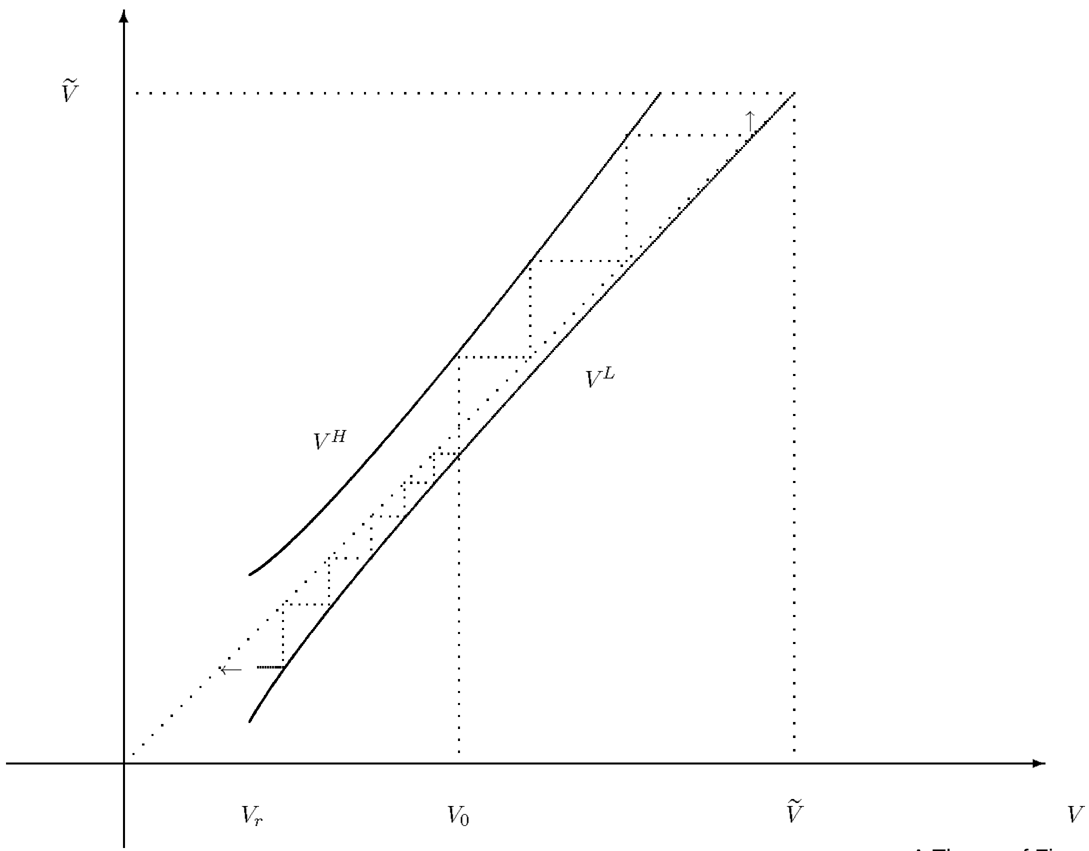
Intuition when $R(k) = \tau$: in this case IC binds, i.e. $R(k) = \delta(V_H - V_L)$. Higher capital advancement requires higher spread in equity values: as standard with moral hazard, the lender wants to make the borrower more sensitive to the realization of output. Given the concavity of $W(V)$, this spread is costly for the investor. Borrowing constraints stem from this trade-off.

Optimal Contract II

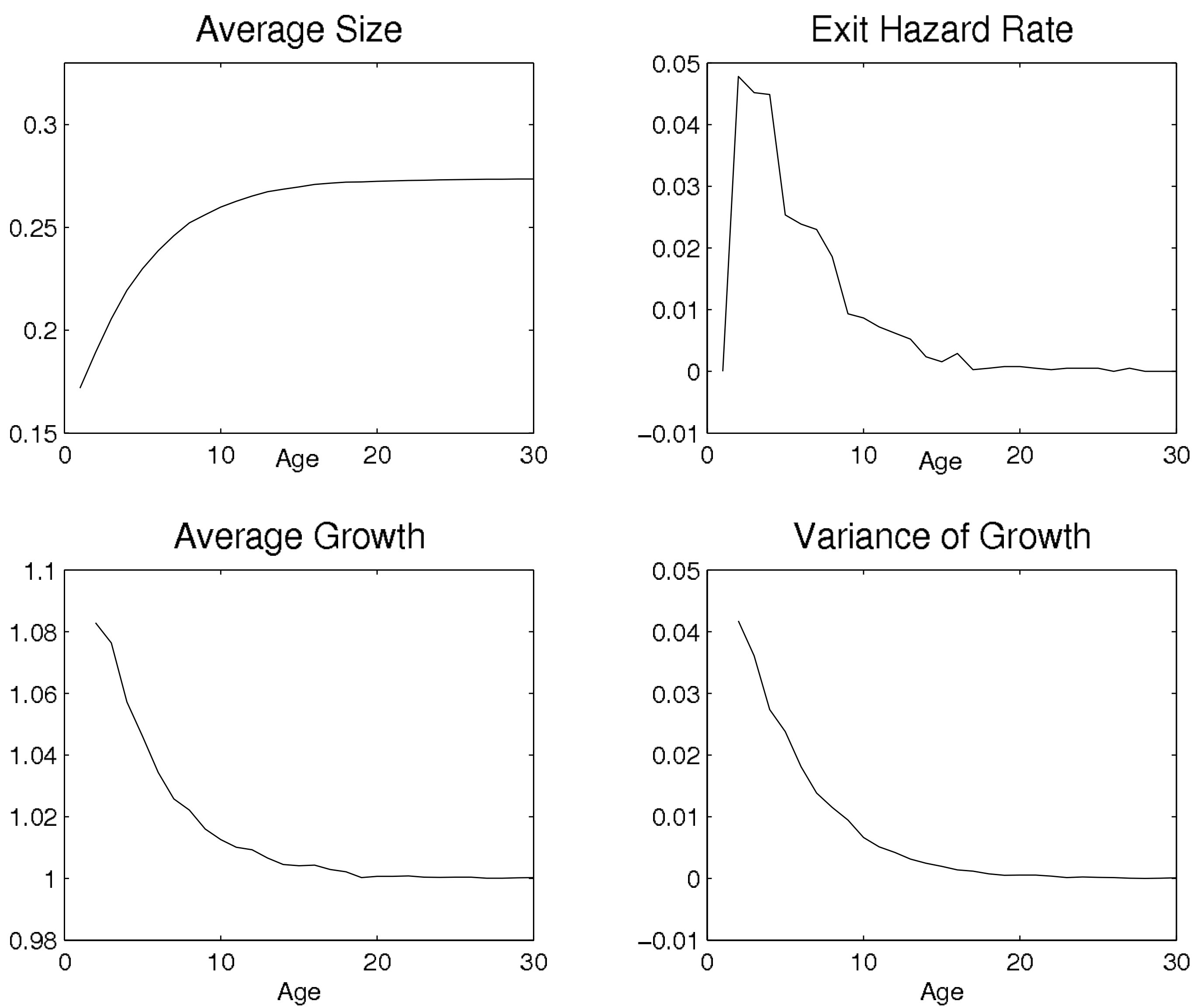
- **Repayment:** For $V < \tilde{V}$, given risk neutrality and the common discount factor δ, it is optimal for both the entrepreneur and the investor to set $R(k) = \tau$ until V reaches $\tilde{V}$ and providing the efficient level of capital k^* becomes incentive compatible. (For $V^H < \tilde{V}$, $R(k) = \tau$ is necessary for optimality).
  Intuition: For given k and V^L, there are infinite combinations of τ and V^H that are feasible and incentive compatible. Instantaneous profits are not affected by the choice. Continuation value, though, is weakly increasing in V^H.

Optimal Contract III

- **Evolution of Equity when $R(k) = \tau$ and $V_r \leq V < \tilde{V}$:**
  - $V^L < V < V^H \quad (V^L = \frac{V - pR(k)}{\delta}$, $V^H = \frac{V + (1-p)R(k)}{\delta}$);
  - $V < pV^H + (1-p)V^L$.



Simulation


Average Size
0.3
0.25
0.2
0.15
0
10
Age
20
30
Exit Hazard Rate
0.05
0.04
0.03
0.02
0.01
0
−0.01
0
10
Age
20
30
Average Growth
1.1
1.08
1.06
1.04
1.02
1
0.98
0
10
20
30
Age
Variance of Growth
0.05
0.04
0.03
0.02
0.01
0
−0.01
0
10
20
30
Age

Conclusions

- **Borrowing constraints** emerge endogenously from the optimal contract; tend to be less stringent the higher the entrepreneur's share of total firm value (V);
- Optimal contract generates **firm dynamics** that match stylized facts qualitatively; in a regression of investment on some controls (Tobin's average q's, cash flows, ...), would expect positive and significative cash flow coefficients.
- **Extensions:**
  - "Hunger";
  - Multiple shocks;
  - General equilibrium.