Search and Rest Unemployment

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Sargent Reading Group Presentation - Greg Kaplan

9 October 2007

Introduction

- 3 labor market states?
- What do people do when they are not working?
- Introduce a 4th state... rest unemployment

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- What do people do when they are not working?
- Introduce a 4th state... rest unemployment
- Can we distinguish, empirically, search from rest?
- Perhaps we only need 2 states after all...?
- Where does labor market risk stem from?

Intermediate Goods - Labor Markets

- Continuum of intermediate goods, $j \in [0,1]$
- Measure $l(j,t)$ of workers, $e(j,t)$ of which are emplyed
- Each employed worker produces $Ax(j,t)$ goods
- Competitive price of good j is $p(j,t)$ and wage, $w(j,t)$
- Idiosyncratic productivity:

$$d\log x(j,t) = \mu_x dt + \sigma_x dz(j,t)$$
$$z(j,t) \text{ Weiner process}$$

- Market j shuts down at rate λ, (Poisson process)
- Replaced by new labor market, j, $x \sim F(x)$

Final Goods

- Competitve final goods sector, CES technology

$$Y\left(t\right)=\left(\int_{0}^{1}q\left(j,t\right)^{\frac{\theta-1}{\theta}}dj\right)^{\frac{\theta}{\theta-1}}$$

- Take prices, $p\left(j,t\right)$, as given, choose demands, $q\left(j,t\right)$:

$$\max_{\{q(j,t)\}_{j}}\left\{Y\left(t\right)-\int_{0}^{1}q\left(j,t\right)p\left(j,t\right)dj\right\}$$

- Demands given by:

$$q\left(j,t\right)=Y\left(t\right)p\left(j,t\right)^{-\theta}$$

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     1.1 $E(t)$ employed $\Rightarrow$ wage $w(j,t)$
     1.2 $U_r(t) = L(t) - E(t)$ rest-unemployed $\Rightarrow$ leisure b_r

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  2. $U_s(t)$ search-unemployed $\Rightarrow$ leisure b_s

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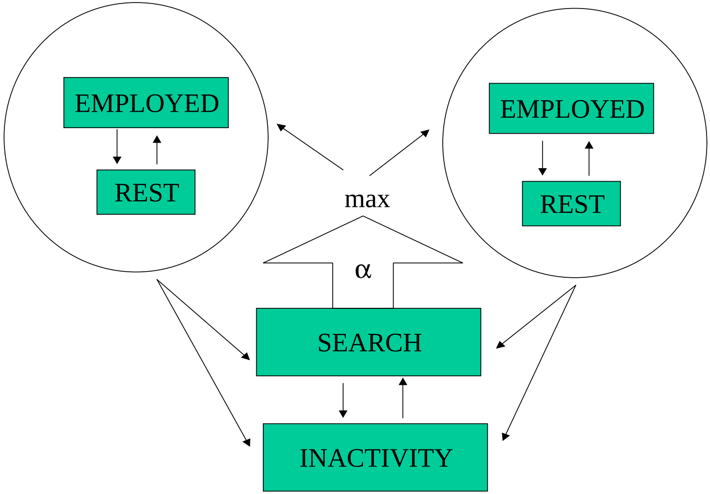
$$\int_0^\infty e^{-\rho t}\left[\begin{array}{c} u(C(t)) + b_i\left[1 - E(t) - U_r(t) - U_s(t)\right] \\ + b_r U_r(t) + b_s U_s(t) \end{array}\right] dt$$

$$b_i \geq b_r, \quad b_i > b_s$$

Labor Market States



Marginal Value of Household Members

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Searching vs Inactive

- Costless switching implies:

$$b_i = b_s + \alpha \left(\bar{v} - \underline{v} \right)$$

- $\rho \underline{v} = b_i$ implies

$$\bar{v} = b_i \left(\frac{1}{\rho} + \frac{b_i - b_s}{b_i \alpha} \right)$$

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Working vs Resting

$$w(j,t)\, u'(C(t)) > b_r \implies \text{all employed}$$
$$w(j,t)\, u'(C(t)) = b_r \implies \text{some employed / some resting}$$
$$w(j,t)\, u'(C(t)) < b_r \implies \text{all resting}$$

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$$\omega \equiv \frac{\log Y + (\theta - 1)\log(Ax) - \log l}{\theta} + \log U'(C)$$

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$W(l,x)$	wage rate	$\frac{1}{U'(C)}\max\{b_r, e^{\omega}\}$
$E(l,x)$	number employed workers	$l\min\left\{1, \frac{e^{\omega}}{b_r}\right\}^{\theta}$

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$$d\omega(j,t) = \frac{\theta-1}{\theta}\mu_x dt + \frac{|\theta-1|}{\theta}\sigma_x dz(j,t)$$

$$v(\omega_0; \underline{\omega}, \bar{\omega}) = E\left[\int_0^\infty e^{-(\rho+\lambda)t}\left(\max\left\{b_r, e^{\omega(t)}\right\} + \lambda \underline{v}\right) | \omega = \omega_0\right]$$

- In addition, we require:

$$v(\omega; \underline{\omega}, \bar{\omega}) \in [\underline{v}, \bar{v}] \quad \forall \omega$$

$$v(\bar{\omega}; \underline{\omega}, \bar{\omega}) = \bar{v}, \qquad v(\underline{\omega}; \underline{\omega}, \bar{\omega}) = \underline{v}$$

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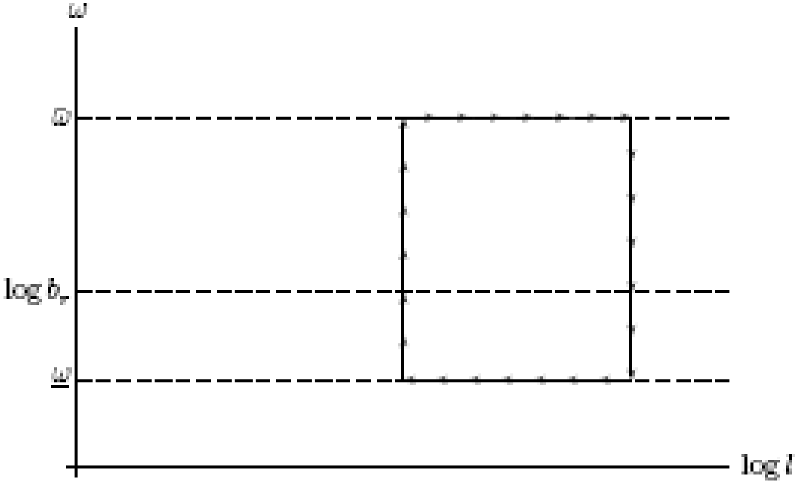
- Hamilton-Jacobi-Bellman equation:

$$(\rho + \lambda) v(\omega; \underline{\omega}, \bar{\omega}) = \max\{b_r, e^{\omega}\} + \lambda \underline{v} + \mu v_\omega(\omega; \underline{\omega}, \bar{\omega}) + \frac{\sigma^2}{2} v_{\omega\omega}(\omega; \underline{\omega}, \bar{\omega})$$
$$v_\omega(\bar{\omega}; \underline{\omega}, \bar{\omega}) = v_\omega(\underline{\omega}; \underline{\omega}, \bar{\omega}) = 0$$

Proposition 1 There exist unique $\underline{\omega}, \bar{\omega}$ such that these hold together.

Labor Market Dynamics



Definition of Equilibrium

An equilbrium is a value function, v, thresholds, $\underline{\omega}, \bar{\omega}$, and consumption C, s.t.:

1. $v, \underline{\omega}, \bar{\omega}$ solve the H-J-B equation
2. individual labor market output sum to aggregate output, Y
3. $Y = C$

Proposition 5 There exists a unique equlibrium

A Limiting Economy

- Assume $\mu = -\frac{\theta\sigma^2}{2}$ and let $\lambda \to 0$
- Let $\hat{\omega} = \max\{\log b_r, \underline{\omega}\}$
- Nice expressions for labor market statistics:

$$L = \frac{Yu'(Y)\, e^{-\hat{\omega}} \left(\overline{\omega} - \underline{\omega}\right)}{e^{(\hat{\omega}-\underline{\omega})} - 1 + \frac{1-e^{-\theta(\hat{\omega}-\underline{\omega})}}{\theta}}$$

$$\frac{U_r}{L} = \frac{\theta\left(\hat{\omega} - \underline{\omega}\right) + e^{-\theta(\hat{\omega}-\underline{\omega})} - 1}{\theta\left(\overline{\omega} - \underline{\omega}\right)} \quad \text{and} \quad \frac{U_s}{L} = \frac{\theta\sigma^2}{2\alpha\left(\overline{\omega} - \underline{\omega}\right)}$$

No Rest Unemployment

$$\frac{U_s}{L} = \frac{1}{2} \times \frac{1}{\alpha} \times \theta\sigma \times \left(\frac{\overline{\omega} - \underline{\omega}}{\sigma}\right)^{-1}$$

- $\frac{1}{\alpha}$: mean duration of unemployment $\Rightarrow$ 2.5
- θ : elasticity of subn between goods markets $\Rightarrow$ 4
- σ : volatility of industry level wages $\Rightarrow$ $sd\,(\Delta \log y)$
- $\frac{\overline{\omega} - \underline{\omega}}{\sigma}$: autocorrelation of industry level wages $\Rightarrow$ $corr\,(\log y_{t-1}, \log y_t)$

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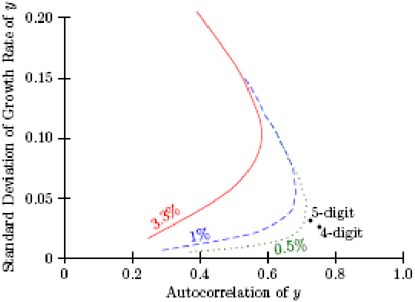

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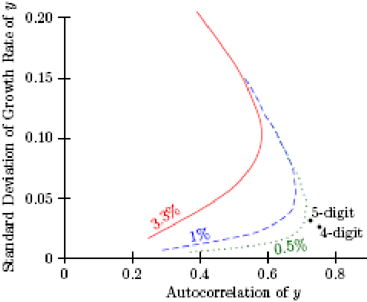

- $\lambda > 0$ can raise unemployment rate to 3.3%
- $\lambda = 0.081 \Rightarrow$ shut down every 12 years, 95% ue spells start at shut down

Reintroducing Rest Unemployment

- Improve fit because:
  1. extra source of unemployment
  2. constant wage region $[\underline{\omega}, \hat{\omega}]$, reduces sd and increases ac

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- Improve fit because:
    1. extra source of unemployment
    2. constant wage region $[\underline{\omega}, \hat{\omega}]$, reduces sd and increases ac
- Match same industry level volatility and auto-correlation:

$$\begin{aligned} \text{Search unemployment} &= 0.5\% \\ \text{Rest unemployment} &= 2.8\% \end{aligned}$$

- Implied values of leisure:
    1. to generate rest unemployment: $\frac{b_r}{b_i} \simeq 1$
    2. to generate low search unemployment: b_s small
- Relationship to Hornstein, Krussel and Violante (2006)

Final Thoughts

- What is the real difference between search and rest?
- What are the big risks that workers face?
- Policy implications?