An Exploration in the Theory of Optimum Income Taxation

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Objectives and Results

- Characterize the optimal income tax schedule when individual productivities are not observed by the planner
- Is there a theoretical basis for progressive or regressive marginal taxes?
- Evaluate optimal trade-off of labor incentives and consumption insurance through use of a non-linear tax schedule

Results

- Optimal marginal tax rate is between 0 and 1.
- Tax schedule is sensitive to distribution of skills and preference specification

Environment

Demographics Static economy with a continuum of consumers, indexed by their labor productivity, n.

Preferences $u(x, y)$ where x is consumption and y is labor supplied. Assume u is cont. diff. with $u_1 > 0$, $u_2 < 0$, u strictly concave.

Individual Productivities $n \sim F$, with $f(n) = F'(n)$. An agent who provides labor, y, produces output $z = ny$

Government Chooses a tax schedule such that an agent who produces z is left with $c(z) = c(ny)$ to consume after tax. Restrict c to be upper semi-continuous.

Timing Government announces function $c(z)$ to which it can commit, then agent observes n and choose y

Consumer Problem

(CP): Choose (x_n, y_n) to maximize $u(x, y)$ subject to $x \leq c(ny)$

$$u_1 n c'(ny) = -u_2 \tag{1}$$

Let $u_n = u(x_n, y_n)$ be indirect utility as a function of n. Then (1) gives:

$$\frac{du_n}{dn} = u_1 y_n c'(ny_n) \tag{2}$$

$$= -\frac{y_n u_2}{n} \tag{3}$$

Consumer Problem (cont'd)

Proposition 1 There exists a number $n_0 \geq 0$ such that $y_n = 0$ if $n \leq n_0$ and $y_n > 0$ if $n > n_0$

Proof Let $m < n$ and $y_m > 0$. Then

$$u[c(my_m), y_m] < u\left[c\left(n\frac{m}{n}y_m\right), \frac{m}{n}y_m\right] \leq u_n$$

So $y_m = 0$ if $y_n = 0$.

Proposition 2 Any function of n, (x_n, y_n) that solves (CP) for some USC function, $c(z)$, also solves (CP) for a non-decreasing USC funtion, $\tilde{c}(z)$ [define $\tilde{c}(z) = \sup_{z' \leq z} c(z')$]
$\rightarrow$ marginal tax rate need not be greater than 100%

Government Problem

Government chooses a function, $c(z)$ to maximise

$$W = \int_n G(u_n) f(n) dn \tag{4}$$

subject to

$$\int_n x_n f(n) dn \leq H \left(\int_n n y_n f(n) dn \right) \tag{5}$$

$$\frac{du_n}{dn} = -\frac{y_n u_2}{n} \tag{6}$$

$$u_n = u(x_n, y_n) \tag{7}$$

Take u_n as state, y_n as control and use Pontriyagin Maximum Principle. What functions, y_n, are admissible and ensure there exists $c(z)$ such that (CP) is satisfied?

A Useful Theorem

Assumption B $V(x,y) = -\frac{yu_2}{u_1}$ is increasing in y for all x

Theorem 1 Under Assumption B, $z_n = ny_n$ maximizes utility for every n under some consumption function, $c(z)$, *iff*

1. z_n is a non decreasing function for $n > 0$
2. $0 \leq z_n < n$ for all $n > 0$

- Intuition: FOC from (CP) can be written as

$$u_1 nc'(ny) = -u_2 \quad (8)$$

$$nyc'(ny) = -\frac{yu_2}{u_1} \quad (9)$$

  choose $c(z)$ so that$c'(z)$ provides correct wedge in MRS

- Implication: We know which functions, y_n, in Govt problem are implementable

Marginal Tax Rates

Define the marginal tax rate, θ, by

$$\theta = \frac{d}{d(wz)}\left[wz - c(z)\right] \tag{10}$$

So

$$w\theta = \frac{d}{dz}\left[wz - c(z)\right] = w + \frac{u_2}{nu_1} \tag{11}$$

$$= \frac{\psi_y}{n^2 f(n)} \int_n^\infty \frac{1 - \lambda G' u_1}{u_1} T_{nm} f(m) dm \geq 0 \tag{12}$$

Proposition 3 Under Assumption B, the tax function, $wz - c(z)$ is non-decreasing function of z

Implication Marginal tax rates lies in $[0, 1]$

Additive Utility

- Let $u_{12} = 0$. Then $V(x, y)$ is increasing y.
- Let $\psi_y = -\frac{\partial}{\partial y} y u_2(x, y)$. Can be roughly interpreted as the increase in the cost of providing the correct incentives as productivity increases.
- Let $v = \left(w + \frac{u_2}{n u_1}\right) / \psi_y$
- The solution to the planning problem - 2 D.E's:

$$\frac{dv}{dn} = -\frac{v}{n}\left(2 + \frac{nf'}{f}\right) - \frac{1}{n^2 u_1} + \frac{\lambda G'}{n^2} \tag{13}$$

$$\frac{du}{dn} = -\frac{y u_2}{n} \tag{14}$$

subject to boundary conditions on v_{n_0} and v_∞

Additive Utility (cont'd)

- Recall from Theorem 1 that we required $\frac{dz}{dn} \geq 0$ to guarantee implementability - needs to be verified ex-post
- Mirrlees provides an algorithm to compute solution

An Example from Mirrlees

- $u(x, y) = \log x + \log(1 - y)$, $G(u) = u$
- $n \sim$ lognormal$(-1, \sigma^2)$
- $\sigma = 0.39, 1$

TABLE I

(*Case* 1)

$\alpha = 1, \beta = 0, \sigma = 0{\cdot}39$, mean $n = 0{\cdot}40$, $X/Z = 0{\cdot}93$.
Full optimum for $X = Z - 0{\cdot}013$: $x^0 = 0{\cdot}19$, $F(x^0) = 0{\cdot}045$.
Partial optimum (income-tax): $x_0 = 0{\cdot}03$, $n_0 = 0{\cdot}04$, $F(n_0) = 0{\cdot}000$.

$F(n)$	x	y	$x(1-y)$	z	full optimum x
0	0·03	0	0·03	0	0·19
0·10	0·10	0·42	0·05	0·09	0·19
0·50	0·16	0·45	0·08	0·17	0·19
0·90	0·25	0·48	0·13	0·29	0·19
0·99	0·38	0·49	0·19	0·45	0·19
Population average	0·17			0·18	0·19

TABLE II

Same case as Table I.

z	x	Average tax rate per cent	Marginal tax rate per cent
0	0·03		23
0·05	0·07	−34	26
0·10	0·10	−5	24
0·20	0·18	9	21
0·30	0·26	13	19
0·40	0·34	14	18
0·50	0·43	15	16

Another Example

- $u(x, y) = \frac{x^{1-\gamma}}{1-\gamma} - \psi \frac{y^{1+\sigma}}{1+\sigma}$
- $n \sim \text{lognormal}(-\frac{1}{2}\nu^2, \nu^2)$
-

γ	1.5
σ	2.5
ν^2	0.05

- Quantitative results are sensitive to functional form and parametrization

Another Example - Results