On Efficient Distribution with Private Information

Andrew Atkeson and Robert E. Lucas Jr.

Objectives and Results

- Characterize efficient distribution of consumption when agents have private idiosyncratic taste shocks in a closed economy.
- Examine whether decentralization of the efficient allocation is possible in this setting

Results

- Immiseration result of Thomas and Worrall (1990) carries over to closed economy
- Full decentralization not possible, however, a "component planners" decentralization is achievable.

Environment

Demographics Continuum of consumers, indexed by their initial expected utility entitlements, w.

Preferences $E(1-\beta)\sum_{t=0}^{\infty}\beta^t V(c_t)\theta_t$

$V : R \to D$ continuous with $V' > 0, V'' < 0$

Individual Taste Shocks $\theta_t \in \Theta = \{\theta^1 \dots \theta^n\}$, θ iid

$\mu(\theta_t) > 0 \; \forall \; \theta_t$ and $E(\theta) = 1$

Θ^t,μ^t: relevant product space and probability measure

Utility Cost $C : D \to R$, $C(v)$ consumption needed to deliver utility $(1-\beta)v$, before taste shock.

Strategies and Plans

- A **reporting strategy** is a sequence of functions, $z = \{z_t(\theta^t)\}$, where $z_t(\theta^t)$ is an agent's report of his taste shock at t
- A **truthful reporting strategy** is $z_t^*(\theta^t) = \theta_t$
- A **plan** is a sequence of functions, $\{u_t(w, z^t)\}$ which deliver time t utility $(1-\beta)u_t(w, z^t)\theta_t$ to an agent w with reporting strategy, $z_t(\theta^t)$
- Let $U(w, u, z) = (1-\beta)E\sum \beta^t u_t[w, z^t(\theta^t)]\theta_t$

Allocations and Attainability

- An **allocation** is a plan that satisfies:
  1. $U(w, u, z^*) \geq U(w, u, z) \; \forall \; w, z$ (IC)
  2. $U(w, u, z^*) = w$ (PK)

Let ψ be a distribution of promised utilities.

- An allocation **attains ψ with resources** y if

$$\int_{D \times \Theta^t} C[u_t(w, \theta^t)] d\mu^t d\psi \leq y \quad \forall t$$

Planning Problem

- For a given ψ, find $\varphi^*(\psi)$, the minimum constant resources needed to attain that distribution with some (incentive compatible) allocation:

$$\varphi^*(\psi) = \inf\{y : \quad \exists \text{ allocation } u \text{ that attains } \psi \text{ with resources } y\}$$

- An allocation is **efficient** if it attains a distribution, ψ, with resources, $\varphi(\psi)$

Equivalent Recursive Representation

- An **allocation rule** is a sequence of pairs of functions, $\{f_t, g_t\}$ that give current utility and a future promised value as functions of current promised value w, and report,z_t:

$$u_t = f_t(w_t, z_t)$$

$$w_{t+1} = g_t(w_t, z_t)$$

  such that:

  1. $(1-\beta)f_t(w,\theta)\theta + \beta g_t(w,\theta) \geq (1-\beta)f_t(w,\hat{\theta})\theta + \beta g_t(w,\hat{\theta}))$ $\forall\hat{\theta} \in \Theta$ (IC)
  2. $\int_\theta [(1-\beta)f_t(w,\theta)\theta + \beta g_t(w,\theta)]d\mu = w$ (PK)

- Lemmas 3.1 and 3.2: $\exists$ an allocation,u, that attains ψ with resources, y iff $\exists$ an allocation rule, f, g, that attains ψ with resources y
- Let $S_g\psi$ be distribution of w induced by g.

A Bellman Equation

$$\varphi^{f,g}(\psi) = \max\left\{\int_{D\times\Theta} C[f(w,\theta)]d\mu d\psi \quad ; \quad \varphi^{f,g}(S_g\psi)\right\}$$

$$\varphi^{*}(\psi) = \inf_{f,g}\max\left\{\int_{D\times\Theta} C[f(w,\theta)]d\mu d\psi \quad ; \quad \varphi^{*}(S_g\psi)\right\}$$

subject to (IC) and (PK)

Lemma 4.2: a strategy for finding a solution If there exists two functions, φ_c, and φ_a, such that for all ψ:

1. $\varphi_c \leq \varphi^* \leq \varphi_a$
2. $lim_{n\to\infty}T^n\varphi_c = lim_{n\to\infty}T^n\varphi_a = \varphi$

then $\varphi^* = \varphi$

Upper and Lower Bounds

Autarky

- constant consumption level regardless of taste shock, such that utility w is delivered:

$$\varphi_a(\psi) = \int_w C(w) d\psi$$

- trivially incentive compatible so $\varphi_a(\psi) \geq \varphi^*(\psi) \quad \forall \psi$

Full Insurance

- constant utility level regardless of taste shock, so that agent is fully insured.
- not incentive compatible but the IC allocation that gives φ^* is feasible for this problem so $\varphi_c(\psi) \leq \varphi^*(\psi) \quad \forall \psi$

Solution

- Solutions only characterized for log, CARA and CRRA utility.
- With $V(c) = \log c$, solution has the form:

$$f(w, \theta) = r(\theta) + w$$

$$g(w, \theta) = h(\theta) + w$$

- Iterating yields

$$w_{t+1} = w_t + h(\theta)$$

$$Var(u_t) = Var(w_0) + (t-1)Var(h(\theta)) + Var(r(\theta))$$

Immiseration

- $E(h(\theta)) < 0 \quad \rightarrow E[w_t + 1 | w_t] < w_t$:
- Sub-martingale convergence theorem, utility drifts towards $-\infty$
- Expected consumption remains constant, variance of consumption is increasing without bound, concave utility must imply that expected utility is decreasing without bound
- Result holds for log of utility in CRRA and CARA case.

Possibilities for Decentralization

1. A Component Planner Decentralization:
   - 1 planner per initial utility level w_0
   - Planners take price sequence q_t as given
   - Each planner delivers an incentive compatible sequence of utilities to his agent to minimize cost given prices
   - Total resources is $\varphi^*(\psi_0)$ i.e. allocation is efficient
2. Full Decentralization with unmonitored bond trading is not possible