Endogenous Borrowing Constraints with Incomplete Markets

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Objective

- Endogenize borrowing constraints used in some incomplete markets models

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- Two ways:
  1. The nonnegative consumption borrowing constraint
  2. The no default borrowing constraint

The model

- **Environment** Discrete time

  Two types of infinitely lived investors denoted by $i = 1, 2$

  One non storable consumption good

  One risk-free one-period bond

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  u^i continuous with $u^{i\prime} > 0$ and $u^{i\prime\prime} < 0$

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- **Budget constraint** $c_t^i + p_t b_t^i \leq y_t^i + b_{t-1}^i$

- **Borrowing constraint** $b_t^i \geq -\underline{B}^i$ where $\underline{B}^i \geq 0$

Equilibrium

An equilibrium for the economy is a set of sequences

$$\{c_t^1, c_t^2, b_t^1, b_t^2, p_t\}_{t=0}^{\infty}$$

such that

1. Each investor in the economy maximizes his expected discounted lifetime utility subject to a stream of budget constraints and borrowing constraints;
2. Markets clear in each period, for each state of the world:

$$c_t^1 + c_t^2 = y_t^1 + y_t^2$$

$$b_t^1 + b_t^2 = 0$$

The nonnegative consumption borrowing constraint

- Implied by the Inada condition $\lim_{c\to 0} u'(c) = \infty$
- If interest rate is constant (Aiyagari, 1994),

$$\underline{B}^i = y^i_{min}/r$$

- If interest rate is stochastic and the economy is stationary,

$$\underline{B}^i = \frac{y^i_{min}}{\max_{\Omega|y^i=y^i_{min}} r(y^i_{min}, -\underline{B}^i)}$$

The nonnegative consumption borrowing constraint

Case of a growth economy with stochastic interest rate:

$$\underline{B}^i = \frac{\tilde{y}^i_{min}}{\max_{\Omega|\tilde{y}^i=\tilde{y}^i_{min}} \left(\frac{r(\tilde{y}^i_{min}, -\underline{B}^i) - g}{1+g} \right)}$$

The no default borrowing constraint

$$\underline{B}^i = \min_{Z_t \in \Omega_Z} \left\{ -\underline{b}^i(Z_t) : \; W^i(Z_t, \underline{b}^i(Z_t)) = V^i(Z_t) \right\}$$

where

$$W_t^i = E_t \left[\sum_{j=0}^{\infty} \beta^j u^i(c_{t+j}^i) \right]$$

and

$$V_t^i = E_t \left[\sum_{j=0}^{\infty} \beta^j u^i(y_{t+j}^i) \right]$$

Existence of a no default borrowing constraint

Proposition 1

Suppose that an equilibrium exists for the economy and the equilibrium asset holdings are nontrivial. If the vector of exogenous variables Z_t takes only a finite number of outcomes, and

$$\lim_{b_{t-1}\to-\infty} W_t^i(Z_t, b_{t-1}) = -\infty$$

then there exists a no default borrowing limit given by

$$\underline{B}^i = \min_{Z_t \in \Omega_Z} \left\{ -\underline{b}^i(Z_t) : \; W^i(Z_t, \underline{b}^i(Z_t)) = V^i(Z_t) \right\}$$

Algorithm

- Approximate the law of motions for the endowment processes by a finite state Markov process
- Guess a borrowing limit
- Solve for an equilibrium interest rate function (nonnegative consumption borrowing limit) or for the value function (no default borrowing limit)
- Use the result to find a new borrowing limit
- Use this new limit to solve for next round equilibrium interest rate function or value function and iterate until the borrowing limit converges

Numerical simulation: The nonnegative borrowing constraint

- $\beta = 0.98$ and $\gamma = 2$
- $\underline{B} = 7.5$ times total current income
- Bond returns

Moment	NC	CM
Mean	0.06123	0.06177
Std. Dev.	0.00884	0.00550

The no default borrowing limit

	β			
γ	$\beta = 0.95$	$\beta = 0.96$	$\beta = 0.97$	$\beta = 0.98$
1.05	0.18439	0.26197	0.39164	0.64680
1.10	0.19346	0.27251	0.40139	0.64900
1.30	0.23185	0.31162	0.43673	0.65868
1.50	0.26624	0.34648	0.46749	0.66927
2.00	0.34132	0.42047	0.53135	0.69721
3.00	0.46000	0.53323	0.62703	0.75146
4.00	0.55093	0.61728	0.69793	0.79817
5.00	0.62220	0.68216	0.75251	0.83665
6.00	0.67913	0.73353	0.79591	0.86820

Statistics of bond returns

	Mean		Std. Deviation	
γ	$\beta = 0.96$	$\beta = 0.98$	$\beta = 0.96$	$\beta = 0.98$
1.05	0.04449	0.03627	0.023184	0.012950
1.10	0.04535	0.03699	0.023785	0.013487
1.30	0.04861	0.03988	0.025910	0.015553
1.50	0.05175	0.04274	0.027935	0.017504
2.00	0.05924	0.04976	0.032297	0.021936
3.00	0.07295	0.06309	0.039107	0.029260
4.00	0.08489	0.07496	0.044122	0.034966
5.00	0.09482	0.08503	0.047728	0.039486
6.00	0.10249	0.09307	0.050249	0.042871

The pricing kernel

- In the standard frictionless consumption-based CAPM with complete markets,

$$p_t = E_t[m_{t+1}] = E_t[IMRS_{t+1}]$$

- In the incomplete markets environment,

$$p_t^i = E_t[IMRS_{t+1}^i] + \frac{\mu_t^i}{u^{i\prime}(c_t^i)}, \quad i = 1, 2$$

$$\mu_t^i > 0, \qquad \text{if } b_t^i = -\underline{B}^i$$

The pricing kernel

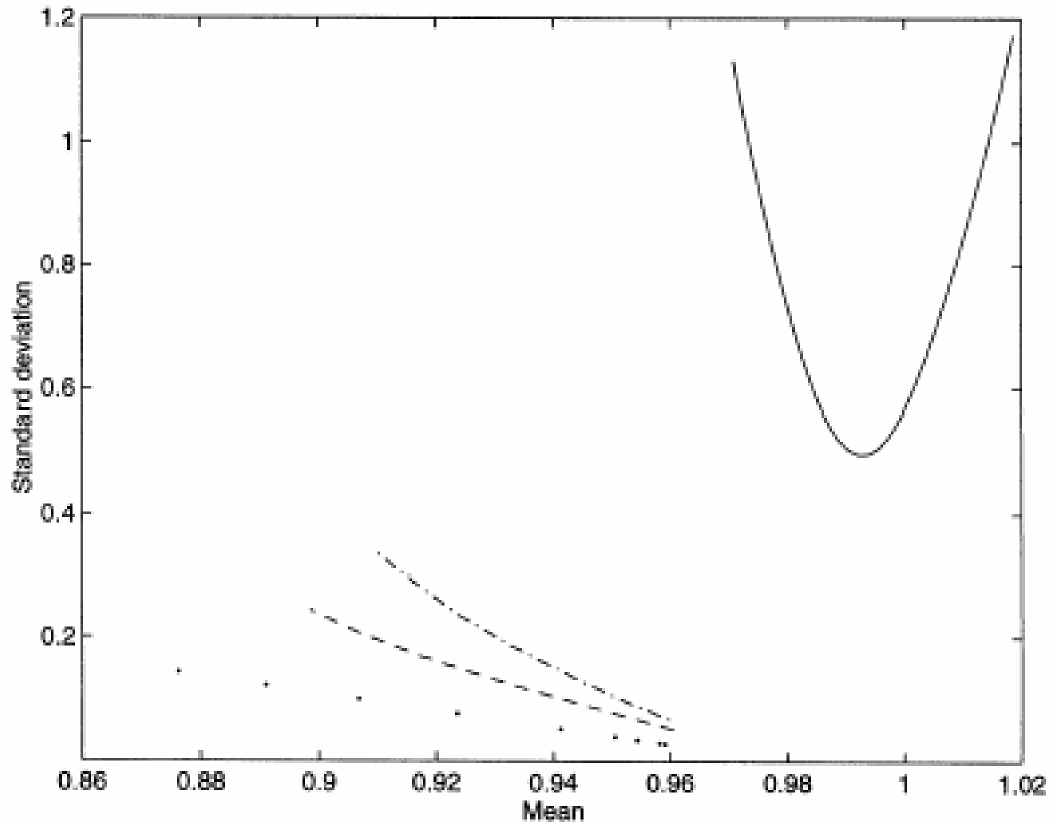

Figure 1. Mean and Standard Deviation Frontier of Stochastic Discount Factors. The solid line is the Hansen-Jagannathan volatility bound constructed using the annual stock and bond returns by compounding quarterly real value-weighted NYSE and three month Treasury bill returns from 1947:Q1 to 1993:Q4 (see Cochrane and Hansen (1992)). The dotted line is the mean and standard deviation locus for a complete market economy. The dashed line is the mean and standard deviation locus for a no-default borrowing constraint. The broken line is the mean and standard deviation locus for a 35 percent ad hoc borrowing constraint. The discount factor (β) is fixed at 0.98.

Summary

- For the parameters values used in this paper, both the nonnegative consumption borrowing constraint and the no default borrowing constraint are much looser than the ad hoc borrowing constraints used in other papers.
- The incomplete markets model with the two types of endogenous borrowing constraints examined in this paper fails to match the first and second moments of bond returns with their observed values.