On Repeated Moral Hazard with Discounting

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Objective and results

- Analyze optimal contracts in an infinitely repeated principal-agent model in which both the principal and the agent discount the future

Results

- History dependence can be summarized by including the agent's conditional expected utility as a state variable.
- There exists a critical value of output in terms of which the optimal contract can be entirely studied.

The model

- Infinitely repeated version of standard agency model
- The **principal** owns a technology that produces y_t at time t. y_t is drawn from $F(y_t|a_t)$ where $a_t \in [\underline{a}, \bar{a}]$ is the action taken at the beginning of t by the **agent** who operates the technology. After y_t has been observed, the agent gets paid I_t.
- The principal has access to perfect credit markets.
- **Moral hazard:** a_t is not observed by the principal.
- **Preferences:**

Agent:

$$W(\sigma_t, I, a) = \sum_{\tau=1}^{\infty} \beta^{\tau-1} \int H\left(a(\sigma_{t+\tau-1}), I(\sigma_{t+\tau})\right) d\pi(\sigma_{t+\tau}; \sigma_t, a)$$

Principal:

$$S(\sigma_t, I, a) = \sum_{\tau=1}^{\infty} \alpha^{\tau-1} \int G\left(y_{t+\tau} - I(\sigma_{t+\tau})\right) d\pi(\sigma_{t+\tau}; \sigma_t, a)$$

The contract design problem

An optimal contract is a sequence of strategies $\{I, a\} = \{I(\sigma_t), a(\sigma_t)\}$ such that I maximizes $S(I, a)$ subject to $W(I, a) \geq \bar{w}$ and $W(\sigma_t, I, a) \geq W(\sigma_t, I, \bar{a})$ for all σ_t and for all $\bar{a}$.

Reduction of the problem

Let w denote the discounted expected utility of the agent at the beginning of the period.

Any solution to the contract design problem can be characterized by 4 functions $a(w),\ I(w,y),\ U(w,y)$ and $V(w)$ which satisfy:

(i)
$$w = \int \{H(a(w), I(w,y)) + \beta U(w,y)\} f(y|a(w))dy$$
$$\geq \int \{H(a, I(w,y)) + \beta U(w,y)\} f(y|a)dy, \quad \forall a \in \mathbf{A}$$

(ii)
$$U(w,y) = \int \{H(a(U(w,y)), I(U(w,y),y'))$$
$$+ \beta U(U(w,y),y')\} f(y'|a(U(w,y)))dy' \quad \text{for all } w, y$$

(iii)
$$V(w) = \int \{G(y - I(w,y)) + \alpha V(U(w,y))\} f(y|a(w))dy \quad \text{for all } w$$

Additional assumptions

A.1 $\alpha = \beta$.

A.2 The principal is risk-neutral, and $G(y - I) = y - I$.

A.3 $H(I, a) = \phi(I) - \psi(a)$,
with $\phi' > 0,\ \phi'' < 0,\ \psi' > 0$ and $\psi'' > 0$.

A.4 $f(y|a)$ is twice continuously differentiable in a.

A.5 The monotone likelihood ratio condition holds,
i.e. $f_a(y|a)/f(y|a)$ is an increasing function of y.

A.6 The first-order approach is valid.

Characteristics of the optimal contract

The problem is to choose $a(w),\ I(w,y)$ and $U(w,y)$ to

$$\max \int \{y - I(y,w) + \beta V(U(w,y))\} f(y|a(w))dy$$

subject to

$$\int \{\phi(I(w,y)) + \beta U(w,y)\} f(y|a(w))dy - \psi(a(w)) = w \quad \text{(PK)}$$

$$\int \{\phi(I(w,y)) + \beta U(w,y)\} f_a(y|a(w))dy - \psi'(a(w)) = 0 \quad \text{(IC)}$$

Characteristics of the optimal contract

Proposition 4.1 The optimal contract satisfies

$$\frac{1}{\phi'(I(w,y))} = \lambda(w) + \mu(w)\frac{f_a(y|a)}{f(y|a)}$$

$$\int \{y - I(w,y) + \beta V(U(w,y))\} f_a(y|a(w))dy$$

$$+\mu(w)\left[\int \{\phi(I(w,y)) + \beta U(w,y)\} f_{aa}(y|a(w))dy - \psi''(a(w))\right] = 0$$

where $\lambda(w)$ is the multiplier on (PK) and $\mu(w)$ is that on (IC).

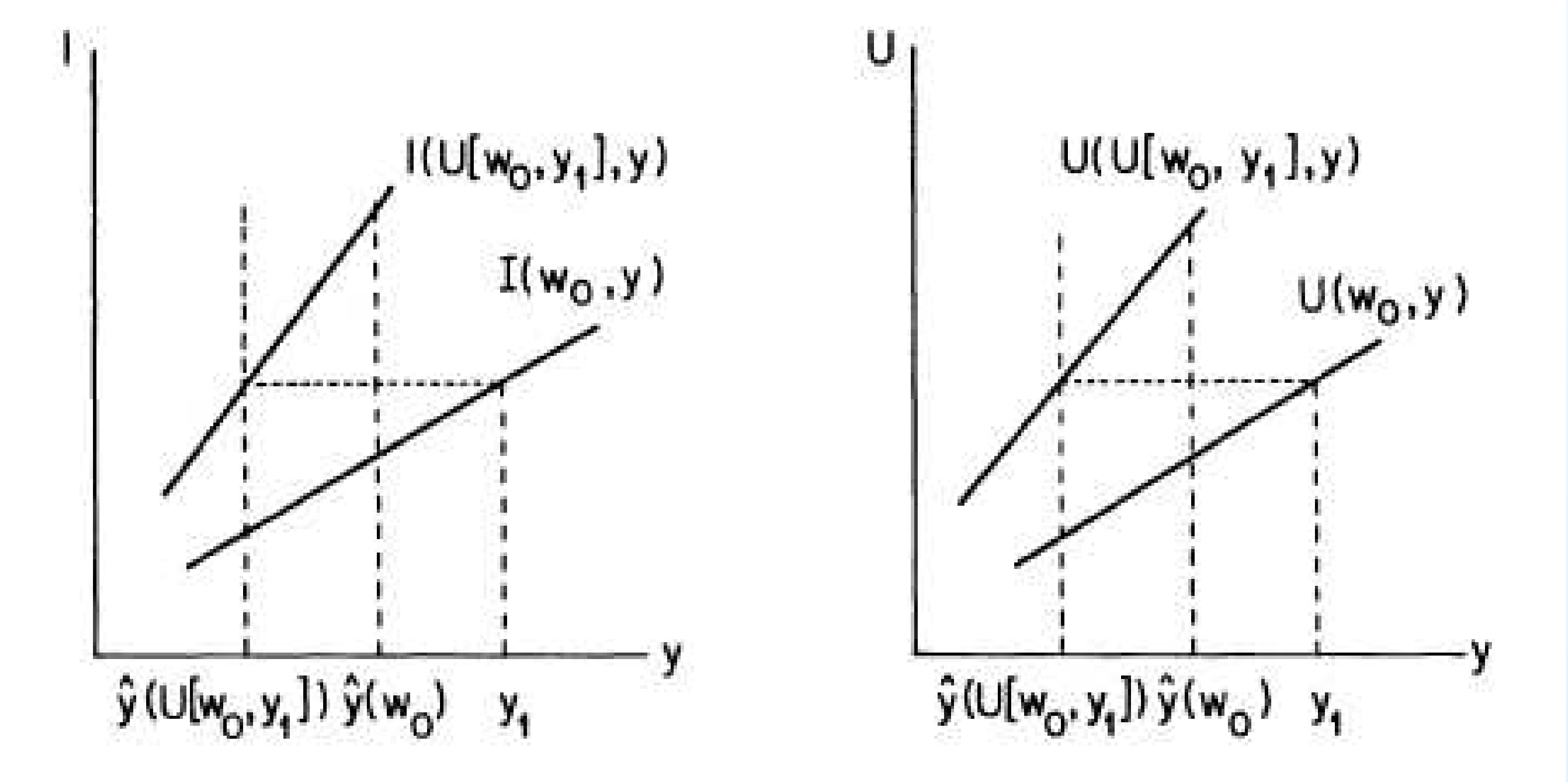
Proposition 4.2 U and V satisfy

$$V'(U(w,y)) = -\frac{1}{\phi'(I(w,y))}$$

Evolution of the contract over time

Proposition 4.3 For every w, there exist an output level $\hat{y}(w)$ such that $w = U(w, \hat{y}(w))$ and $f_a(\hat{y}(w)|a(w)) = 0$.

Proposition 4.5 MLRC implies $I_y > 0$ and $U_y > 0$.



Changes in the critical level of output

Assume that $\hat{y}$ increases when a increases and focus on the sign of a'.

Proposition 4.6 If $\phi'(I) < 1$ for all feasible compensations I, then $a'(w) > 0$. Similarly, if $\phi'(I) > 1$ for all feasible I, then $a'(w) < 0$. If ϕ' is sometimes greater and sometimes less than one, the sign of a' is not determinate.

Proposition 4.7 Suppose at some point in time the principal has promised the agent an increase in future discounted expected utility. If $a'(w) < 0$, then

$$\int \phi'(I(w, y)) I_w(w, y) f(y|a) dy > 0$$

Covariance property of discounted expected utilities

Proposition 4.9

For the case of $\phi' < 1$, in which $a' > 0$,

$$cov(V'(U), \phi'(I)I_w + \beta U_w) < 0.$$

Hence, when changes in w lead to an increased average marginal payoff to the agent, the marginal payoff to the principal decreases on average.

Since V is strictly concave, this implies that the average $U(w, y)$ has increased.