Sequential Equilibria in a Ramsey Tax Model

Christopher Phelan, Ennio Stachetti

Sep 25 2007

Introduction

- Puts the optimal government policy problem into a game theoretic framework.
- Model it as a strategic game between a government and a continuum of households.
- The government behaves strategically while the households respond competitively to the governments actions.
- The governments strategy serves an important role - describing the public's rule for forecasting the government's behaviour and the government will confirm such expectations in equilibrium.

Introduction

- APS - Switch away from strategies and focus on one period outcomes and continuation values.
- Think of the government as choosing a period action and a continuation value (reputation) that it will want to confirm in equilibrium.
- They show that keeping track of the marginal value of capital and of the government's continuation value suffices here to solve for the class of games in hand.
- They solve for the entire value set and they study whether the Chamley-Judd result holds under non-commitment.

The Game

- Benevolent government and a continuum of households, endowed with a unit of leisure each period.
- Single consumption good produced with CRS technology f(K,L). Agents start with the same level of capital stock k_0.
- Beginning of the period the outcome x_t of a public random variable X_t uniform on [0,1] is publicly observed.
- Subsequently, the government chooses tax rates $\{\tau_{kt}, \tau_{lt}\}$ on capital and labour.
- Then the agents choose their period consumption, labour and next period capital. Period t utility is given by $u(c_t, l_t) + g(G_t)$.

The Game

- Capital fully depreciates each period. The government uses the proceedings to finance a public good.
- The assumptions on g(.), f(.), and u(.) are standard in a one sector growth model.
- Per capita government revenue is:

$$G_t = \tau_{k,t} p_{k,t} K_t + \tau_{l,t} p_{l,t} L_t$$

- Income to household:

$$y_t = (1 - \tau_{k,t}) p_{k,t} K_t + (1 - \tau_{l,t}) p_{l,t} L_t$$

- Denote the game $\Gamma(\mathrm{k}_0)$, and $\Gamma(\mathrm{k}_0, x_0)$ is the subgame after the realization of x_0 but before the govt sets taxes.
- Focus only on symmetric strategy equilibria.

Players' Strategies

- Let $\tau_t = (\tau_{k,t}, \tau_{l,t}) \in \underline{[\tau, \overline{\tau}]^2}$ be the govt action at t.
- An action for the household is a pair $a_t = (l_t, \theta_t) \in [0,1]^2$ of labour and saving choice at time t.
- A public history at t is a collection $\{x^t, \tau^t, a^t\}$
- Strategies are symmetric across agents and no agent can individually affect the game path. Since we ignore simultaneous deviations in studying SE, public history can be reduced to $h^t = \{x^t, \tau^t, a^t\}$
- Define the govt strategy and household strategy as:

$$\tau_t = \sigma_G(t)(h^{t-1}, x_t)$$

$$a_t = \sigma_C(t)(h^t); h^t = (h^{t-1}, x_t, \tau_t)$$

Competitive Equilibria

- First characterize the CE equilibria of the dynamic economy for an arbitrarily specified tax policy.
- The government does not have commitment power. Along the equilibrium path agents act as if such policy was in place.
- Let $q = (l_t, c_t, k_{t+1})(x^t)$ denote CE of $\Gamma(k_0|\tau)$.
- $p_{k,t}(x^t) = (1 - \tau_{k,t}(x^t))f_k(k_t(x^{t-1}), l_t(x^t))$
- $p_{l,t}(x^t) = (1 - \tau_{l,t}(x^t))f_l(k_t(x^{t-1}), l_t(x^t))$ denote the after tax prices.
- The expected marginal value of capital is:

$$m_{t+1}(x^t) = E_{x_{t+1}}[p_{k,t+1}(x^{t+1})u_c(l_{t+1}(x^{t+1}), c_{t+1}(x^{t+1}))|x^t]$$

Competitive Equilibria

- Consider the static economy with augmented utility function $u(c_t, l_t) + \beta * m_{t+1} * k_{t+1}$, where m_{t+1} exogenous.
- $CE(k_t, \tau_t, k_{t+1})$ is the set of static CE of this economy.
- Main result: q is a CE of $\Gamma(k_0|\tau)$ iff for all t>0 , x^t, $(l_t, c_t, k_{t+1})(x^t) \in CE(k_t(x^{t-1}), \tau_t(x^t), m_{t+1}(x^t))$

Sequential Equilibria

- A symmetric strategy profile σ for $\Gamma(k_0)$ generates a random outcome path $(l_t, c_t, k_{t+1})(x^t)$.
- The value of $\sigma \in \Sigma$ is the government's normalized total discounted payoff:

$$\Sigma_G(k_0, \sigma) = (1-\beta)E[\sum_{t=0}^{\infty} \beta^t [u(c_t, l_t) + g(G_t)]]. \quad (1)$$

- Also, define $\sigma|_{x_0}$ as the strategy profile generated after the first realization of the public variable.
- Definition: A symmetric strategy profile σ is an SSE for $\Gamma(k_0)$ if for all t, h^{t-1}, current capital k_t and $x_t \in [0,1]$:

Sequential Equilibria

- (i) For any deviation strategy γ from the government:

$$\Gamma_G(k_t, \sigma|_{(h^{t-1},x_t)}) \geq \Gamma_G(k_t, (\sigma_C|_{(h^{t-1},x_t)}, \gamma|_{x_t}))$$

- (ii) For any $\widehat{\tau_0}$, where $(\widehat{\tau}, \widehat{q})$ is the tax-policy and allocation generated by $\sigma|_{(h^{t-1},x_t,\widehat{\tau_0})}$ the agents respond competitively. That is, $\widehat{q}$ is a CE for $\Gamma(k_0|\widehat{\tau})$.
- A single hhd deviation is not detectable and does not affect the eqm path. The rest of agents continue to play the CE of $\Gamma(k_0|\tau)$.
- Therefore, it suffices to make his strategy only as a function of public history. Whatever the household does after the deviation is irrelevant to the game, and not optimal to him.

Solving for the eqm value set

- Define the ex-ante and ex-post eqm value correspondences $V : [\underline{k}, \overline{k}] \mapsto R^2$, and $V^R : [\underline{k}, \overline{k}] \mapsto R^2$ where:
- (i) $V(k_0) = \{\Gamma(k_0, \sigma) : \sigma$ is an SSE of $\Gamma(k_0)\}$
- (ii) $V^R(k_0) = \{\Gamma(k_0, \sigma|_{x_0}) : \sigma$ is an SSE of $\Gamma(k_0, \sigma|_{x_0})\}$
- So, $V(k_0)$ is the collection of all pairs (m,v) such that m is the expected marginal value of capital and v is the expected lifetime payoff of the government.
- We want to calculate $V(k_0)$.

Solving for the eqm value set

- Consider a compact and convex valued correspondence $W : [\underline{k}, \overline{k}] \mapsto R^2$
- A vector $\epsilon = (\tau, l, c, k_+, m_+, v_+)$ is consistent wrt to W at k_0 if $(l, c, k_+) \in CE(k, \tau, m_+)$,$k_+ \in [\underline{k}, \overline{k}]$, and $(m_+, v_+) \in W(k_+)$.
- The vector ϵ is admissible wrt to W at k, if it is consistent wrt to W at k, and $\Sigma(k, \epsilon) \geq X(k)$. We can think of this as the govt incentive constraint. If the government deviates to a new tax rate, then the hhd beliefs are manipulated so that they coordinate on the worst possible future payoff to the government.
- Let $\Psi^G(k, \epsilon) =$ lifetime govt payoff, $\Psi^C(k, \epsilon) =$ hhd marginal value of capital.

Solving for the eqm value set

- For the value correspondence W, let:
- $B(W)(k)$=co($\{\Psi(k,\epsilon)|\epsilon$ is admissible wrt W at k$\}$)
- So, think of W as giving the continuation payoffs for next period, and ϵ the current period action supporting such payoffs.
- Of course in equilibrium such continuation payoffs need to be eqm payoffs themselves, and should also be supported by some other consistent and admissible vector.
- Self Generation: W is self-generating if W(k) $\subset$ B(W)(k) for all k. One can show that in this case B(W) $\subset$ V.

Solving for the eqm value set

- Main Theorem: B(V) = V, and V is the largest fixed point of the B() operator.
- Algorithm: Start with V ⊂ B(W) ⊂ W and keep iterating until $W_{n+1}{=}B(W_n)$ converges to V.

Steady states and equilibria

- $\epsilon = (\tau^s, l^s, c^s, k^s, m^s, v^s)$ is a steady state vector if it is a CE of the game starting at k^s with taxes fixed at τ^s.
- If such vector is also admissible then it gives the constant path of play of $\Gamma(k^s)$.
- Result: If a strategy σ of Γ is a best strategy (on the upper boundary of V), and if σ converges to a steady state vector ϵ^s then either (i) $\tau^k = 0$, (ii) $v^s =$the worst possible eqm payoff given k^s.

Example

- The steady state level of k with commitment is k=1.178. We see from Fig.1 that this steady state cannot be supported as an SSE since the two locii do not intersect. Moreover, from any initial capital level we cannot support the Chamley-Judd state.
- The best SSE converges to steady state k=0.743. In this case L(k), and V(k) intersect once, and the government incentive constraint is binding (therefore $\tau_k > 0$). Here, $\tau =$ (0.27,0.26).
- Figure 4 and 5 give the time path of capital and tax rates for initial k=0.054,0.743,1.178. They converge at the steady level k=0.743, $\tau = 0.27$.

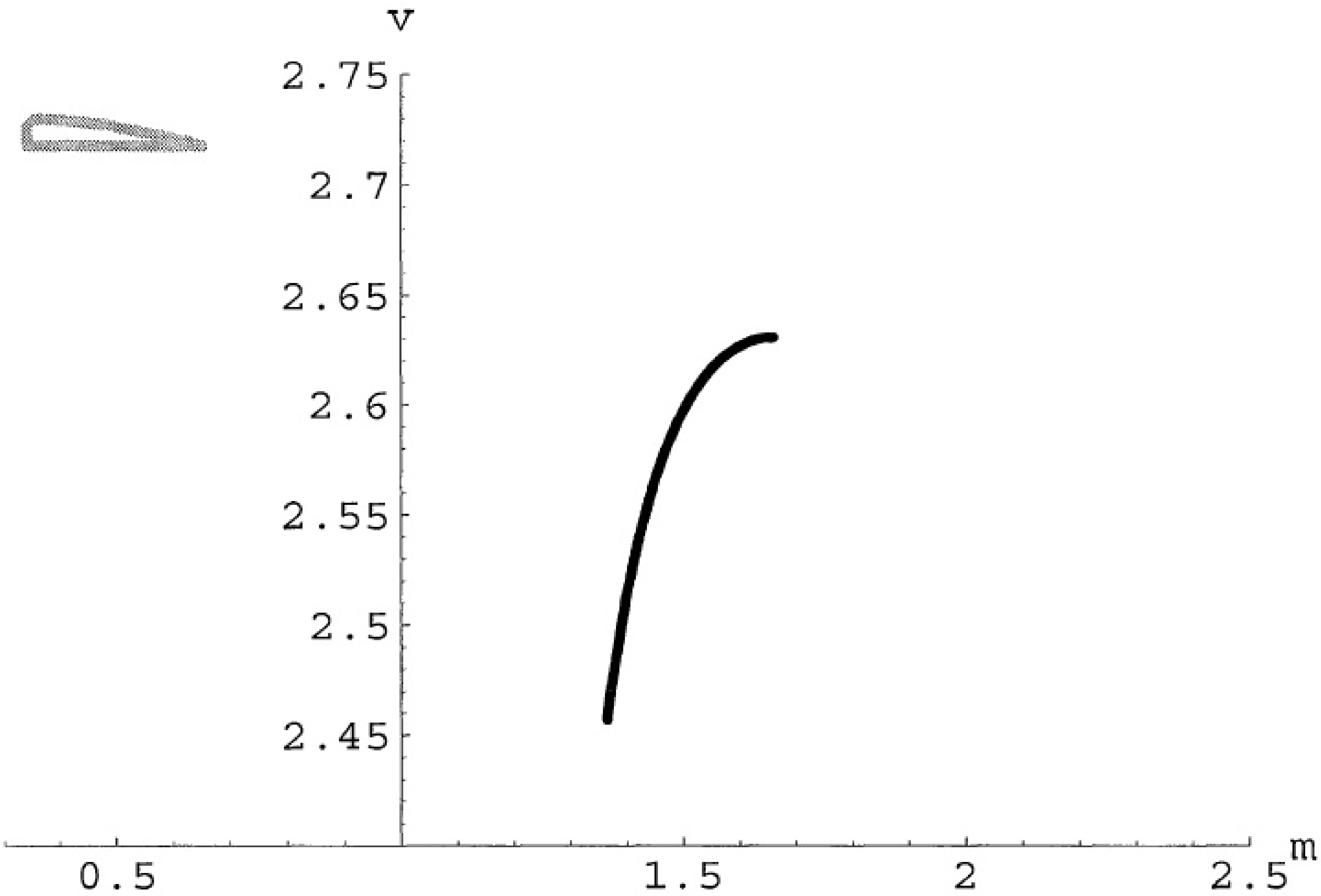

FIGURE 1.—Equilibrium value correspondence and steady states for $k = 1.178$.

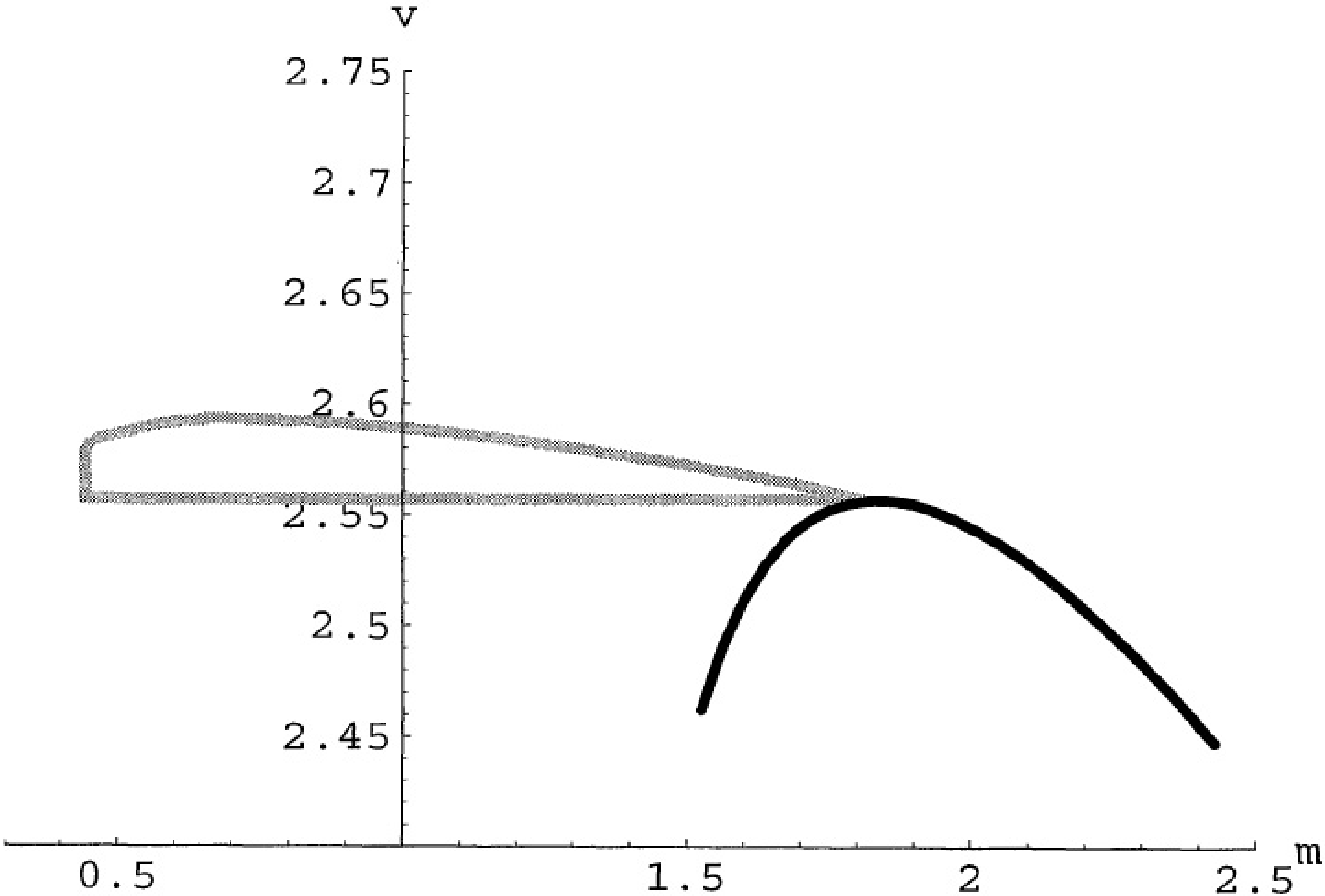

FIGURE 3.—Equilibrium value correspondence and steady states for $k = .743$.

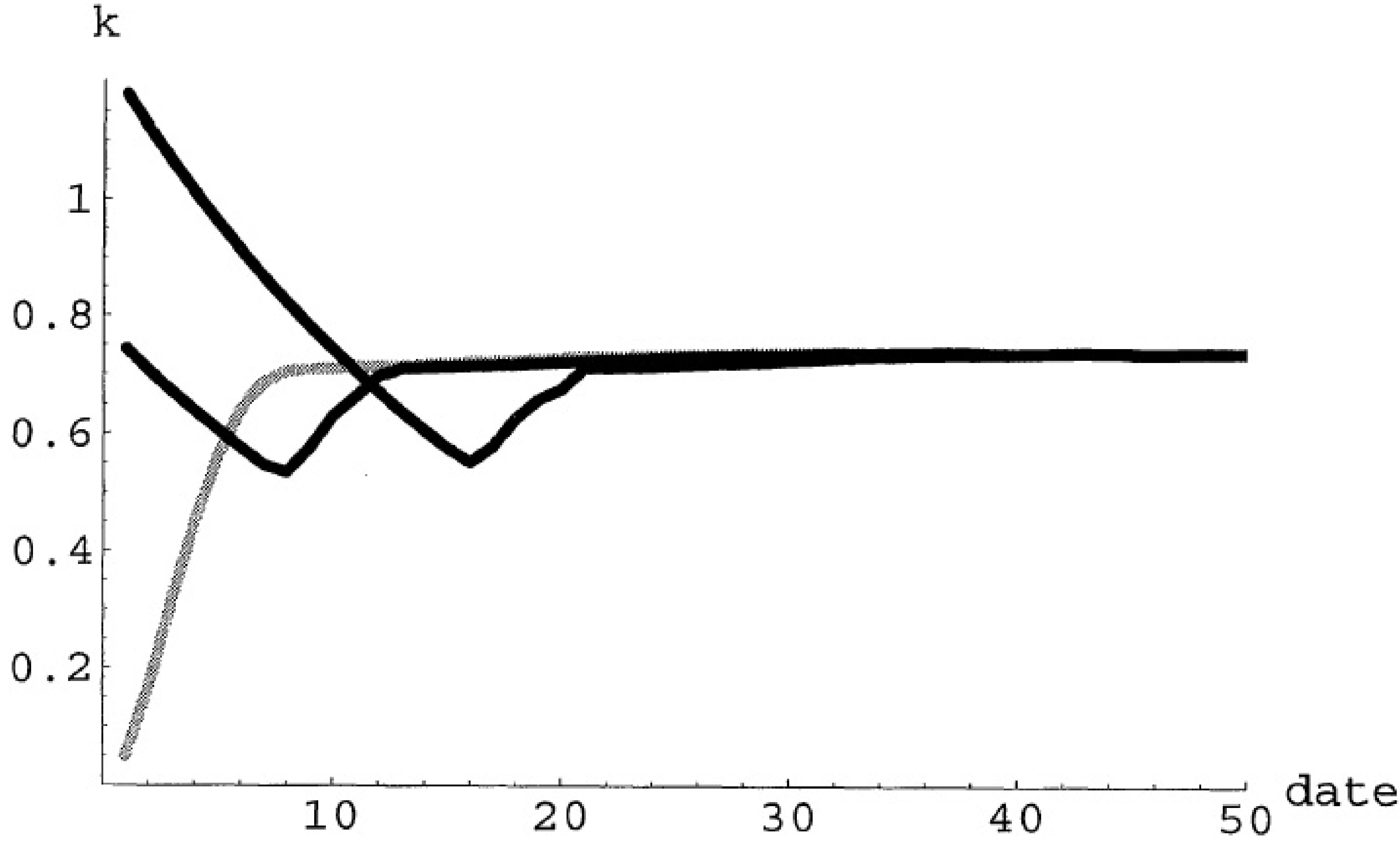

FIGURE 4.—Time paths of capital from $k = \{.054, .743, 1.178\}$.

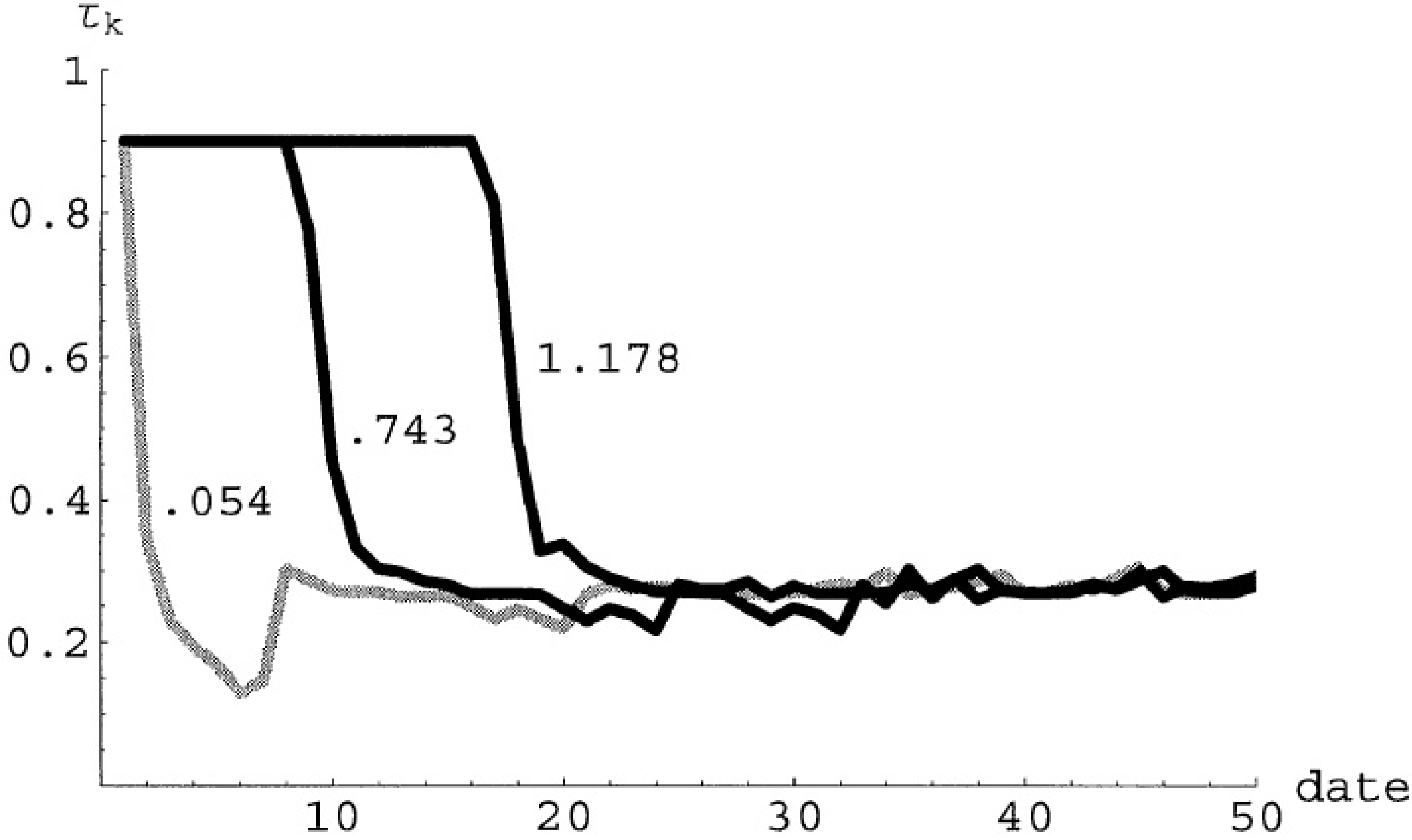

FIGURE 5.—Time paths of capital taxes from $k = \{.054, .743, 1.178\}$.