Monetary policy and price level determinacy in a CIA economy

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Oct 2 2007

Introduction

- Considers determinacy of equilibria in a CIA economy.
- Compare two monetary regimes: Constant money growth rate and interest rate pegging.
- In the first regime, indeterminacy of perfect foresight equilibria and sunspot equilibria prevail.
- In the second regime, only unique steady state equilibria occur.

The Model

- Representative consumer maximises expected value of:

$$\sum_{t=0}^{\infty} \beta^t V(c_{1t}, c_{2t})$$

- c_{1t} and c_{2t} denote consumption of cash and credit goods.
- Consumer endowed with y $>$ 0 units of productive capacity, to be used in production of cash and credit goods.
- V is twice continously differentiable and satisfies:
  (A1) V_1, $V_2 > 0$; V is strictly concave.
  (A2) $\widehat{c_1} = \text{argmax } V(c_1, y - c_1)$ satisfies $0 < \widehat{c_1} < y$.

The Model

- Hhd enters period with some net wealth, then splits up into a buyer and seller.
- Buyer enters the financial mkt- chooses portfolio of one period contingent claims (possibly dependent upon sunspots).
- Then he enters goods mkt - chooses mix of cash and credit goods. If he buys on credit he pays the seller next period.
- Seller stands ready to sell y goods on cash and credit at price p_t.
- No specific assumption about the nature of the sunspot process. Denote information set at t by I_t.

Budget Constraints

- Given the processes $\{p_t, r_{t+1}, M_t, H_t\}$ the household's budget constraint a.d.a.c, is given by:

$$M_t^d + E_t[r_{t+1} B_{t+1}^d] < W_t + H_t$$

$$p_t c_{1t} \leq M_t^d$$

$$c_{1t}, c_{2t} \geq 0$$

$$W_{t+1} = M_t^d + p_t(y - c_{1t} - c_{2t}) + B_{t+1}^d$$

$$W_{t+1} \geq -q_{t+1}^{-1} \sum_{j=1}^{\infty} E_{t+1}[Y_{t+j+1}]$$

- where $Y_{t+j+1} = q_{t+j+1} p_{t+j} y + q_{t+j} H_{t+j}$, is the limit on the hhd ability to borrow.

Monetary-Fiscal Regime and REE

- Also, the government budget constraint is given a.d.a.c:

$$M_t + E_t[r_{t+1}B_{t+1}] = M_{t-1} + B_t + H_t$$

- Defn: A rational expectations equilibrium (REE) is a set of stochastic processes $\{p_t, r_t, R_t, c_{1t}, c_{2t}, M_t^d, B_t^d, M_t, B_t, H_t\}$ such that:

(i) $\{p_t, r_{t+1}, M_t, B_{t+1}, H_t\}$ are consistent with the policy regime specification

(ii) the variables $\{c_{1t}, c_{2t}, M_t^d, B_{t+1}^d\}$ are an optimal plan given processes for prices, transfers and initial wealth W_0

(iii) markets clear

Some preliminary results

1. The demand for securities is not defined unless $R_t \geq 1$.
2. The set of consumption and money plans for the consumer is equivalent to the present value budget constraint plus CIA:

$$\sum_{t=0}^{\infty} E_t[X_{t+1}] \leq \sum_{t=0}^{\infty} E_t[Y_{t+1}] + W_0$$

$$X_{t+1} = q_t p_t c_{1t} + q_{t+1} p_t c_{2t} + (q_t - q_{t+1})(M_t^d - p_t c_{1t})$$

3. A plan for consumption and money holdings is optimal iff:

$$V_1(c_{1t}, c_{2t}) = R_t V_2(c_{1t}, c_{2t})$$

$$V_1(c_{1t}, c_{2t}) r_{t+1} = \beta(p_t / p_{t+1}) V_1(c_{1t+1}, c_{2t+1})$$

CIA holds with equality unless $\mathrm{R}_t = 1$.

Some preliminary results

4 The set of processes $\{p_t, r_{t+1}, R_t, c_{1t}, c_{2t}, M_t, B_{t+1}, H_t\}$ is an REE iff:

(i) the variables $\{p_t, c_{1t}, c_{2t}, M_t\}$ satisfy:

$$c_{1t} = min(M_t/p_t, \widehat{c_1}); c_{2t} = y - c_{1t}$$

(ii) the variables $\{p_t, M_t\}$ satisfy:

$$M_t^{-1}F(M_t/p_t) = \beta * E_t[M_{t+1}^{-1}G(M_{t+1}/p_{t+1})]$$

$$F(z) = zV_2^*(min\{z, \widehat{c_1}\}); G(z) = zV_1^*(min\{z, \widehat{c_1}\})$$

(iii) The budget constraint is satisfied a.d.a.c:

$$\sum_{j=0}^{\infty} E_t[(q_{t+j} - q_{t+j+1})M_{t+j}] \leq \sum_{j=0}^{\infty} E_t[q_{t+j}H_{t+j}] + q_t W_t$$

Constant money growth

- From 4, suffices to consider processes $\{p_t, r_t, M_t, B_t, H_t\}$ that satisfy (ii),(iii), and:

$$r_{t+1} = \beta(M_t/M_{t+1})[G(M_{t+1}/p_{t+1})/G(M_t/p_t)]$$

- FIRST CASE: Money grows exogenously with growth rate π $>$ -1.
- Calculating the equilibria reduces to finding solution to:

$$F(z_t) = (\beta/1+\pi)E_t[G(z_{t+1}]; z_t = M_t/p_t$$

$$\lim_{T\to\infty} \beta^T E_t[G(z_T)] = 0$$

- Note that under this regime net securities are always zero.

Steady States

- If $\pi = \beta$-1, then any z$\geq \widehat{c_1}$ is a steady state monetary eqm. Agents will consume $\widehat{c_1}$ every period in cash goods and carry the rest of the money into next period to pay for the credit goods (graph 2)
- If $\pi < \beta$-1, no steady state exists. The government is taxing away resources at such a rate that the households real balances eventually go to zero (graph 3)

Steady States

- If $\pi > \beta$-1, then a sufficient condition for existence of a SS is:

$$\lim_{c \to 0} [V_2^* / V_1^*] > (\beta/1+\pi)$$

The MU of consuming cash goods is higher than that of credit goods close to zero. The agent would like to save some real balances each period. At z=$\widehat{c_1}$, the two MU's equal, so he will choose some intermediate steady balances (graph 4)

Indeterminacy of equilibria

(A4) The following assumption states that real balances do not play an essential role:

$$\lim_{c \to 0} cV_1^*(c) = 0;$$

- If the preferences satisfy all of the above then there exists a continuum of perfect foresight equilibria (PFE) and sunspot equilibria (SE) such that real money balances approach zero asymptotically
- PFE($\pi \geq 0$) - In this case the hhd expects increased transfers over time, and hence it wants to spend more. After buying in cash, it will demand more in credit. Sellers refuse to sell unless the price is too high, since they get paid next period. This brings an upward spiral in prices.

Indeterminacy of equilibria

- PFE(β-1 $\leq \pi < 0$)- Hhd expects increased taxes over time, and wants to save more today. Initially their money balances are so low, that they will buy a lot in credit. This will keep prices up, and make whatever they save worth even less tomorrow.(figure 5)
- SE - If balances today are z_t then we can pick values for cash balances $\{z_{t+1}\}$ depending on the sunspots tomorrow such that they satisfy the intertemporal optimality constraint and $z_{t+1} < z_t$. Construct a stochastic process for $\{z_t\}$ by proceeding recursively this way.

Indeterminacy of equilibria

- If $\pi < \beta - 1$ real balances go to zero asymptotically, if eqm exists. The government is taxing at such a high rate tha consumers substitute into credit more and more as time goes. However, sellers tomorrow will also pay even higher taxes so they will keep prices high. The economy will eventually be demonetized.
- For $\lim_{c \to 0} cV_1^*(c) > 0$, then self-fulfilling inflations are impossible. However, the results remain the same regarding self-fulfilling deflations for $\beta < \pi < 0$. For $\pi \geq 0$, self-fulfilling deflations are impossible, and therefore the stationary eqm is the unique one, if it exists.

Conclusions

- Money growth rates close to the Friedman rule will bring about indeterminacy of equilibria with low levels of welfare
- Conflict between choosing a money growth rate resulting in high welfare and one that results in a unique equilibrium.
- See whether interest rate pegging fares better.

Interest rate peg

- A regime that fixes a constant nominal interest rate R, together with real net transfers h
- The set of processes $\{z_t, M_t, W_t\}$ is an REE if they satisfy:

$$R = G(z_t)/F(z_t)$$

$$W_{t+1} = RW_t + [Rhz_t^{-1} - (R-1)]M_t$$

$$(\frac{R-1}{R})\sum_{j=0}^{\infty}\beta^j E_t[G(z_{t+j})] = h\sum_{j=0}^{\infty}\beta^j E_t[G(z_{t+j})/z_{t+j}] + G(z_t)\frac{W_t}{M_t}$$

Interest rate peg

- Suppose there exists z s.t R(z)=R > 1. Then there is a unique SS equilibrium where:

$$W_t = W_0(1+\pi)^t; \pi = (\beta R - 1)$$

$$M_t = \mu M_0(1+\pi)^t$$

$$B_{t+1} = (\beta R - \mu)W_t$$

$$p_t = \frac{\mu}{z}W_t$$

$$c_{1t} = z; c_{2t} = y - z$$

Interest rate peg

- Let R = 1. Then there is a unique solution $\{p_t, r_{t+1}, W_t, c_t\}$. However the money and security holdings are indeterminate.

$$W_t = W_0 \beta^t$$

$$p_t = \frac{1-\beta}{-h} W_t$$

$$c_{1t} = \widehat{c_1}; c_{2t} = y - \widehat{c_1}$$

$$M_t \geq \frac{1-\beta}{-h} \widehat{c_1} W_t$$

$$B_{t+1} = W_t - M_t$$

Conclusions

- Any interest rate peg results in a unique SS equilibrium.
- Such a regime results in a steady state allocation and prices that can also be reached with a constant growth rate of money regime.
- However, the latter regime can result in indeterminate equilibria with low welfare. For example, R $< \beta^{-1}$ maps to π < 0, which has indeterminate equilibria for sure.
- Hence, in the case when the welfare of the hhd is highest (including the optimal steady state), interest rate peg can ensure the desired eqm while constant money growth rule cannot.