Technological Growth, Asset Pricing and Consumption Risk

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Introduction

- Contemporaneous consumption growth shows little correlation with excess returns $\Rightarrow$ Empirical failure of CAPM
- Consumption over longer horizons (3 years) shows high correlation with excess return (Parker and Julliard, JPE 2005) $\Rightarrow$ long horizon versions of CAPM perform better
- This paper tries to rationalize the new well documented fact in a GE framework with a non standard production sector

The model: Timing

- There is a sequence of different technological epochs N, $N \in (-\infty ..., -1, 0, 1 ... + \infty)$, that arrive at the Poisson rate $\lambda > 0$
- At the beginning of epoch N a large scale technological change occurs and each firm draws a tree specific productivity $\zeta(i_{j,N}) \backsim U(0,1)$ i.i.d. across epochs and $\zeta(i) = \zeta_0(1-i)^\nu$, $i \in [0,1] \Rightarrow$ heterogeneity across firms
- Within each epoch, a firm decides if to adopt the new technology, paying a fixed cost, or not. Once the epoch changes the firm loses the option to plant a tree of any previous epochs
- At each time t a "small", disembodied aggregate shock θ_t hits the entire economy, $\frac{d\theta_t}{\theta_t} = \mu dt + \sigma dB_t$

The model: Technology

- There is a continuum of firms indexed by $j \in [0,1]$
- Earnings stream of a tree planted in N:

$$Y_{N,i,t} = (\bar{A})^N \zeta(i_N)\theta_t, \qquad (\bar{A})^N\text{: vintage effect, } \bar{A} > 1 \tag{1}$$

- Total earnings:

$$Y_{j,t} = \sum_{n=-\infty}^{N} \bar{A}^n \zeta(i_{j,n}) 1_{(\tilde{X}_{j,n}=1)} \theta_t = X_{j,t}\theta_t \tag{2}$$

- Total earnings (output) in the economy at t:

$$Y_t = \int_0^1 Y_{j,t}\,dj = \int_0^1 X_{j,t}\,dj\,\theta_t = X_t\theta_t \tag{3}$$

The model: Technology

- Share price: $P_{N,j,t} = P^A_{j,t} + P^o_{N,j,t} + P^f_{N,t}$

$$
\begin{aligned}
P^A_{j,t} &= X_{j,t}\Big(E_t \int_t^\infty \frac{H_s}{H_t}\theta_s ds\Big) \\
P^o_{N,j,t} &= \sup_\tau E_t\Big\{1_{(\tau<\tau_{N+1})}\Big[\Big(\bar{A}^N\zeta(i_{j,N})\int_\tau^\infty \frac{H_s}{H_t}\theta_s ds\Big) - \frac{H_\tau}{H_t}q_\tau\Big]\Big\} \\
P^f_{N,t} &= E_t\Big(\sum_{n=N+1}^\infty \frac{H_{\tau_n}}{H_t}P^o_{N,j,\tau_n}\Big)
\end{aligned}
$$

- The problem of the firm is to choose the optimal stopping time τ within each epoch

Optimal Stopping Time

$$\tau^*_{j,N} = \inf_{\tau_N \leq t \leq \tau_{N+1}} \left[t : \frac{\theta_t}{M_{\tau_N}} \geq \frac{\Xi}{\zeta(i_{N,j})} \right] \qquad \Xi > 0 \tag{4}$$

- No firm will find it optimal to install a tree at the beginning of the new epoch
- The firms with the more productive trees will start planting first
- Strong correlation effect due to the continuity of ζ and θ

Preferences

- There is a continuum of consumers/gardeners aggregated in a single representative agent that owns all the firms
- $U_t = U(C_t, M_t^C) \qquad M_t^C = \max_{s \le t}\{C_s\}$
- $U_C > 0$, $U_{CC} < 0$, $U_{M^C} < 0$ (envy), $U_{CM^C} > 0$ (catching up with the Joneses)
- Consumer's problem

$$\max_{C_s, dl_s} E_t \Big[\int_t^\infty e^{-\rho(s-t)} U(C_s, M_s^C) ds - \int_t^\infty \underbrace{e^{-\rho(s-t)} U_C(s) \eta(s)}_{\text{fixed disutility per tree planted}} dls \Big]$$

$$\text{s.t. } E_t \Big(\int_t^\infty \frac{H_s}{H_t} C_s ds \Big) \le \int_0^1 P_{N,j,t} dj + E_t \Big(\sum_{n=-\infty}^{N_t} \int_t^\infty \frac{H_s}{H_t} q_s dls \Big)$$

Equilibrium

A competitive equilibrium for this economy is a set of stochastic processes $\{C_t, X_t, K_{n,t}, dl_t, q_t\}$ s.t.

- C_t and dl_t solve the optimization problem of the household
- Firms determine the optimal time to plant a tree by solving the problem described before
- The goods market clears : $C_t = Y_t$ for all $t \geq 0$
- The gardening market clears

$$dl_t = dK_{n,t} \qquad K_{n,t} = \int_0^1 1_{(\tilde{X}_{j,n}=1)} dj$$

- The markets for all assets clear

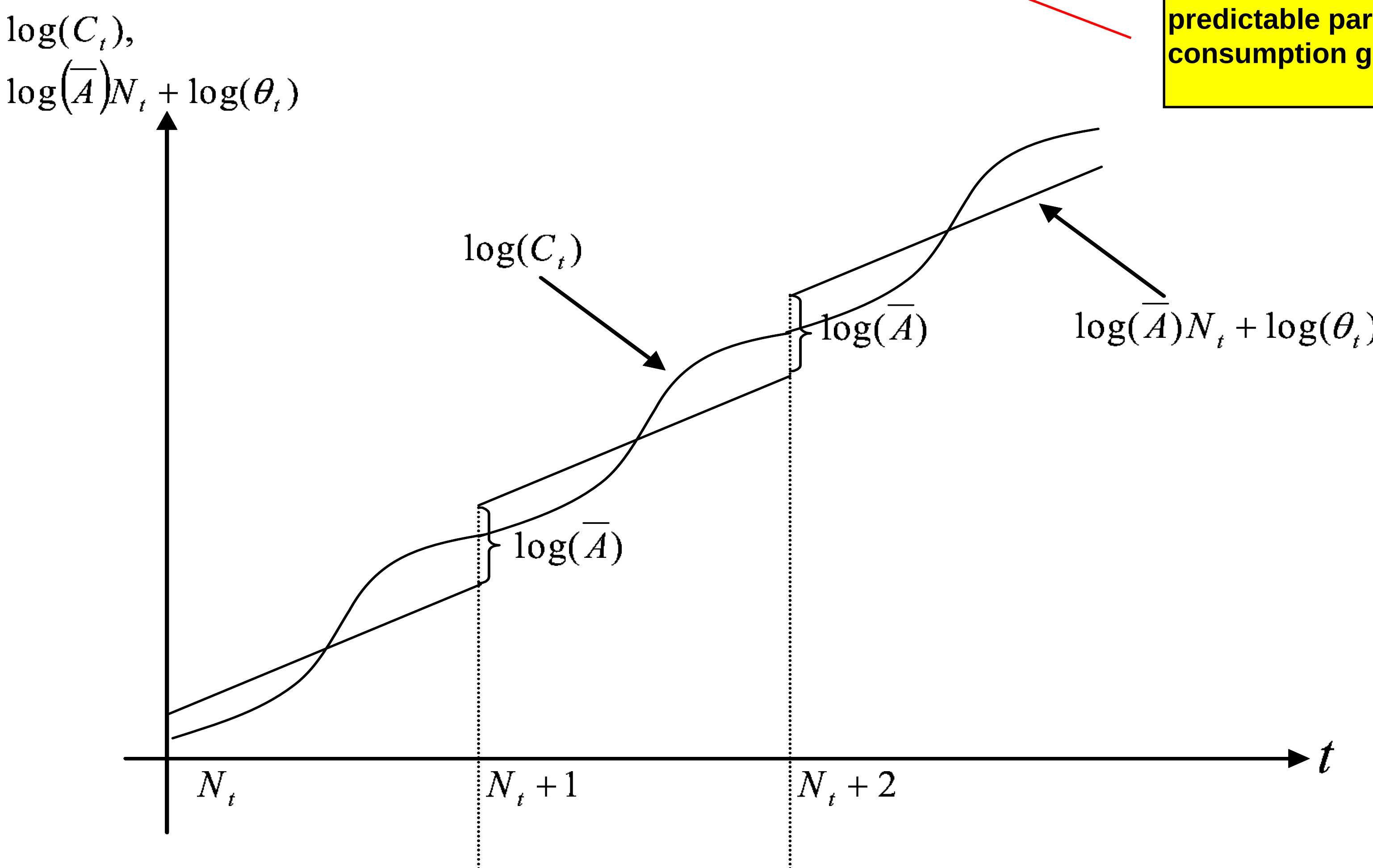
Parametrization

- $U(C_t, M_t^C) = (M_t^C)^\gamma \frac{C_t^{1-\gamma}}{1-\gamma} = \frac{(\frac{C_t}{M_t^C})^{-\gamma} C_t}{1-\gamma} \qquad \gamma > 1$
  - SDF unaffected by investment decisions:
    $U_C = (\frac{C_t}{M_t^C})^{-\gamma} = (\frac{\theta_t}{M_t})^{-\gamma} \qquad M_t = \max_{s \le t} \theta_s$
- $q_t = q\bar{A}^N M_{\tau,N}$
- $M_{\tau,N}$ =historical maximum at the start of new epoch
  - the compensation for gardening must share the same trend as aggregate output
  - the amount of gardening services must be stationary
  - gardening fee must be constant within each epoch
  - the cost of planting a tree must be such that at the beginning of each epoch no firms has incentive to adopt the new technology

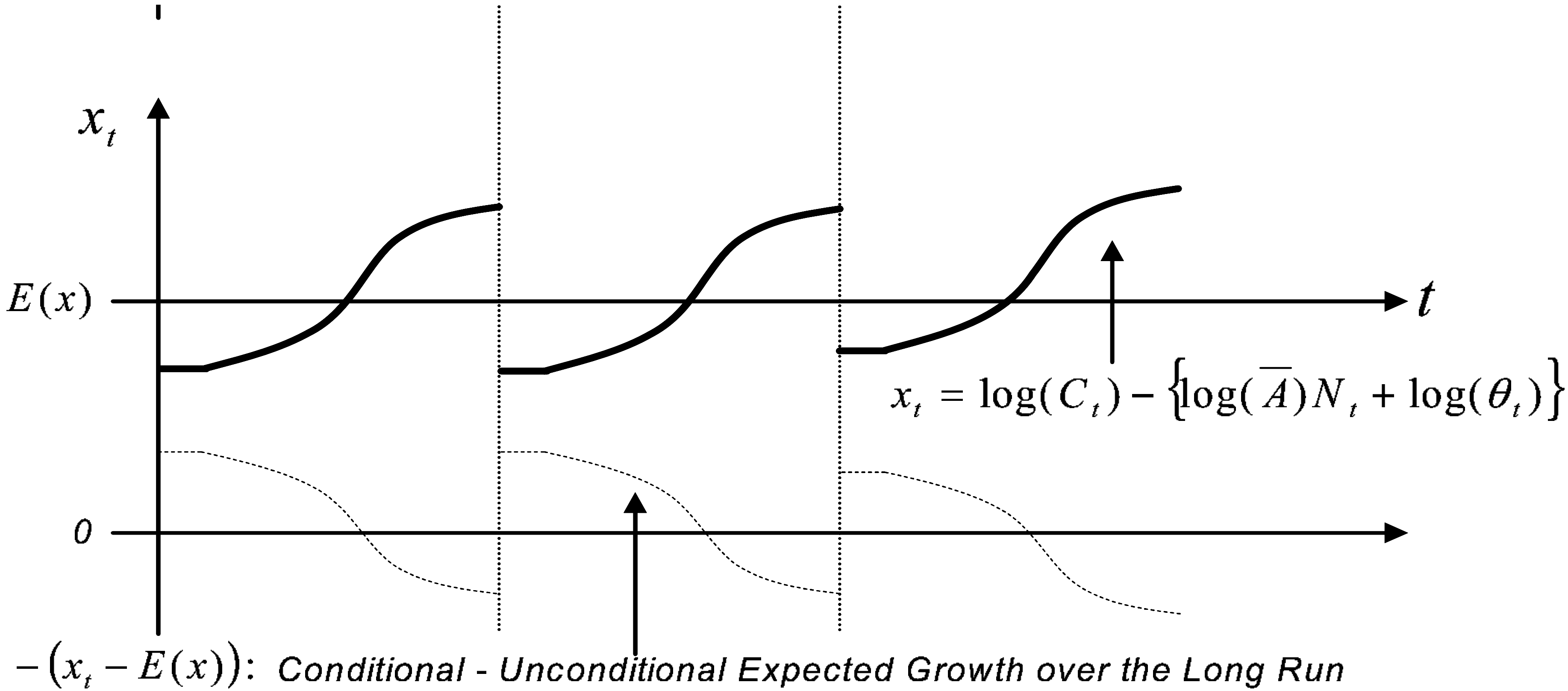
Consumption-Output Dynamics

$$Y_t = C_t = X_t\theta_t = \frac{X_t}{\bar{A}^N}\bar{A}^N\theta_t \Rightarrow \log C_t = x_t + N\log\bar{A} + \log\theta_t$$

predictable part of consumption growth



Consumption-Output Dynamics



Calibration and Unconditional Moments

μ	0.010	γ	7	$\zeta(0)$	1
σ	0.033	ρ	0.05	v	2
λ	0.050	$\bar{A}$	1.55	q	32

	Data	Model
Mean of consumption growth	0.022	0.030
Volatility of consumption growth	0.033	0.033
Mean of 1-year zero coupon yield	0.018	0.029
Volatility of 1-year zero coupon yield	0.030	0.066
Mean of Equity Premium	0.042	0.036
Volatility of Equity Premium	0.177	0.208
Mean (log) Price to Earnings Ratio	3.091	3.496
Volatility of (log) Price to Earnings Ratio	0.280	0.335
Mean of Book to Market	0.668	0.707
Volatility of Book to Market	0.230	0.316

Countercyclical Variation in Expected Returns

- Growth options value over aggregate stock market value

$$w_t^{o+f} = \frac{\int_{K_{N_t}}^1 P_{N,j,t}^o dj + P_{N,t}^f}{\int_0^1 P_{N,j,t}^A dj + \int_{K_{N_t}}^1 P_{N,j,t}^o dj + P_{N,t}^f}$$

- **Lemma 1:** $w_t^{o+f} = \frac{e^{-x_t}}{f(\frac{\theta_t}{M_t}, \frac{M_t}{M_{\tau_N}}) + e^{-x_t}} \Rightarrow w_x < 0$
- Expected excess return

$$\mu - r = (1 - w_t^{o+f})(\mu^A - r) + w_t^{o+f}\left(\frac{w_t^o}{w_t^{o+f}}(\mu^o - r) + \frac{w_t^f}{w_t^{o+f}}(\mu^f - r)\right)$$

- **Lemma 2:** $(\mu^o - r) > (\mu^f - r) > (\mu^A - r)$

Cross Section Predictability

- Growth options value over firm's value

$$w_t^{j,o+f} = \frac{P^o_{N,j,t} + P^f_{N,t}}{P^A_{N,j,t} + P^o_{N,j,t} + P^f_{N,t}}$$

- Assume $P_{N,j,t} > P_{N,i,t}$ and $P^o_{N,j,t} = P^o_{N,i,t} = 0$, it follows that:

$$w_t^{j,o+f} = \frac{P^f_{N,t}}{P_{N,j,t}} < \frac{P^f_{N,t}}{P_{N,i,t}} = w_t^{j,o+f}$$

- Using lemma 2:

$$\begin{aligned} \mu^j - r &= (1 - w_t^{j,o+f})(\mu^A - r) + w_t^{j,o+f}(\mu^f - r) < \\ &< (1 - w_t^{i,o+f})(\mu^A - r) + w_t^{i,o+f}(\mu^f - r) < \mu^i - r \end{aligned}$$

Consumption Risk

- Unconditional CAPM:

$$E\mu_t^j - r = -\gamma cov(\frac{dP_t^j}{P_t^j}; \frac{d\theta_t}{\theta_t})$$

- The econometrician's CAPM

$$\begin{aligned} E\mu_t^j - r &= -\gamma cov\left(R_t^j; \triangle \log C_t\right) \\ \triangle \log C_t &= \triangle \log \theta_t + \triangle[x_t - N \log \bar{A}] \end{aligned}$$

- **Negative correlation b/w $\triangle[x_t - N \log \bar{A}]$ and expected returns!!!**:

$$cov\left(R_t^j; \triangle \log C_t\right) < cov\left(R_t^j; \triangle \log \theta_t\right)$$

  Downward biased estimate of $cov\left(R_t^j; \triangle \log \theta_t\right) \Rightarrow$ upward biased estimate of γ

Long Run Consumption Growth Performs Better