Long-Term Contracting with Markovian Consumers

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Introduction

- This paper describes the properties of the optimal contract under asymmetric information between a monopolist and a consumer whose preferences evolve over time according to a Markov Process
- The main properties of the optimal contract are :
  - "Truly" dynamic structure
  - Asymptotic efficiency
  - Non stationary and infinite memory
  - Renegotiation proofness in pure strategies

The model: preferences

- There are a buyer and a seller, both risk neutral and same discount rate δ
- Per period utility of the buyer is:

$$\theta_t q_t - p_t$$

- Per period utility of the seller is:

$$p_t - \frac{1}{2} q_t^2$$

- θ_t, the marginal benefit for the buyer, has a markovian structure

The model: preferences

- The transition matrix for θ is:

$$P = \begin{bmatrix} p(\theta_L|\theta_L) & p(\theta_H|\theta_L) \\ p(\theta_L|\theta_H) & p(\theta_H|\theta_H) \end{bmatrix}$$

- 2 states, high and low, and $\Delta\theta = \theta_H - \theta_L > 0$
- Positive correlation: $p(\theta_H|\theta_H) \geq p(\theta_H|\theta_L)$ and $p(\theta_L|\theta_L) \geq p(\theta_L|\theta_H)$
- The prior of the seller is $\mu = (\mu_L, \mu_H)$

The model: technology

- One side commitment
  - The buyer can always walk away if his reservation utility falls below zero
  - The seller commits to the contract he offers

What is a contract?

- By virtue of the revelation principle, a contract is a price-quantity pair:

$$< \mathbf{p}, \mathbf{q} >= [p_t(\hat{\theta}|h_t), q_t(\hat{\theta}|h_t)]_{t=1}^{\infty}$$

- where:
  - h_t is the public history up to time t
  - $\hat{\theta}$ is the type revealed at time t
- in this setting we define:
  - a strategy for the seller as offering a contract $< \mathbf{p}, \mathbf{q} >$
  - a strategy for the buyer as a function mapping $h_t^C \mapsto b(h_t^C)$

The optimal contract

The seller chooses a contract $< \mathbf{p}, \mathbf{q} >$ to maximise:

$$\begin{aligned}\Pi^{seller} &= \mu_H \Big[p(\theta_H | h_1) - q^2(\theta_H | h_1)/2 + \delta E[\Pi(\theta | h_1, \theta_H) | \theta_t = \theta_H] \Big] \\ &+ \mu_L \Big[p(\theta_L | h_1) - q^2(\theta_L | h_1)/2 + \delta E[\Pi(\theta | h_1, \theta_L) | \theta_t = \theta_L] \Big]\end{aligned}$$

subject to:

$$\begin{aligned}(\mathbf{IC_{h_t}}(\theta_\mathbf{i})) : \quad & \theta_i q(\theta_i | h_t) - p(\theta_i | h_t) + \delta E[U(\theta | h_t), \theta_i) | \theta_t = \theta_i] \\ & \geq \theta_i q(\theta_j | h_t) - p(\theta_j | h_t) + \delta E[U(\theta | h_t), \theta_j) | \theta_t = \theta_i]\end{aligned}$$

$\mathbf{IR_{h_t}}(\theta_\mathbf{L})$ and $\mathbf{IR_{h_t}}(\theta_\mathbf{H})$

$$\forall h_t, \forall i \neq j, i, j \in \{H, L\}$$

Solution strategy

- Assume $T = 2$ and IC^H not binding after $h_2 = \theta_L$.
  - Reduce the rent at $t = 2$ and the price that the low type pays at $t = 1$ such that IR^L binds at $t = 1$
  - Outside option of high type at $t = 1$ is reduced. Why? High type more optimistic about future realizations of his type:
    - $\Delta p_1 = Pr(\theta_H|\theta_L)\Delta R_2$
    - $Pr(\theta_H|\theta_H)\Delta R_2 - \Delta p_1 = (Pr(\theta_H|\theta_H) - Pr(\theta_H|\theta_L))\Delta R_2 < 0$
  - Expected profits would be larger without affecting any constraint
  - The contract is not optimal
- By induction, we can generalize the above result to the T periods case, $T > 2$
- Similar argument to show that IR^L binds

Proposition 1

- By using the inductive argument, we assume the above conditions w.l.o.g and solve a "relaxed" version of the problem
- **Proposition 1:** at any time, the optimal contract is characterized by the supply function:

$$q_t^*(\theta|h_t) = \begin{cases} \theta_H & \text{if } \theta = \theta_H \\ \theta_L - \Delta\theta \frac{\mu_H}{\mu_L} \left[\frac{p(\theta_H|\theta_H) - p(\theta_H|\theta_L)}{p(\theta_L|\theta_L)} \right]^{t-1} & \text{if } \theta = \theta_L \text{ , } h_t = h_t^L \\ \theta_L & \text{if } \theta = \theta_L \text{ , } h_t \in H_t \setminus h_t^L \end{cases}$$

- As $t \to \infty$ the optimal contract converges to the efficient contract along any possible history since

$$\left[\frac{p(\theta_H|\theta_H) - p(\theta_H|\theta_L)}{p(\theta_L|\theta_L)} \right]^{t-1} = \left[1 - \frac{p(\theta_L|\theta_H)}{p(\theta_L|\theta_L)} \right]^{t-1} \to \infty$$

Implications

- Static case:
  - $q^* = \theta_H$ if $\theta = \theta_H$
  - $q^* = \theta_L - \frac{\mu_H}{\mu L}(\theta_H - \theta_L)$ if $\theta = \theta_L$
  - **No memory:** the seller commits not to use the information gathered from consumer's choices
- Dynamic case:
  - Non stationary
  - **Infinite memory:** the seller uses the information gathered in a dynamic way
  - *Efficiency invasion*
  - Asymptotic efficiency

Intuition

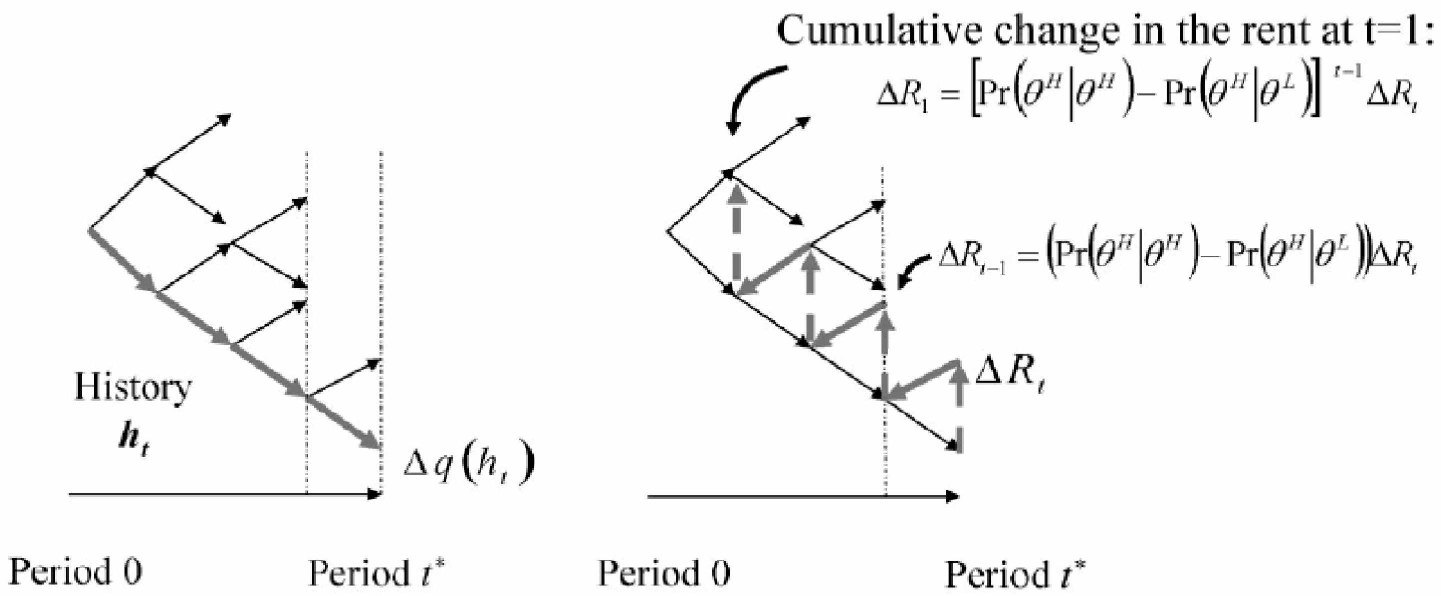

FIGURE 1. MARGINAL COST AND BENEFIT OF A CHANGE IN THE QUANTITY OFFERED AFTER HISTORY h_t

- Distorsion only along the lowest branch because IC^H has a dynamic structure

Intuition

- price for the low type at $(t-1)$: $\Delta p_{t-1} = Pr(\theta_H|\theta_L)\Delta R_t$
- Change in rent at $(t-1)$ for the high type:

$$Pr(\theta_H|\theta_H)\Delta R_t - \Delta p_{t-1} = (Pr(\theta_H|\theta_H) - Pr(\theta_H|\theta_L))\Delta R_t \geq 0$$

going back $t-1$ periods the change in rent will be proportional to

$$(Pr(\theta_H|\theta_H) - Pr(\theta_H|\theta_L))^{t-1}$$

- the change in expected profits at time zero will be proportional to

$$Pr(\theta_L|\theta_L)^{t-1}\mu_L$$

- marginal cost - marginal benefit ratio at time t is proportional to

$$(Pr(\theta_H|\theta_H) - Pr(\theta_H|\theta_L))^{t-1}/Pr(\theta_L|\theta_L)^{t-1}$$

What is the state?

- Very simple binary state for this economy

$$X_t = X(\theta_t, X_{t-1}) = \begin{cases} 1 & \text{if } X_{t-1} = 1 \text{ and } \theta_t = \theta_L \\ 0 & \text{otherwise} \end{cases}$$

- We can rewrite the optimal solution as

$$q_t^*(X_t) = \theta_t - \Delta\theta \frac{\mu_H}{\mu_L} X_t \left[\frac{p(\theta_H|\theta_H) - p(\theta_H|\theta_L)}{p(\theta_L|\theta_L)} \right]^{t-1}$$

Another interesting result

- Because of the markovian consumer, for big T the distribution of types will converge to the stationary distribution
- The seller and the buyer can predict the distribution og types in the far future almost in the same manner
- The discount factor matters:
  - δ is high then seller can separate the types by paying a very little rent
  - $\delta \rightarrow 1$ the seller will pay no rent at all and his average utility will be the first best solution. The buyer average utility will be zero

Last slide

- There are other interesting results concerning:
  - Property rights
  - Dynamics of monetary payments
  - Renegotiation proofness
- It will be interesting to study cases with more than 2 types
- What if risk aversion?
- Extension in other settings implying optimal contracting