A Unified Framework for Monetary Theory and Policy Analysis

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Motivation

The goal is to build an *analytically tractable* model based on explicit market frictions that motivate the existence of money to study monetary policy

The Economic Environment

- time is discrete
- a continuum of infinitely lived agents (discount factor β)
- each period is divided into two subperiods: "day" and "night"
- during the "day" agents can produce a special good. This good comes in many varieties and each agent only gains utility from consuming a subset of these special goods. It is assumed that agents do not gain utility from consuming the special good they can produce themselves
- during the "day" the agents interact in a decentralized market (anonymous bilateral matching)
- during the "night" agents can produce a general good which they can trade in a centralized (Walrasian) market
- neither good is storable. However, in this economy intrinsically worthless fiat money exists that is storable

The Economic Environment

- utility is given by:

$$\mathcal{U}(x, h, X, H) = u(x) - c(h) + U(X) - H \tag{1}$$

  $u(0) = c(0) = 0$
- Probabilities in decentralized market:
  1. α probability of meeting another agent during one day
  2. δ double coincidence of wants (agents can directly trade goods)
  3. σ single coincidence of wants
- total money stock fixed at M (for now)
- let $F_t(.)$ be distribution of money holdings across agents at the start of "day" t
- let $G_t(.)$ be distribution of money holdings across agents at the start of "night" t
- ϕ_t is the price of money

Bargaining in the decentralized Market

- some more notation:
  1. $V_t(m)$ value function of agent at the beginning of "day" t
  2. $W_t(m)$ value function of agent at the beginning of "night" t
  3. $q_t(m, \tilde{m})$ amount of specialized good traded in single coincidence meeting
  4. $d_t(m, \tilde{m})$ money traded in single coincidence meeting
  5. $B_t(m, \tilde{m})$ payoff in double coincidence meeting
- double coincidence of wants:
  symmetric Nash Bargaining with the continuation values as threat point takes place
  one can show that no money changes hands and
  $B_t(m, \tilde{m}) = u(q*) - c(q*) + W_t(m)$ where $q*$ is defined as the solution to $u'(q*) = c'(q*)$

Bargaining in the decentralized Market

Generalized NB when two agents want to trade money for a good

$$(q,d) = \arg\max[u(q)+W_t(m-d)-W_t(m)]^{\theta}[-c(q)+W_t(\tilde{m}+d)-W_t(\tilde{m})]^{1-\theta} \quad (2)$$

q and d turn out to depend only on the money holdings of the buyer, not the seller

Value Functions

$$V_t(m) = \alpha\sigma \int [u(q_t(m,\tilde{m})) + W_t(m - d_t(m,\tilde{m}))]dF_t(\tilde{m}) \tag{3}$$
$$+\alpha\sigma \int -c(q_t(\tilde{m},m)) + W_t(m + d_t(\tilde{m},m))]dF_t(\tilde{m})$$
$$+\alpha\delta \int B_t(m,\tilde{m})dF_t(\tilde{m}) + (1 - 2\alpha\sigma - \alpha\delta)W_t(m)$$

$$W_t(m) = \max_{X,H,m'} [U(X) - H + \beta V_{t+1}(m')] \tag{4}$$

subject to

$$X = H + \phi_t m - \phi_t m' \tag{5}$$
$$X \geq 0 \tag{6}$$
$$0 \leq H \leq \overline{H} \tag{7}$$

Equilibrium Definition

An equilibrium consists of the objects $(V_t, W_t, X_t, H_t, m'_t, q_t, d_t, \phi_t, F_t, G_t) \forall t$ such that

- X_t, H_t, m' solve the agents' problem in the centralized market and result in value function W_t
- q_t, d_t and B_t are the result of the Nash bargaining in the decentralized market and result in V_t
- $\phi_t > 0$
- good and money markets clear
- F_t, G_t are consistent with initial distributions and the evolution of money holdings implied by the other equilibrium objects

Results

- $\phi_t \geq \beta\phi_{t+1}$, i.e. the minimal inflation rate in this economy is $\phi_t/\phi_{t+1} = \beta$, which is the Friedman rule
- for θ near 1 there is a unique solution for m', i.e. $F_t$is degenerate
- authors also provide alternative conditions which hold for any θ
- solution then reduces to the following difference equation:

$$z(q_t) = \beta z(q_{t+1}) \left[\alpha\sigma u'(q_{t+1})(z'(q_{t+1}))^{-1} + 1 - \alpha\sigma\right] \tag{8}$$

- $$z(q) = \frac{\theta c(q)u'(q) + (1-\theta)u(q)c'(q)}{\theta u'(q) + (1-\theta)c'(q)} \tag{9}$$

- in steady state:

$$\frac{u'(q)}{z'(q)} = 1 + \frac{1-\beta}{\alpha\sigma\beta} \tag{10}$$

Changes in the Money Supply

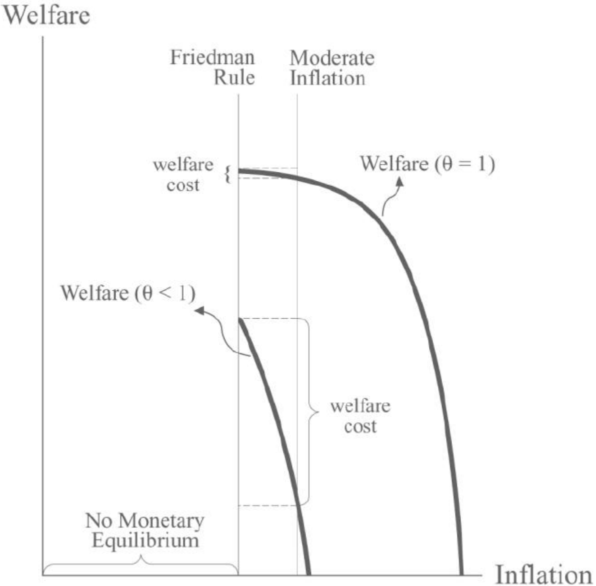
$$z(q_t)/M_t = \beta z(q_{t+1})/M_{t+1}\left[\alpha\sigma u'(q_{t+1})(z'(q_{t+1}))^{-1} + 1 - \alpha\sigma\right] \tag{11}$$

Also, suppose

$$M_{t+1} = (1+\tau)M_t \tag{12}$$

Then, in steady state (for real variables):

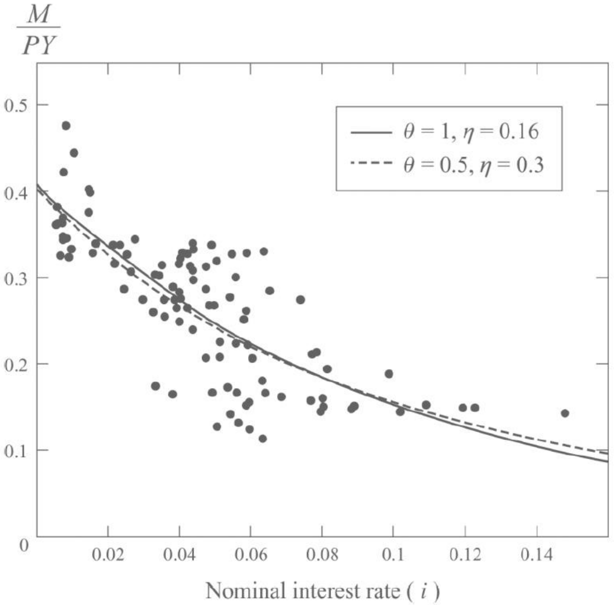
$$\frac{u'(q)}{z'(q)} = 1 + \frac{1-\beta+\tau}{\alpha\sigma\beta} = 1 + \frac{i}{\alpha\sigma} \tag{13}$$



Welfare
Friedman Rule
Moderate Inflation
welfare cost
Welfare (θ = 1)
Welfare (θ < 1)
welfare cost
No Monetary Equilibrium
Inflation

Quantitative Analysis

- $u(q) = [(q+b)^{1-\eta} - b^{1-\eta}]/(1-\eta)$
- $U(X) = B\log(X)$
- $c(h) = h$
- $r = 0.04$
- $\alpha = 1, \delta = 0$
- $M/(PY) = z(q)/B + \sigma z(q)$
- (η, B, σ) are set to fit the money demand data
- three possible values of $\theta : (1, 0.5, 0.343)$



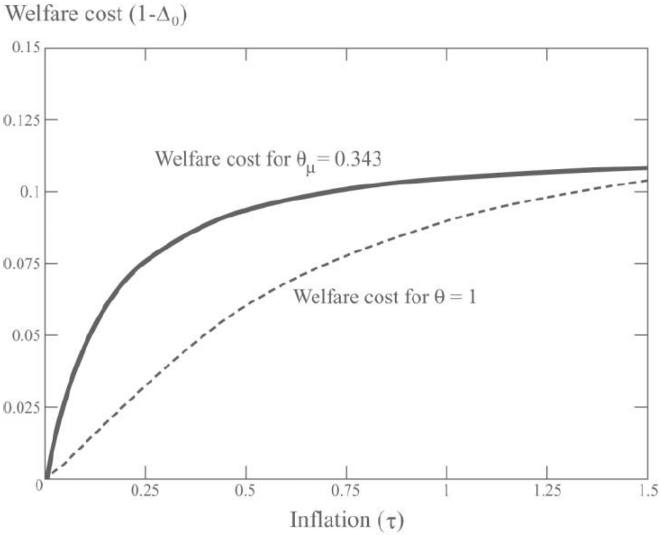
M/PY
0.5
0.4
0.3
0.2
0.1
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
θ = 1, η = 0.16
θ = 0.5, η = 0.3
Nominal interest rate (i)

Welfare Analysis

- steady state utility:
  $(1-\beta)V(\tau) = U(X*) - X* + \alpha\sigma[u(q(\tau)) - q(\tau)]$
- Experiment:
  $(1-\beta)V_{\Delta}(0) = U(X*\Delta) - X* + \alpha\sigma[u(q(0)\Delta) - q(\tau)]$
  want to solve for Δ_0 such that $V_{\Delta_0}(0) = V(\tau)$ for $\tau = 0.1$

Annual Model (1900–2000)

	$\theta = 1$		$\theta = .5$	$\theta_\mu = .343$	$\theta = 1$
	$\sigma = .31$ $\eta = .27$ $B = 2.13$ (1)	$\sigma = .50$ $\eta = .16$ $B = 1.97$ (2)	$\sigma = .50$ $\eta = .30$ $B = 1.91$ (3)	$\sigma = .50$ $\eta = .39$ $B = 1.78$ (4)	$\sigma = .50$ $\eta = .39$ $B = 1.78$ (5)
$q(\tau)$	.243	.206	.143	.094	.522
$q(0)$	.638	.618	.442	.296	.821
$q(\tau^F)$	1.000	1.000	.779	.568	1.000
$1 - \Delta_0$	.014	.014	.032	.046	.012
$1 - \Delta_F$	.016	.016	.041	.068	.013


Welfare cost (1-Δ0)
0.15
0.125
0.1
0.075
0.05
0.025
0
Welfare cost for θμ = 0.343
Welfare cost for θ = 1
0.25
0.5
0.75
1
1.25
1.5
Inflation (τ)