Self-Enforcing Wage Contracts

Jonathan Thomas and Tim Worrall

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- Each agent treats the spot market wage parametrically.

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Assumption 3: States are iid such that $a < \mathrm{w}(1) < \mathrm{w}(S) < 1 \leq b$. The probability of state s is $p(s)$ where $\sum_{s=1}^{S} p(s) = 1$.

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For any δ and any history h_t , the **net future benefit** to the worker and the firm are:

$$U(\delta; h_t) = u(\omega(h_t)) - u(\mathrm{w}(s_t)) + E \sum_{\tau=t+1}^{\infty} \alpha^{\tau-1} [u(\omega(h_\tau)) - u(\mathrm{w}(s_\tau))]$$

$$V(\delta; h_t) = \mathrm{w}(s_t) - \omega(h_t) + E \sum_{\tau=t+1}^{\infty} \alpha^{\tau-1} [\mathrm{w}(s_\tau) - \omega(h_\tau)]$$

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δ is **self-enforcing** $\Leftrightarrow$ $U(\delta;\, h_t) \geq 0$ and $V(\delta;\, h_t) \geq 0$.

δ is **efficient** if there exists no other self-enforcing contract which offers both parties at least as much expected utility and one party strictly more.

An **optimal contract** is an efficient contract which distributes the gains from the contract between the firm and the worker according to the market situation at date $t = 0$.

Optimum Contracts

Let U_s^t be a feasible value of the worker's net future utility.

The Pareto-frontier is given by:

$$f_s(U_s^t) = sup_{\omega(s),(U_q^{t+1})_{q=1}^S} \mathrm{w}(s) - \omega(s) + \alpha E f_q(U_q^{t+1}) \quad (1)$$

subject to:

- $U_q^{t+1} \geq 0$ and $f_q(U_q^{t+1}) \geq 0, \qquad q = 1, 2, \ldots, S$
- $u(\omega(s)) - u(\mathrm{w}(s)) + \alpha E U_q^{t+1} \geq U_s^t.$

Getting to a Solution

$$f_s(U_s^t) = sup_{\omega(s),(U_q^{t+1})_{q=1}^S} \mathrm{w}(s) - \omega(s) + \alpha E f_q(U_q^{t+1}) \tag{1}$$

$$\underbrace{U_q^{t+1} \geq 0}_{\{\alpha p(q)\phi_q\}_{q=1}^S} \qquad \underbrace{f_q(U_q^{t+1}) \geq 0}_{\{\alpha p(q)\psi_q\}_{q=1}^S} \qquad \underbrace{u(\omega(s)) - u(\mathrm{w}(s)) + \alpha E U_q^{t+1} \geq U_s^t}_{\lambda_s}.$$

FOCs and Envelope Condition:

$$\begin{aligned} -\lambda_s &= (1+\psi_q) f_q'(U_q^{t+1}) + \phi_q \\ -\lambda_s &= -1/u'(\omega(s)) \\ -\lambda_s &= f_s'(U_t^t). \end{aligned}$$

$\Rightarrow$

- $U_s^t \in [0, \overline{U}_s]$
- $\forall (h_{t-1}, s), \omega(h_{t-1}, s) \in [\underline{\omega}_s, \overline{\omega}_s]$

The Wage Process

$$\omega(h_t,\, q) = \begin{cases} \underline{\omega}_q & \text{if } \omega(h_t) > \underline{\omega}_q \\ \omega(h_t) & \text{if } \underline{\omega}_q \geq \omega(h_t) \geq \underline{\omega}_q \\ \underline{\omega}_q & \text{if } \omega(h_t) < \underline{\omega}_q . \end{cases}$$

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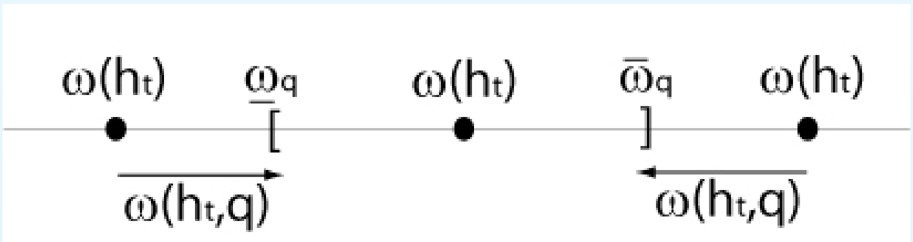

Spot Market Wages and Wage Intervals

Results:

- For $k > q, \quad \overline{\omega}_k > \overline{\omega}_q$ and $\underline{\omega}_k > \underline{\omega}_q$.
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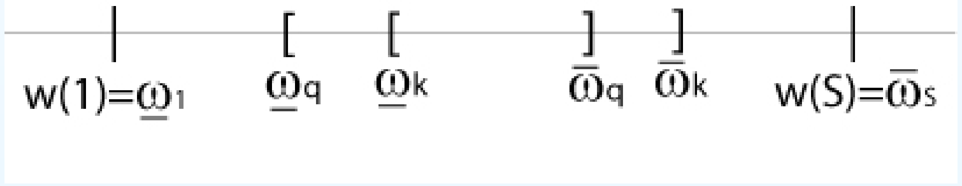

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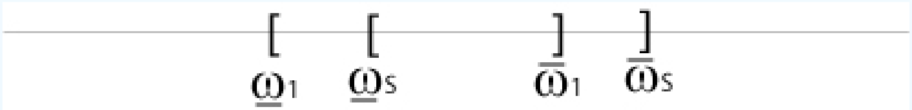

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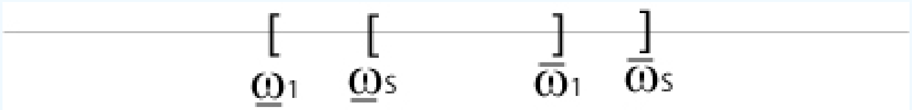

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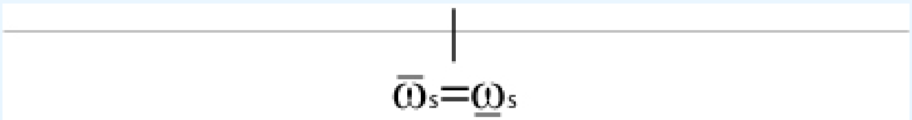

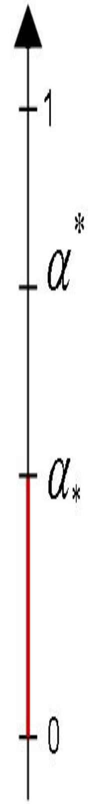

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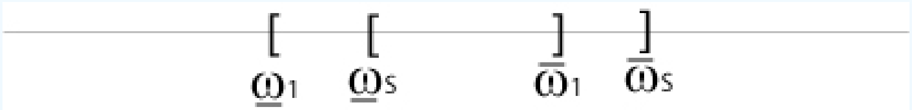

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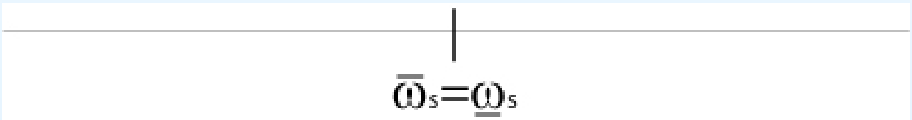

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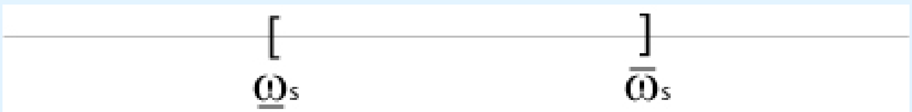
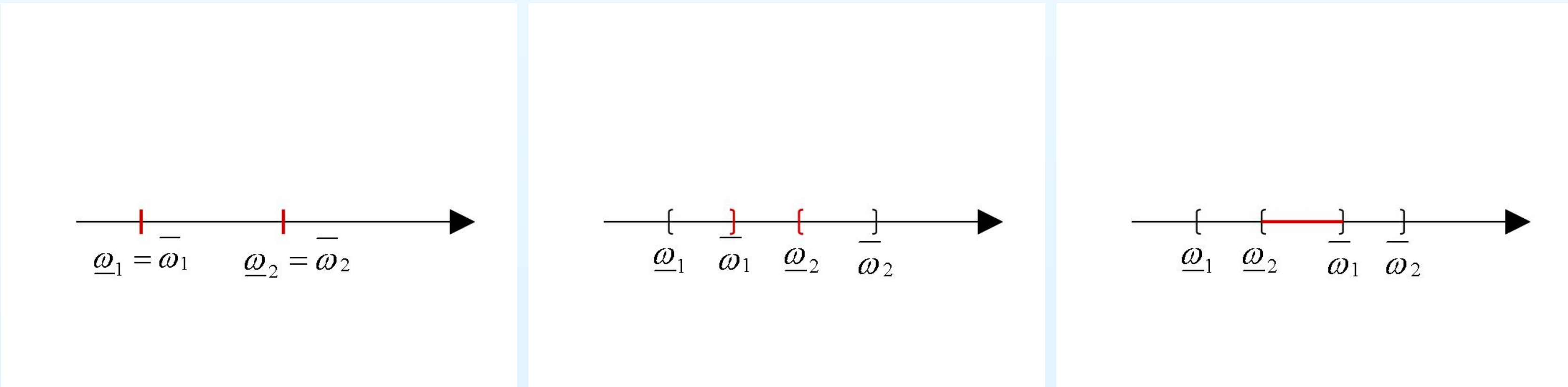


An Example

Two possible spot wages, $\mathrm{w}(1),\ \mathrm{w}(2)$ and suppose the worker can extract all the surplus from the firm.

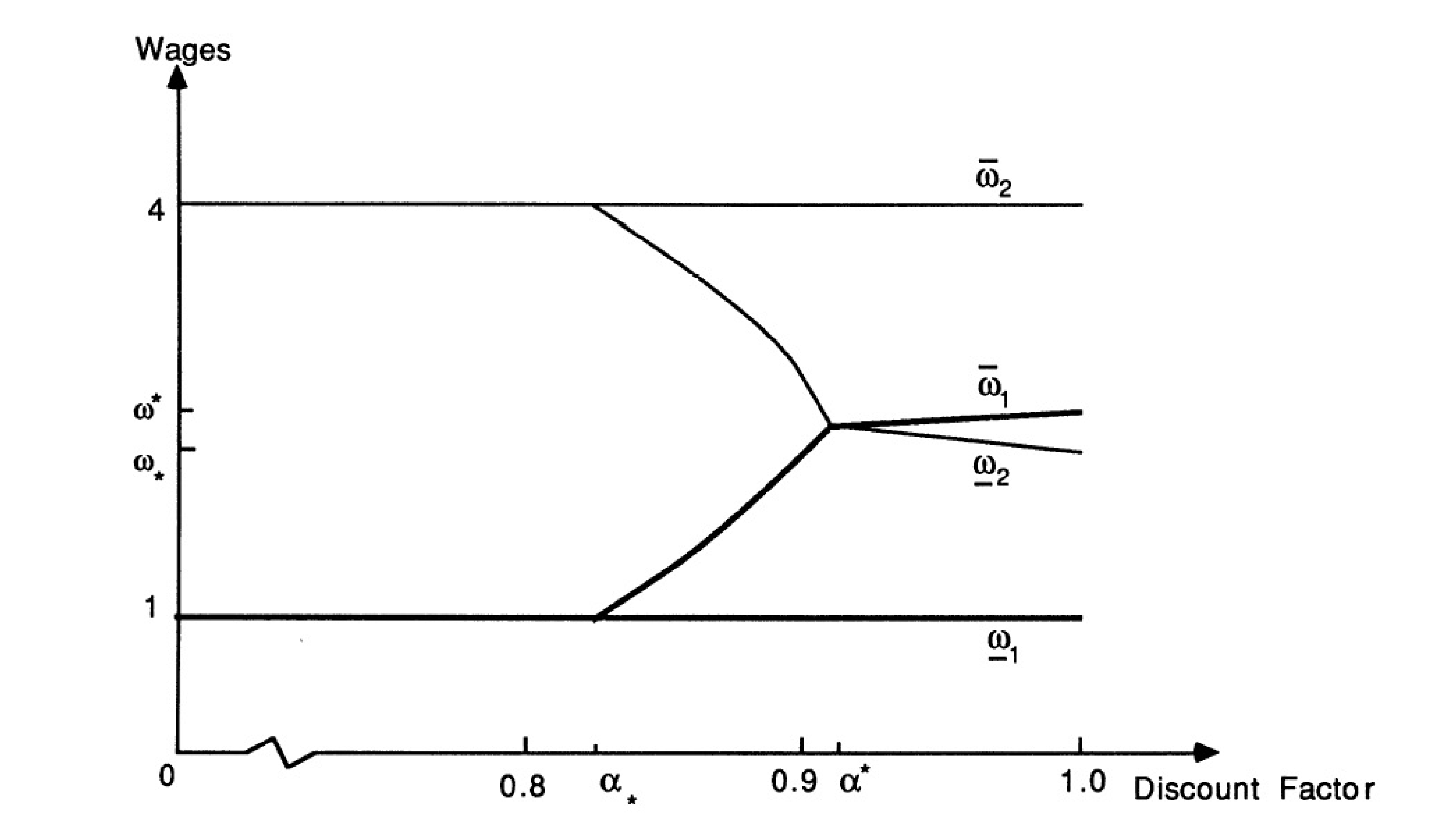
Three cases:

- $\alpha \in [0, \alpha_\star]$: the only feasible contract replicates the spot market outcome.
- $\alpha \in (\alpha_\star,\ \alpha^\star)$: the two intervals $[\underline{\omega}_1,\ \overline{\omega}_1]$ and $[\underline{\omega}_2,\ \overline{\omega}_2]$ are disjoint.
- $\alpha \in (\alpha^\star,\ 1)$: the two intervals overlap, so that $\overline{\omega}_1$ is paid unless state two is the only state to have occurred, in which case $\overline{\omega}_2$ is paid.



A Numerical Example

$$u(\omega) = \sqrt{\omega}, \quad \mathrm{w}(1) = 1, \quad \mathrm{w}(2) = 4, \quad p(1) = p(2) = \frac{1}{2}$$



Conclusion

With low mobility costs and high enforcement costs, wage contracts must be self-enforcing so neither the firm nor the worker have incentive to renege.

Thomas and Worrall provide a complete characterization of the optimal self-enforcing contract:

- Wages are sticky: they are less variable than spot wages and positively serially correlated.
- The updating rule: around each spot wage is a time invariant interval, and the contract wage changes each period by the smallest amount necessary to bring it into the current interval.