Self-Enforcing Wage Contracts

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- Each agent treats the spot market wage parametrically.
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**Assumption 3:** States are iid such that \( a < w(1) < w(S) < 1 \leq b \). The probability of state \( s \) is \( p(s) \) where \( \sum_{s=1}^{S} p(s) = 1 \).
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For any \( \delta \) and any history \( h_t \), the net future benefit to the worker and the firm are:

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U(\delta; h_t) = u(\omega(h_t)) - u(w(s_t)) + \mathbb{E} \sum_{\tau=t+1}^{\infty} \alpha^{\tau-1} \left[ u(\omega(h_\tau)) - u(w(s_\tau)) \right]
\]

\[
V(\delta; h_t) = w(s_t) - \omega(h_t) + \mathbb{E} \sum_{\tau=t+1}^{\infty} \alpha^{\tau-1} \left[ w(s_\tau) - \omega(h_\tau) \right]
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\( \delta \) is self-enforcing \( \iff \) \( U(\delta; h_t) \geq 0 \) and \( V(\delta; h_t) \geq 0 \).

\( \delta \) is efficient if there exists no other self-enforcing contract which offers both parties at least as much expected utility and one party strictly more.

An optimal contract is an efficient contract which distributes the gains from the contract between the firm and the worker according to the market situation at date \( t = 0 \).
Optimum Contracts

Let $U_s^t$ be a feasible value of the worker’s net future utility.

The Pareto-frontier is given by:

$$f_s(U_s^t) = \sup_{\omega(s),(U_q^{t+1})_{q=1}^S} \left(w(s) - \omega(s) + \alpha E f_q(U_q^{t+1}) \right)$$

subject to:

1. $U_q^{t+1} \geq 0$ and $f_q(U_q^{t+1}) \geq 0$, $q = 1, 2, \ldots, S$
2. $u(\omega(s)) - u(w(s)) + \alpha E U_q^{t+1} \geq U_s^t$. 
Getting to a Solution

\[ f_s(U^t_s) = \sup_{\omega(s), (U^t_{q+1})_q} w(s) - \omega(s) + \alpha E f_q(U^t_{q+1}) \]  

(1)

\[
\begin{align*}
U^t_{q+1} & \geq 0 \\
\{\alpha p(q) \phi_q\}_q & \geq 0 \\
\{\alpha p(q) \psi_q\}_q & \geq 0 \\
\lambda_s & = U^t_t \geq U^t_s.
\end{align*}
\]

FOCs and Envelope Condition:

\[
-\lambda_s = (1 + \psi_q) f_q'(U^t_{q+1}) + \phi_q
\]

\[
-\lambda_s = -1/u'(\omega(s))
\]

\[
-\lambda_s = f_s'(U^t_t).
\]

⇒

- \( U^t_s \in [0, \bar{U}_s] \)
- \( \forall(h_{t-1}, s), \omega(h_{t-1}, s) \in [\omega_s, \bar{\omega}_s] \)
The Wage Process

\[
\omega(h_t, q) = \begin{cases} 
\omega_q & \text{if } \omega(h_t) > \omega_q \\
\omega(h_t) & \text{if } \omega_q \geq \omega(h_t) \geq \omega_q \\
\omega_q & \text{if } \omega(h_t) < \omega_q.
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So the wage depends only on the current spot market wage and the contract wage in the previous period.

The interval end-points corresponds to giving the worker and firm a zero gain starting from state \( q \).
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Spot Market Wages and Wage Intervals

Results:

- For $k > q$, $\omega_k > \omega_q$ and $\omega_k > \omega_q$.

- $w_s \in [\omega_s, \overline{\omega}_s]$, $w(1) = \omega_1$, $w(S) = \overline{\omega}_S$. 
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\[ 
\begin{array}{cccccc}
   & w(1)=\omega_1 & [ & \omega_q & [ & \omega_k & ] & ] & [ & \overline{\omega}_q & [ & \overline{\omega}_k & ] & ] & w(S)=\overline{\omega}_S \\
\end{array} \]
Comparison Statics

Results:

- $\forall s, \overline{w}_s (\underline{w}_s)$ is non-decreasing (increasing) in $\alpha$. 
Comparative Statics

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• $\forall S, \overline{w}_S \ (\underline{w}_S)$ is non-decreasing (increasing) in $\alpha$.

• $\exists \alpha^* < 1 : \forall \alpha \in (\alpha^*, 1), \overline{w}_1 > \underline{w}_S$. 
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• \( \exists \alpha_* \in (0, \alpha^*) : \)

  ◦ \( \forall \alpha \in (0, \alpha_*], \ \bar{\omega}_s = \omega_s \) for each \( s \)
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• \( \exists \alpha^* < 1 : \forall \alpha \in (\alpha^*, 1), \overline{\omega}_1 > \omega_S. \)

• \( \exists \alpha^*_\ast \in (0, \alpha^*) : \)
  \( \forall \alpha \in (0, \alpha^*_\ast], \overline{\omega}_s = \omega_s \) for each \( s \)
  \( \forall \alpha \in (\alpha^*_\ast, 1), \overline{\omega}_s > \omega_s \) for each \( s \).
An Example

Two possible spot wages, $w(1)$, $w(2)$ and suppose the worker can extract all the surplus from the firm.

Three cases:

- $\alpha \in [0, \alpha^*]$: the only feasible contract replicates the spot market outcome.
- $\alpha \in (\alpha^*, \alpha^*)$: the two intervals $[\omega_1, \overline{\omega}_1]$ and $[\omega_2, \overline{\omega}_2]$ are disjoint.
- $\alpha \in (\alpha^*, 1)$: the two intervals overlap, so that $\overline{\omega}_1$ is paid unless state two is the only state to have occurred, in which case $\overline{\omega}_2$ is paid.
A Numerical Example

\[ u(\omega) = \sqrt{\omega}, \quad w(1) = 1, \quad w(2) = 4, \quad p(1) = p(2) = \frac{1}{2} \]
Conclusion

With low mobility costs and high enforcement costs, wage contracts must be self-enforcing so neither the firm nor the worker have incentive to renege.

Thomas and Worrall provide a complete characterization of the optimal self-enforcing contract:

- Wages are sticky: they are less variable than spot wages and positively serially correlated.

- The updating rule: around each spot wage is a time invariant interval, and the contract wage changes each period by the smallest amount necessary to bring it into the current interval.