Nominal Debt as a Burden on Monetary Policy

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About the Objective of the Paper

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- **What?**

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- **Why?**

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- **How?**

About the Objective of the Paper

- **What?** Time inconsistency of optimal monetary policy
- **Why?** Nominal debt is associated with the incentive to inflate the economy but is it always so? What about indexed debt?
- **How?** 2 debt regimes: nominal *vs.* indexed debt
  2 policy regimes: with and without commitment

Model Economy

- Two types of agents: government and a representative household.
  - Government: issues debt B_t to finance exogenous public consumption g.
  - Households: $\max \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \alpha n_t \right]$, where $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$
- Production economy with resource constraint: $c_t + g \leq n_t$ and cash-in-advance constraint: $P_t c_t \leq M_t$.

Budget Constraints

- with nominal debt

$$\begin{aligned} M_{t+1} + B_{t+1} &\leq M_t - P_t c_t + B_t (1 + i_t) + P_t n_t \\ M^g_{t+1} + B^g_{t+1} &\geq M^g_t + P_t g + B^g_t (1 + i_t) \end{aligned}$$

- with indexed debt $\left(\text{where } \frac{B_t(1+i_t)}{P_t} \equiv b_t\right)$

$$\begin{aligned} M_{t+1} + \frac{b_{t+1}}{1 + i_{t+1}} P_{t+1} &\leq M_t - P_t c_t + b_t P_t + P_t n_t \\ M^g_{t+1} + \frac{b^g_{t+1}}{1 + i_{t+1}} P_{t+1} &\geq M^g_t + P_t g + b^g_t P_t \end{aligned}$$

Competitive Equilibrium w/ Nominal *vs.* Indexed Debt

Definition. A competitive equilibrium for an economy with nominal (indexed) debt is a government policy, $\left\{M_{t+1}^g, B_{t+1}^g\right\}_{t=0}^{\infty}$ $\left(\left\{M_{t+1}^g, b_{t+1}^g\right\}_{t=0}^{\infty}\right)$, an allocation $\{M_{t+1}, B_{t+1}, c_t, n_t\}_{t=0}^{\infty}$ $\left(\{M_{t+1}, b_{t+1}, c_t, n_t\}_{t=0}^{\infty}\right)$ and a price vector, $\{P_t, i_{t+1}\}_{t=0}^{\infty}$ such that

(i) given M_0^g and $B_0^g(1+i_0)$, and g, the government policy and the price vector satisfy the government budget constraints with nominal (indexed debt) together with no-Ponzi game condition;

(ii) given M_0 and $B_0(1+i_0)$ and the price vector, household's allocation maximizes utility, subject to the cash-in-advance constraint, the household budget constraint with nominal (indexed) debt, and the no-Ponzi game condition;

(iii) all markets clear, that is: $M_{t+1}^g = M_{t+1}$, $B_{t+1}^g = B_{t+1}$ $(b_{t+1}^g = b_{t+1})$, and g and $\{c_t, n_t\}_{t=0}^{\infty}$ satisfy the economy's resource constraint for every $t \geq 0$.

Competitive Equilibrium: Results

The competitive equilibrium is characterized by the following conditions

$$\frac{u_c\left(c_{t+1}\right)}{\alpha} = 1 + i_{t+1} \quad \text{for } t \geq 0$$

$$1 + i_{t+1} = \beta^{-1}\frac{P_{t+1}}{P_t} \text{ for } t \geq 0$$

$$c_t = \frac{M_t}{P_t}$$

$$\lim_{T\to\infty}\beta^T\left(\frac{M_{T+1}+B_{T+1}}{P_T}\right) = 0$$

Furthermore, the government budget constraint, and the resource constraint hold with equality.

Ramsey Problem with Indexed Debt

Definition. A full commitment Ramsey equilibrium with indexed debt is a competitive equilibrium such that $\{c_t\}$ solves

$$\max \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t\right) - \alpha\left(c_t + g\right)\right]$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \beta^t \left[c_{t+1} u_c\left(c_{t+1}\right) \frac{\beta}{\alpha} - \left(c_t + g\right)\right] = b_0$$

- The optimal solution: $c_{t+1} = c_1$ for $t \geq 0$.
- However, for the initial period: $u_c\left(c_0\right) - \alpha = \frac{u_c(c_1) - \alpha}{1 - \frac{u_c(c_1)}{\alpha}(1-\sigma)}$ and unless $\sigma = 1$ the solution is time inconsistent $\left(c_0 \neq c_1 = c_{t+1}\right)$.

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  $\Rightarrow$*"intertemporal elasticity effect"*

Ramsey Problem with Nominal Debt

Definition. A full commitment Ramsey equilibrium with nominal debt is a competitive equilibrium such that $\{c_t\}$ solves

$$\max \sum_{t=0}^{\infty} \beta^t \left[u\left(c_t\right) - \alpha \left(c_t + g\right) \right]$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \beta^t \left[c_{t+1} u_c\left(c_{t+1}\right) \frac{\beta}{\alpha} - \left(c_t + g\right) \right] = z_0 c_0$$

where $z_0 = \frac{B_0(1+i_0)}{M_0}$

- The optimal solution: $c_{t+1} = c_1$ for $t \geq 0$.
- However, for the initial period: $\frac{u_c(c_0)-\alpha}{1+z_0} = \frac{u_c(c_1)-\alpha}{1-\frac{u_c(c_1)}{\alpha}(1-\sigma)}$ and if $\sigma = 1$ but $z_0 > 0$ the solution is time inconsistent $\left(c_0 \neq c_1 = c_{t+1}\right)$.

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- However, for the initial period: $\frac{u_c(c_0)-\alpha}{1+z_0} = \frac{u_c(c_1)-\alpha}{1-\frac{u_c(c_1)}{\alpha}(1-\sigma)}$ and if $\sigma = 1$ but $z_0 > 0$ the solution is time inconsistent $(c_0 \neq c_1 = c_{t+1})$.

  $\Rightarrow$*"intertemporal elasticity effect"* + *"nominal effect"*

Welfare Comparison: Ramsey Problem with Indexed *vs.* Nominal Debt

Proposition 1. Consider two economies with full commitment with an initial money stock M_0. One of them has initial nominal debt $B_0 (1 + i_0) > 0$ and the other has initial indexed debt b_0. Suppose $b_0 = \frac{B_0(1+i_0)}{P_0}$, where P_0 is the period zero price in the economy with nominal debt. Then, the economy with nominal debt always gives lower welfare independently of the value σ.

Markov-Perfect Equilibria with Indexed Debt

Definition A Markov-perfect monetary equilibrium with indexed debt is a value function $V(b)$ and policy functions $C(b)$ and $\mathbf{b}'(b)$ such that $c = C(b)$ and $b' = \mathbf{b}'(b)$ solve

$$V(b) = \max_{c,b'} \left\{ u(c) - \alpha(c+g) + \beta V(b') \right\}$$

$$\text{s.t.} \quad C(b')\, u_c(C(b')) \frac{\beta}{\alpha} + \beta b' = c + g + b$$

- First-order conditions imply

$$\frac{u_c(c')}{\alpha} - 1 = \left(\frac{u_c(c)}{\alpha} - 1 \right) \left[1 + \frac{u_c(C(b'))}{\alpha} C_b(b')(1-\sigma) \right]$$

Markov-Perfect Equilibria with Nominal Debt

Definition A Markov-perfect monetary equilibrium with nominal debt is a value function $V(z)$ and policy functions $C(z)$ and $\mathbf{z}'(z)$ such that $c = C(z)$ and $z' = \mathbf{z}'(z)$ solve

$$V(z) = \max_{c,z'} \left\{ u(c) - \alpha(c+g) + \beta V(z') \right\}$$

$$\text{s.t.} \quad C(z')\, u_c(C(z')) \frac{\beta}{\alpha} + \beta z' C(z') = zc + c + g$$

- First-order conditions imply

$$\frac{\frac{u_c(c')}{\alpha} - 1}{1+z'} = \frac{\left(\frac{u_c(c)}{\alpha} - 1\right)}{1+z} \left\{ 1 + \varepsilon_c(z') \left[1 + \varepsilon_c(z') \frac{u_c(C(z'))}{z'\alpha}(1-\sigma) \right] \right\}$$

with $\varepsilon_c(z) = \frac{zC_z(z)}{C(z)}$.

Welfare Comparison: Markov-Perfect Equilibrium with Indexed *vs.* Nominal Debt

Proposition 2. Consider two economies without commitment with an initial money stock M_0. One of them has initial nominal debt $B_0(1+i_0) > 0$ and the other has initial indexed debt b_0. Suppose $b_0 = \frac{B_0(1+i_0)}{P_0}$, where P_0 is the period zero price in the economy with nominal debt. Then,
(i) If $\sigma = 1$, the welfare in the economy with indexed debt is higher than in the economy with nominal debt
(ii) If $\sigma \neq 1$, the welfare in the economy with nominal debt can be higher, or lower, than in the economy with indexed debt, depending on b_0

Conclusions

Nominal Debt: A Burden or a Blessing?

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  - under full commitment
  - without commitment if $\sigma = 1$
  - without commitment if $\sigma \neq 1$ and with the initial level of debt being high enough

Conclusions

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  - without commitment if $\sigma \neq 1$ and with the initial level of debt being high enough
- ...a blessing
  - without commitment if $\sigma \neq 1$ and the level of debt is not too high

Conclusions

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- ...a burden
  - under full commitment
  - without commitment if $\sigma = 1$
  - without commitment if $\sigma \neq 1$ and with the initial level of debt being high enough
- ...a blessing
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Interplay: Fiscal and Monetary Authorities?