Quantitative Analysis of International Lending with Asymmetric Information

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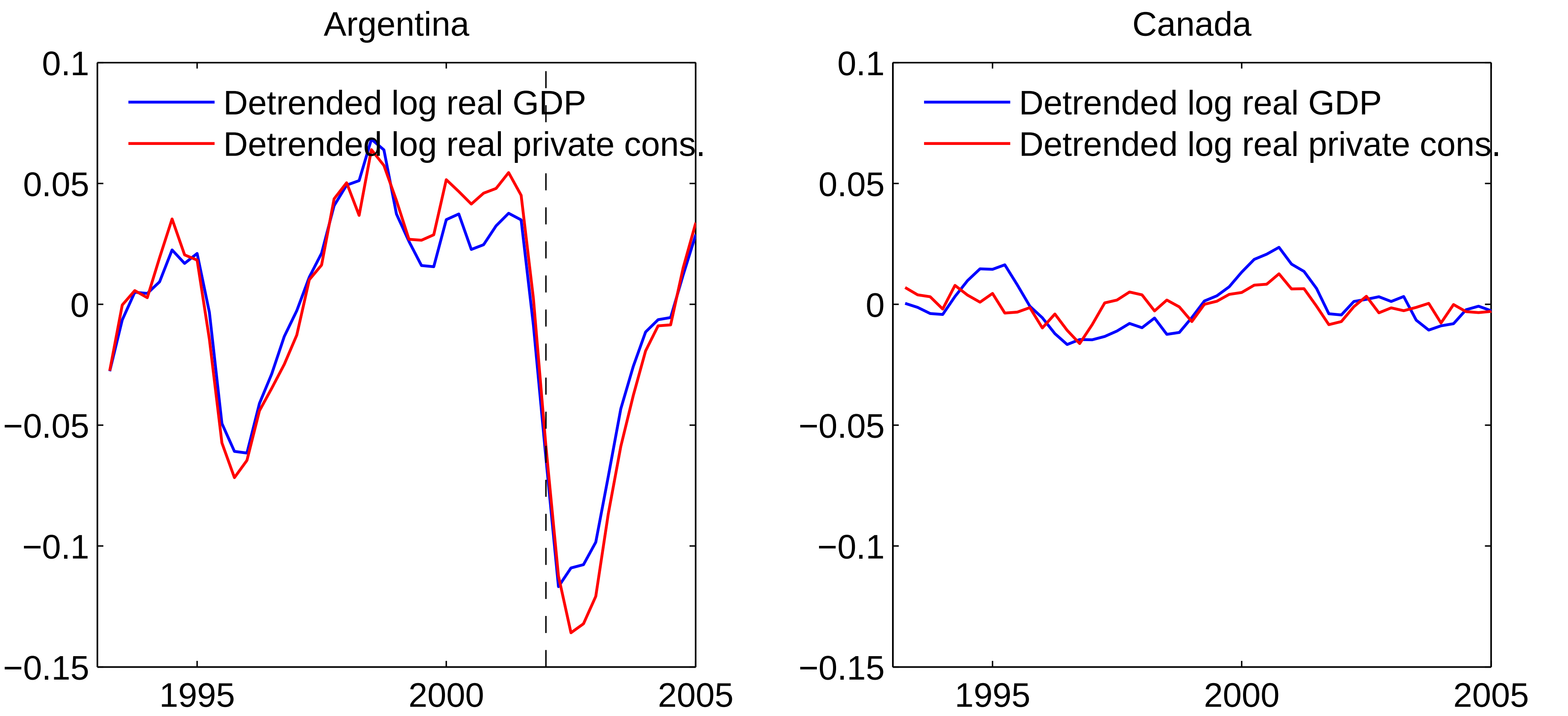
Motivation

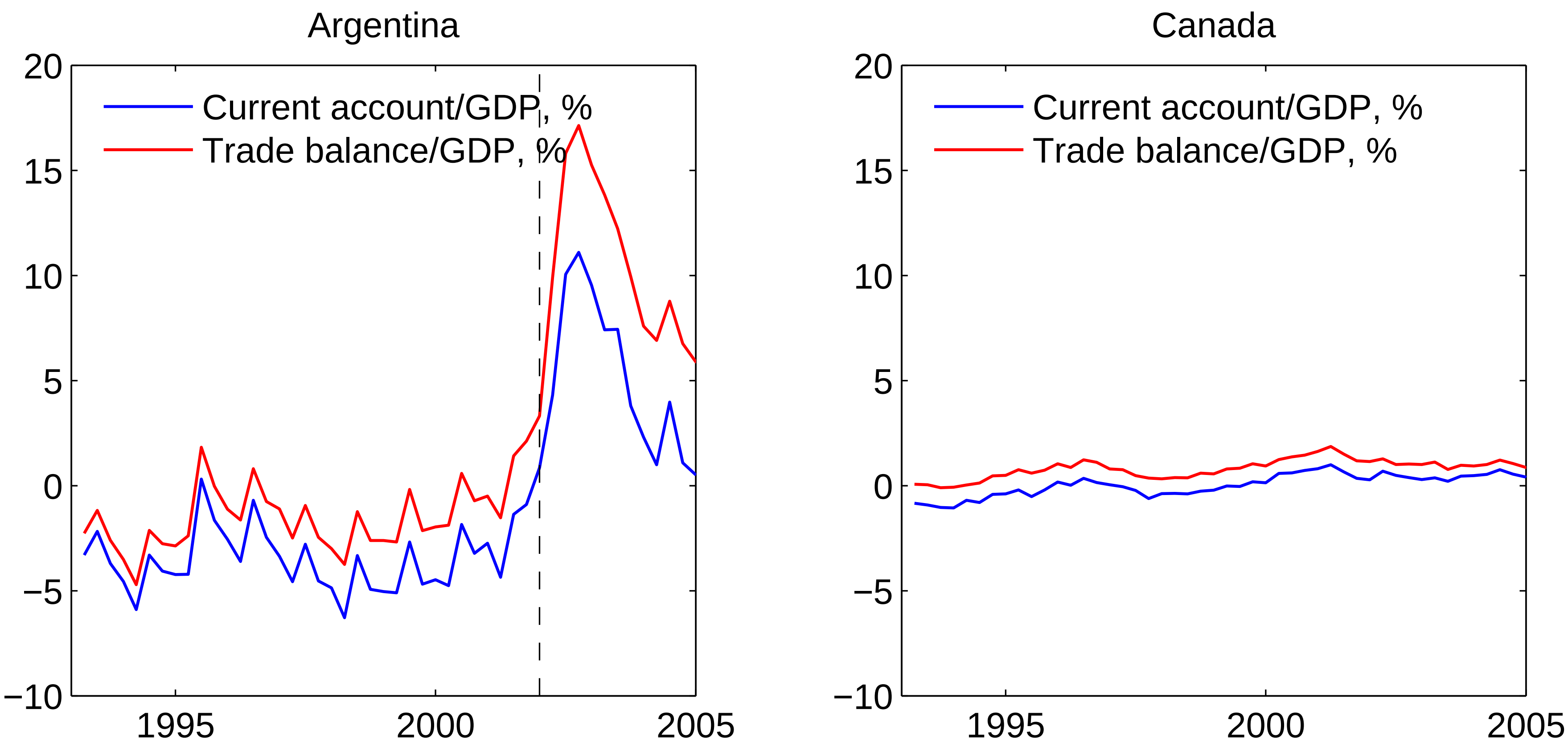
- Emerging economies:
  1. Renewed interest after crises in Russia, Mexico, Argentina
  2. Unable to share risk
  3. Capital outflows in distress states
  4. Counter-cyclical interest rates
- Theory:
  1. Limited enforcement + Incomplete asset structure (Eaton-Gersowitz ’81, Arellano ’02, Yue’ 05)
  2. Moral hazard (Gertler-Rogoff ’90, Atkeson ’91)
- This work:
  – no restrictions on asset structure, constrained efficient contract
  – theoretical and numeric analysis of economies with moral hazard

Fact 1: Inability to share risk



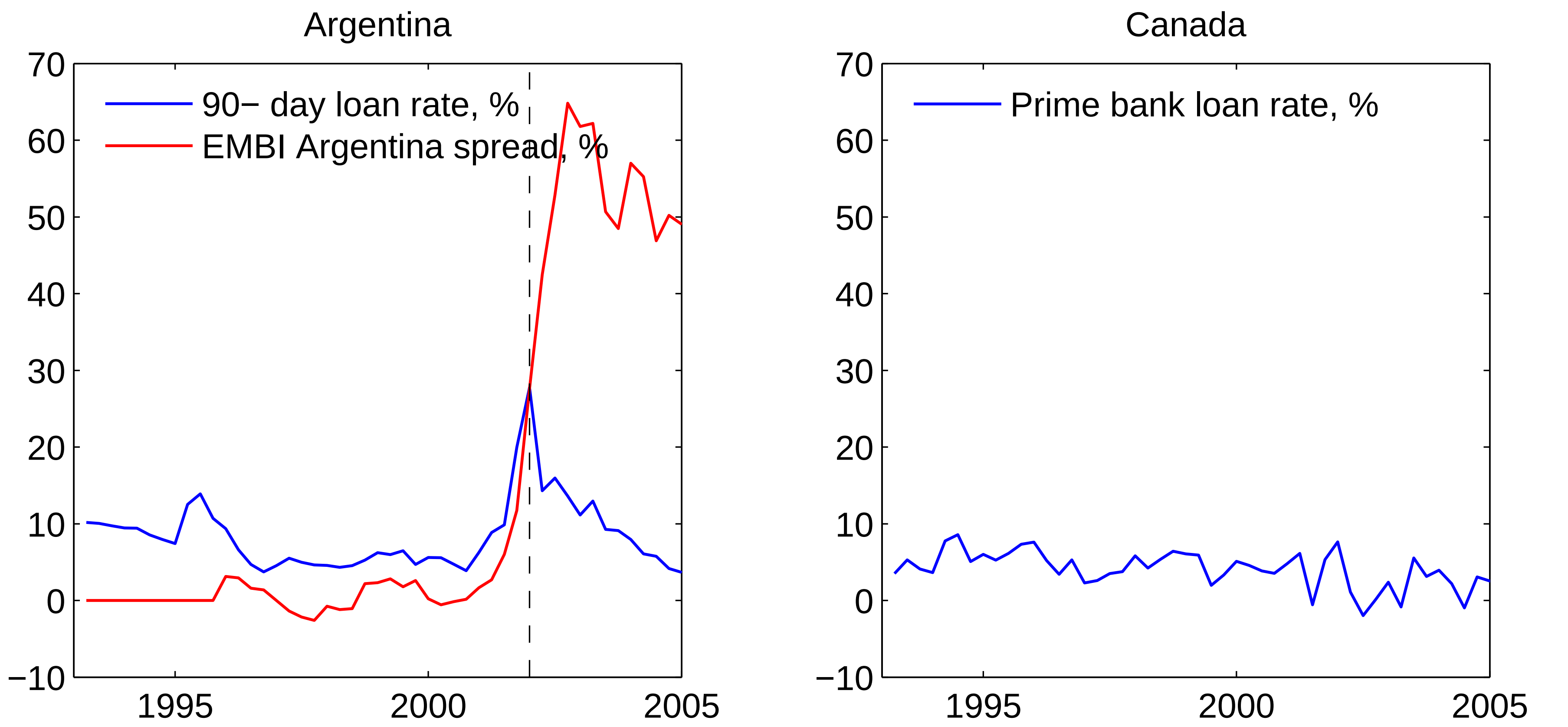
Argentina: Consumption and Investment Path

Fact 2: Volatile and counter-cyclical current account



Argentina: Current Account and Trade Balance

Fact 3: Volatile and counter-cyclical interest rates



Data moments

Country	$\sigma(c)/\sigma(y)$	$\rho(c,y)$	$E(r)$	$\sigma(r)$	$\rho(r,y)$	$\rho(ca,y)$
Argentina, ’93-’02	1.1135	0.9710	8.0142	4.7683	-0.5635	-0.6520
Argentina	1.1500	0.9858	7.8575	4.8028	-0.6726	-0.8146
Canada	0.5611	0.5857	3.9358	2.3941	0.0406	0.3541
AD economy	0	0	0	0	0	≈ -1

Questions Asked

1. Do asymmetric information and limited enforcement indeed generate capital outflows?
2. Which friction is needed for capital outflows: asymmetric information/limited enforcement/both?
3. Is the model empirically relevant?

The Model

Economic Environment

- Infinite horizon, discrete time $t \geqslant 0$;
- Single good at each date;
- Borrower
  - Preferences

$$V^b = E_0 \sum_{t=1}^{\infty} \beta^t u(c_t), \beta \in (0,1), u'(c) > 0, u''(c) < 0$$

  - Endowment, Q_0 of date-0 good
  - Investment technology, $y_{t+1} \sim g(I_t)$

- Sequence of ‘two-period’ lenders
  - Preferences of lender born at date t

$$V^{l,t} = c_t + \beta E_t[c_{t+1}]$$

  - Endowment, M units of date-t and $t+1$ good

Investment Technology

Investing I units of today good yields Y units of tomorrow good

- Finite support

$$Y \in \{Y_1, Y_2, .., Y_n\}, Y_1 < Y_2 < .. < Y_n;$$

- No learning

$$g(Y_i|I) > 0;$$

- Distribution

$$Y|I \sim g, \quad g(Y_i|I) = \lambda(I)g_0(Y_i) + (1 - \lambda(I))g_1(Y_i),$$
$$\lambda : R_+ \rightarrow [0,1], \lambda'(I) > 0, \lambda''(I) < 0;$$

- Monotone LR

$$g_0(Y_i)/g_1(Y_i) \text{ increasing in } i.$$

Contract

Borrower signs a contract $\mathcal{C}$ with creditor

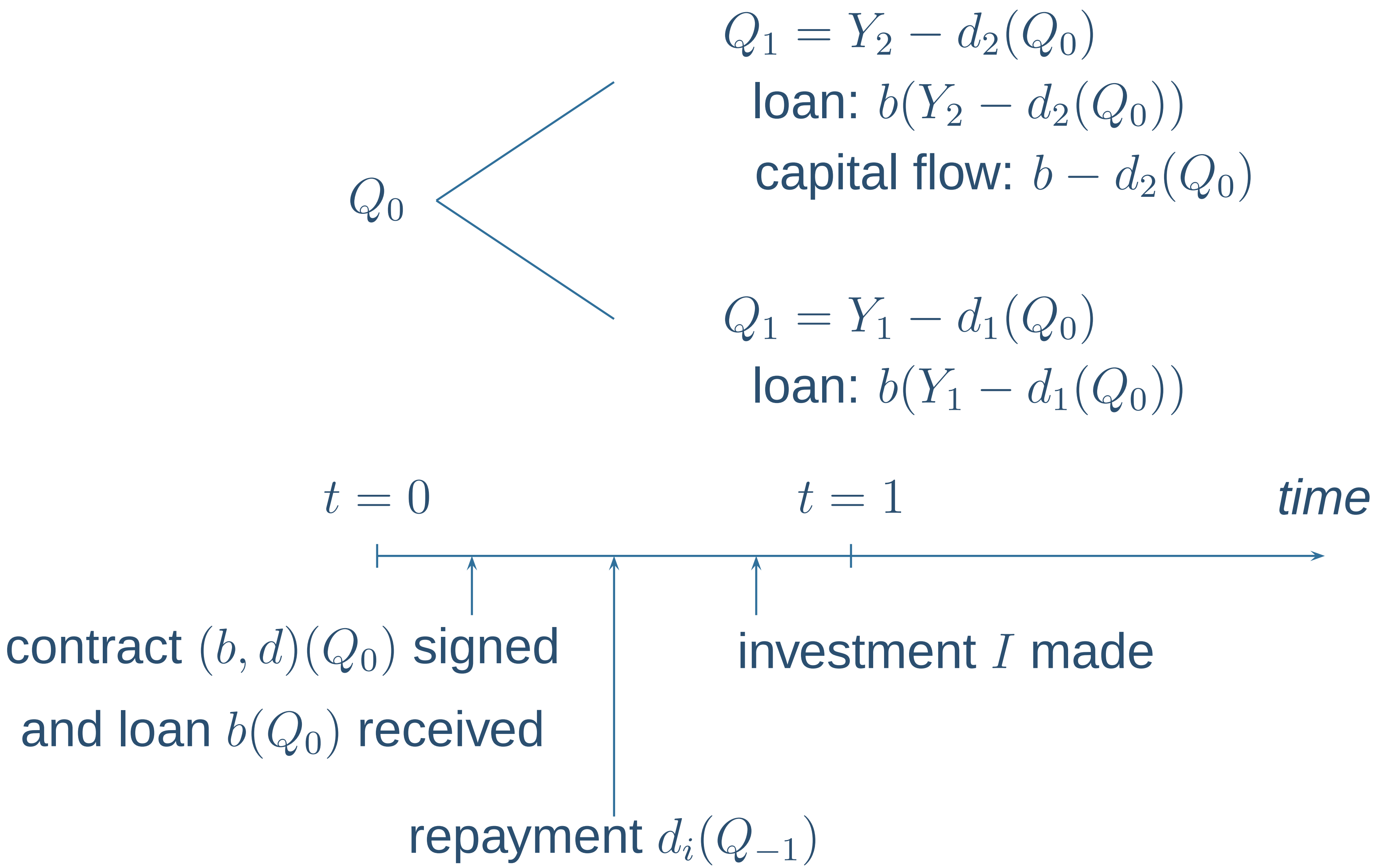
Ideally $\mathcal{C}$ specifies:

– **Loan amount** b provided by creditor,
– **Repayment schedule** $(d_1, ..., d_n)$, where d_i is the amount to be repaid by the borrower in state i,
– **Investment** I made by the borrower.

Usually 3 frictions are studied:

– **Incomplete asset structure**: repayment schedule is non-state contingent: $(d_1, ..., d_n) = (d, .., d)$.
– **Limited enforcement**: borrower or both creditor and borrower cannot commit to abide with the terms of the contract.
– **Asymmetric information**: when investment I is non-*verifiable* and thus cannot be part of a contract.

Time Line



$Q_1 = Y_2 - d_2(Q_0)$
loan: $b(Y_2 - d_2(Q_0))$
capital flow: $b - d_2(Q_0)$
Q_0
$Q_1 = Y_1 - d_1(Q_0)$
loan: $b(Y_1 - d_1(Q_0))$
$t = 0$
$t = 1$
time
contract $(b, d)(Q_0)$ signed
and loan $b(Q_0)$ received
investment I made
repayment $d_i(Q_{-1})$

Dynamic Programming

Autarky

Let $V_{\text{aut}}(Q)$ be optimal value to borrower with Q units of good living in autarky.

$$V_{\text{aut}}(Q) = \max_{I \in [0,Q]} \left\{ u(Q - I) + \beta \sum_{i=1}^{n} g(Y_i | I) V_{\text{aut}}(Y_i) \right\}$$

First-order condition is

$$-u'(Q - I) + \beta \lambda'(I) \sum_{i=1}^{n} \Delta g_i V_{\text{aut}}(Y_i) = 0.$$

Arrow-Debreu Economy

Lending and saving are limited by $M \gg 0$.

Let $V_{\text{AD}}(Q)$ be optimal value for borrower with net worth Q living in AD economy with limit M.

$$V_{\text{AD}}(Q) = \max_{I,b,d} \left\{ u(Q + b - I) + \beta \sum_{i=1}^{n} g(Y_i|I) V_{\text{AD}}(Y_i - d_i) \right\}$$

subject to creditor's rationality constraint

$$b \leqslant \beta \sum_{i=1}^{n} g(Y_i|I) d_i$$

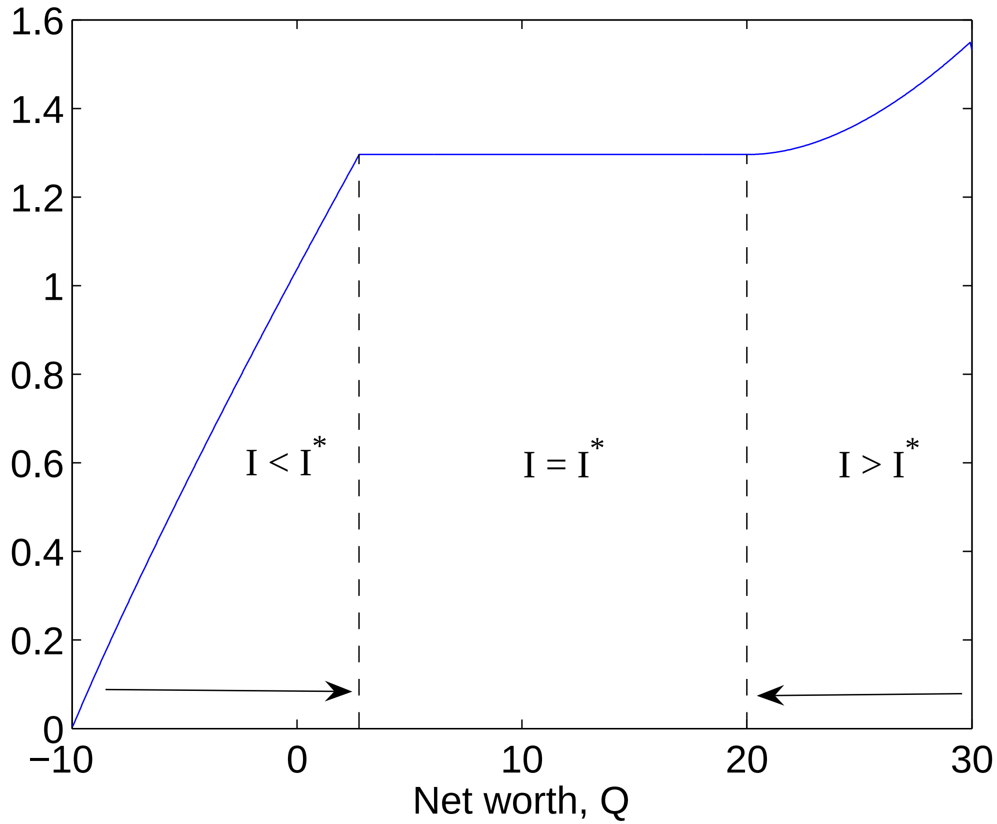
and

$$b, -d_i \leqslant M, \forall i.$$

Unconditionally Optimal Investment Level

Unconditionally optimal investment level I^* satisfies

$$1/\beta = \lambda'(I^*) \sum_{i=1}^{n} \Delta g_i Y_i$$



Asymmetric Information Economy

Let $V_{\text{mh}}(Q)$ be optimal value to borrower living in private information economy when his assets are Q .

$$V_{\text{mh}}(Q) = \max_{b,d} \left\{ u(Q + b - I) + \beta \sum_{i=1}^{n} g(Y_i|I) V_{\text{mh}}(Y_i - d_i) \right\}$$

subject to creditor's rationality condition

$$b \leqslant \beta \sum_i g(Y_i|I) d_i,$$

borrower's incentive constraint

$$I = \arg \max_{I \in [0, Q+b]} \left\{ u(Q + b - I) + \beta \sum_i g(Y_i|I) V_{\text{mh}}(Y_i - d_i) \right\}$$

and

$$b, -d_i \leqslant M, \forall i.$$

Special case: Risk Neutral Borrower

Using $u(C) = C$

$$V_{\text{mh}}(Q) = \max_{b,d} \left\{ Q + b - I + \beta \sum_{i=1}^{n} g(Y_i|I) V_{\text{mh}}(Y_i - d_i) \right\} \text{ s.t.}$$

$$b \leqslant \beta \sum_{i=1}^{n} g(Y_i|I) d_i$$

$$0 = -1 + \beta \lambda'(I) \sum_{i=1}^{n} \Delta g_i V_{\text{mh}}(Y_i - d_i).$$

By the envelope theorem $V'_{\text{mh}}(Q) = 1$ and $V_{\text{mh}}(Q) = Q + const$.
Borrower's individual rationality constraint simplifies to

$$1/\beta = \lambda'(I) \sum_{i=1}^{n} \Delta g_i \cdot (Y_i - d_i).$$

$$I = I^* \Leftrightarrow d_i = d, \forall i$$

Optimality Condition

Optimal contract satisfies

$$V'_{\mathsf{mh}}(Q) = V'_{\mathsf{mh}}(Y_i - d_i)\left(1 + \mu\frac{\lambda'(I)\Delta g_i}{g(Y_i|I)}\right).$$

By assumptions on g_0, g_1, λ

$$\frac{\lambda'(I)\Delta g_i}{g(Y_i|I)}$$

is an increasing function of state. Therefore, $Y_i - d_i$ is an increasing function of state.

Note that

$$V'_{\mathsf{mh}}(Q) = E[V'_{\mathsf{mh}}(Y_i - d_i)] + \mu\lambda'(I)\sum_{i=1}^{n}\Delta g_i V'_{\mathsf{mh}}(Y_i - d_i) < E[V'_{\mathsf{mh}}(Y_i - d_i)].$$

So when $V''' < 0$ then borrower is impoverished over time.

Properties of Optimal Contract

Property 1. *Next period's net worth* $Y_i - d_i(Q)$ *is an increasing function of state* i *for any initial* $Q \in \mathcal{Q}$.

Property 2. *Value of repayments* $\beta \sum_{i=1}^{n} g_i(I) d_i(Q)$ *is increasing in investment for any initial* $Q \in \mathcal{Q}$. *Mathematically,* $\beta \lambda'(I) \sum_{i=1}^{n} \Delta g_i d_i(Q) > 0, \forall Q \in \mathcal{Q}$.

Corollary. *In a two-state economy, repayment schedule is an increasing function of state:* $d_1(Q) < d_2(Q), \forall Q \in \mathcal{Q}$.

Limited Enforcement Economy

Let $V_{\text{lc}}(Q)$ be optimal value to borrower under optimal contract when his assets are Q .

$$V_{\text{lc}}(Q) = \max_{I,b,d} \left\{ u(Q + b - I) + \beta \sum_{i=1}^{n} g(Y_i|I) V_{\text{lc}}(Y_i - d_i) \right\}$$

subject to creditor's rationality constraint

$$b \leqslant \beta \sum_{i} g(Y_i|I) d_i$$

and borrower's participation constraints

$$V_{\text{aut}}(Y_i) \leqslant V_{\text{lc}}(Y_i - d_i), \forall i.$$

Note: Participation constraint can be written as

$$d_i \leqslant Y_i - V_{\text{lc}}^{-1}(V_{\text{aut}}(Y_i)), \forall i.$$

Properties of Optimal Contract

Contract can be characterized analytically.
Property 1. *Next period's net worth* $Y_i - d_i(Q)$ *is an increasing function of state* i *for any initial* $Q \in \mathcal{Q}$.
Moreover, $\exists \{Q_j\}_{j=1}^n$*: if* $k = \arg\max(Q_j : Q_j < Q)$ *then* $d_j = Y_j - Q, \forall j \leqslant k$ *and* $d_j = Y_j - Q_j, \forall j > k$.

Property 2. *Repayment* d_i *is monotone in state* i*:* $d_k(Q) > d_i(Q), \forall k > i, Q \in \mathcal{Q}$.

Property 3. *Investment is an increasing function of net worth* Q.

Property 4. *Contract features endogenous persistence: after adverse shock probability of adverse shock increases; after favorable shock probability of favorable shock increases.*

Moral Hazard + Limited Enforcement Economy

Let $V_{\text{lc}}(Q)$ be optimal value to borrower under optimal contract when his assets are Q .

$$V_{\text{Atk}}(Q) = \max_{b,d} \left\{ u(Q+b-I) + \beta \sum_{i=1}^{n} g(Y_i|I) V_{\text{Atk}}(Y_i - d_i) \right\}$$

subject to creditor's rationality constraint

$$b \leqslant \beta \sum_i g(Y_i|I) d_i$$

borrower's incentive constraint

$$I = \arg \max_{I \in [0, Q+b]} \left\{ u(Q+b-I) + \beta \sum_i g(Y_i|I) V_{\text{Atk}}(Y_i - d_i) \right\}$$

and borrower's participation constraints

$$d_i \leqslant Y_i - V_{\text{Atk}}^{-1}(V_{\text{aut}}(Y_i)), \forall i.$$

Properties of Optimal Contract

Property 1. *Next period's net worth* $Y_i - d_i(Q)$ *is an increasing function of state* i *for any initial* $Q \in \mathcal{Q}$*.*

Property 2. *Value of repayments* $\beta \sum_{i=1}^{n} g_i(I) d_i(Q)$ *is increasing in investment for any initial* $Q \in \mathcal{Q}$*. Mathematically,* $\beta \lambda'(I) \sum_{i=1}^{n} \Delta g_i d_i(Q) > 0, \forall Q \in \mathcal{Q}$*.*

Corollary. *In a two-state economy, repayment schedule is an increasing function of state:* $d_1(Q) < d_2(Q), \forall Q \in \mathcal{Q}$*.*

Let $\underline{Q}$ be lower bound of stationary distribution of net worth:

$$\underline{Q} = Y_1 - d_1(\underline{Q}), \quad V_{\text{Atk}}(\underline{Q}) \geqslant V_{\text{aut}}(Y_1)$$

Property 3. *If limited enforcement constraint binds in state 1 at* $\underline{Q}$ *then it must bind in all other states.*

Property 4. *If limited enforcement constraint binds in state 1 at* $\underline{Q}$ *then capital flow in state 1 at* $\underline{Q}$ *is zero.*

Proof of Property 3

Lemma A. *If $V_{\text{Atk}}(Q) = V_{\text{aut}}(Q')$ for some sustainable levels of net worth Q, Q' then $c_{Atk}(Q) \leqslant c_{aut}(Q')$. Exact equality holds only when all enforcement constraints bind at $Q : V_{\text{Atk}}(Y_i - d_i(Q)) = V_{\text{aut}}(Y_i), \forall i \in \{1, .., n\}$.*

Lemma B. *Suppose $V_{\text{Atk}}(Q) = V_{\text{aut}}(Q')$ for some Q, Q'. Then $V_{\text{Atk}}(Q + x) > V_{\text{aut}}(Q' + x)$ and $V_{\text{Atk}}(Q - x) < V_{\text{aut}}(Q' - x)$ for all $x > 0$.*

Because $V'_{\text{Atk}}(Q) = u'(c) - \gamma u''(c) \geqslant u'(c)$

Lemma C. *$V'_{\text{Atk}}(\underline{Q}) = V'_{\text{aut}}(Y_1)$.*

Otherwise could reduce $d_1(\underline{Q})$ and improve welfare.

Proposition. *At the lowest sustainable level of net worth $\underline{Q}$, if enforcement constraint binds in state 1 then enforcement constraints must bind in all states.*

No Capital Outflows

If enforcement constraint in state 1 binds then

$$V_{\text{aut}}(Y_1) = V_{\text{Atk}}(\underline{Q}) = \max_I \left\{ u(\underline{Q} + b(\underline{Q}) - I) + \beta \sum_{i=1}^{n} g_i(I) V_{\text{Atk}}(Y_i - d_i(\underline{Q})) \right\}$$

$$= \max_I \left\{ u(\underline{Q} + b(\underline{Q}) - I) + \beta \sum_{i=1}^{n} g_i(I) V_{\text{aut}}(Y_i) \right\} = V_{\text{aut}}(\underline{Q} + b(\underline{Q})).$$

Therefore

$$Y_1 = \underline{Q} + b(\underline{Q}) = Y_1 - d_1(\underline{Q}) + b(\underline{Q})$$

or

$$\textit{capital flow} = b(\underline{Q}) - d_1(\underline{Q}) = 0$$

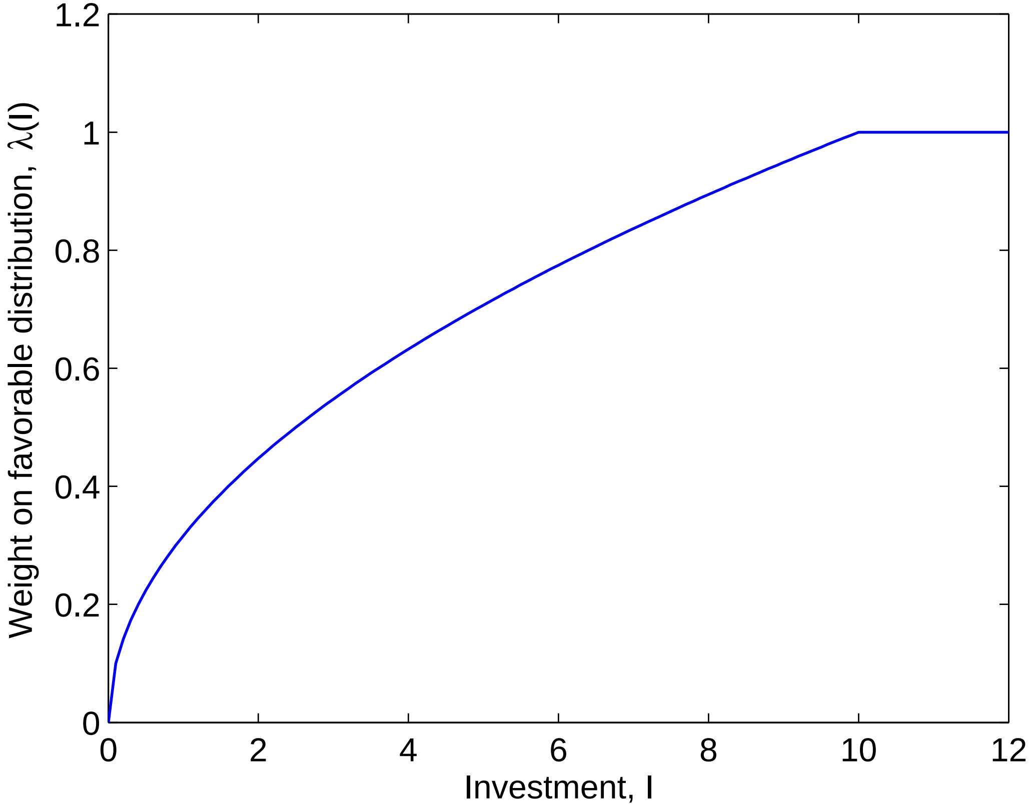
Numeric Example: 2-state Economy

Table 1: CRRA utility specification

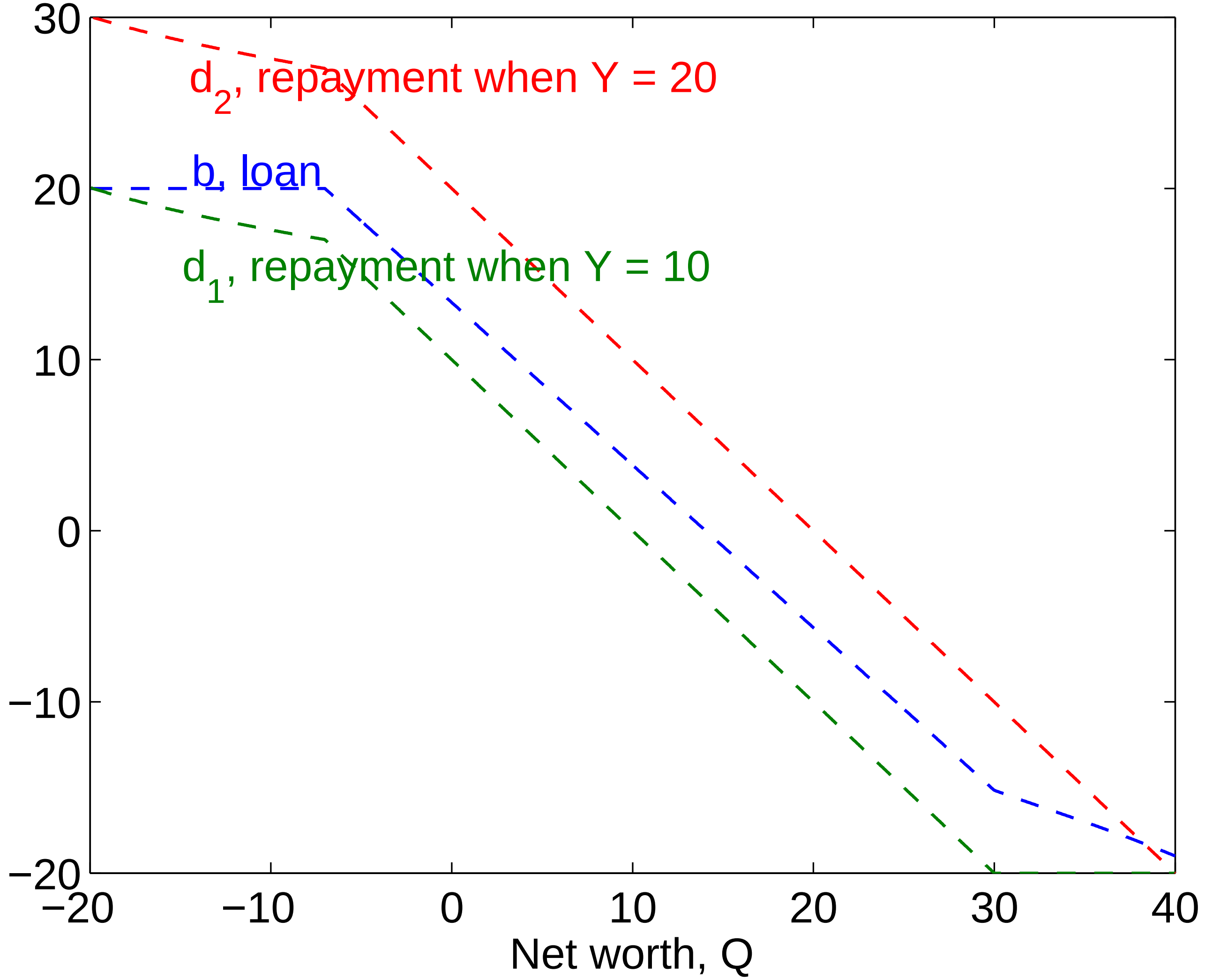
discounting	$\beta = 0.95$
endowments	$Y_1 = 10, Y_2 = 20, M = 20$
preferences	$u(c) = \ln(c)$
investment	$\lambda(I) = min(\sqrt{I/10}, 1)$
uncertainty	$g_0 = (0.1, 0.9)$
	$g_1 = (0.9, 0.1)$

Distribution of Output

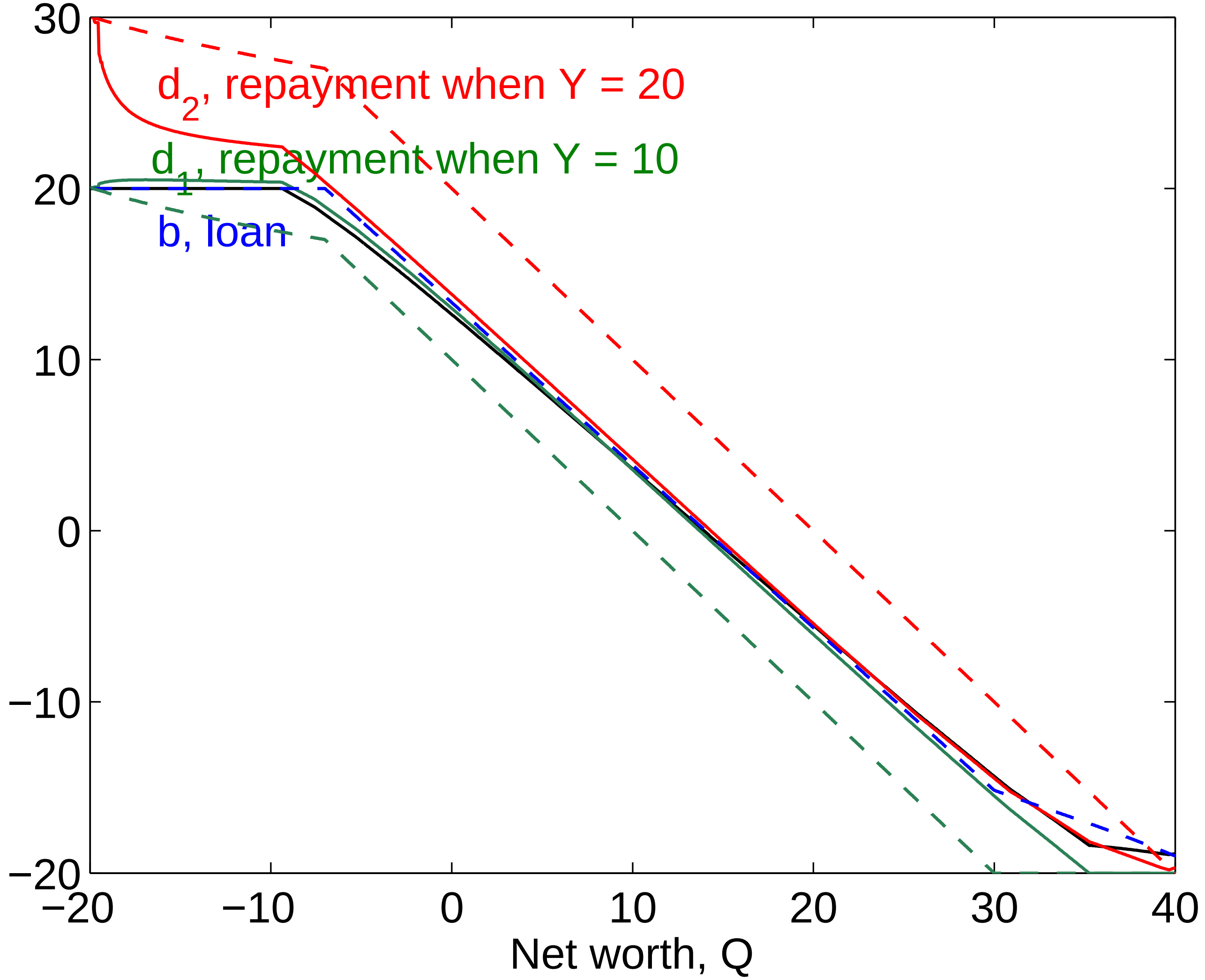
$$g_i(I) = \lambda(I)\begin{bmatrix} 0.1 \\ 0.9 \end{bmatrix} + (1-\lambda(I))\begin{bmatrix} 0.9 \\ 0.1 \end{bmatrix}, \quad \lambda(I) = min(\sqrt{I/10}, 1)$$



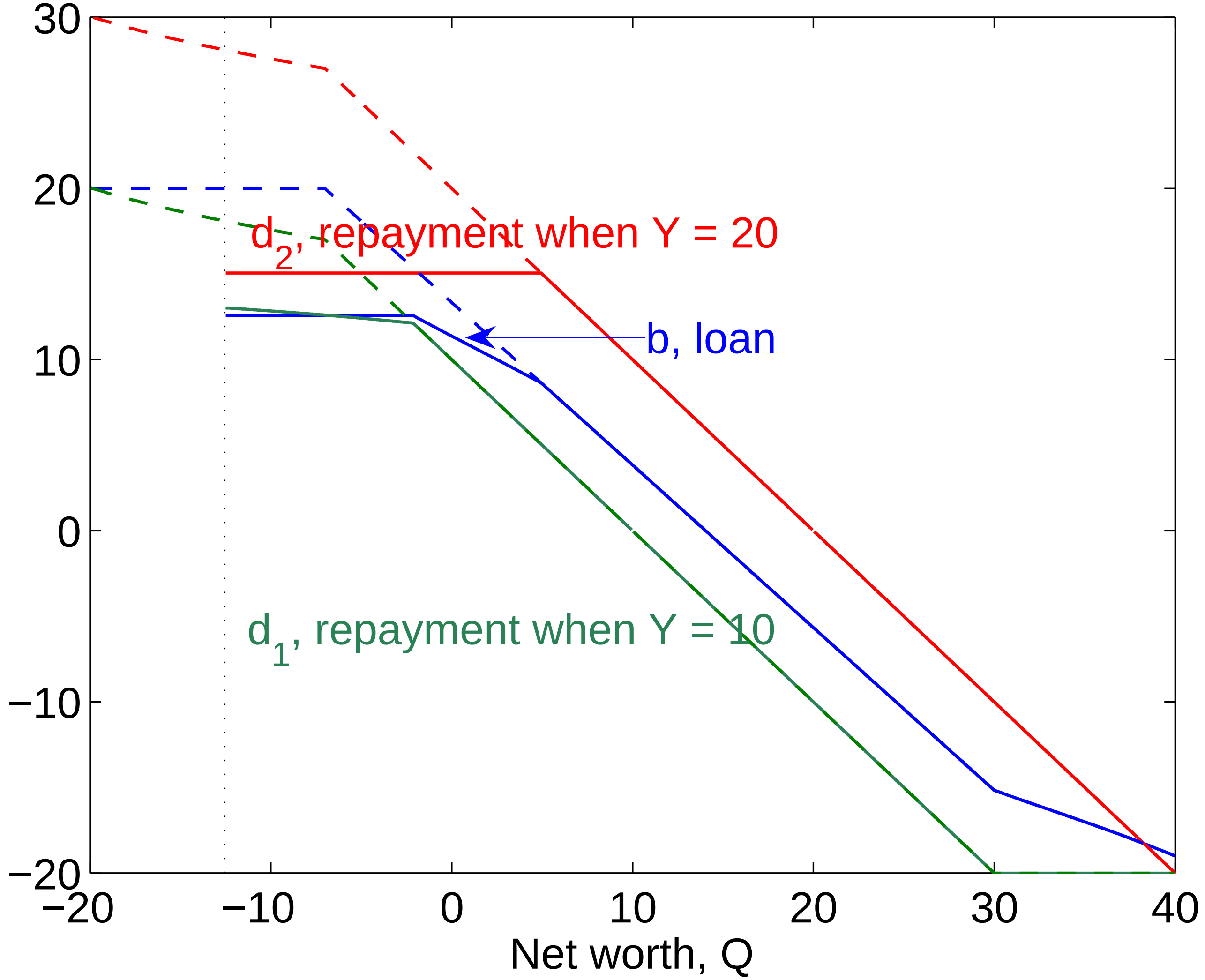
Contract in Arrow-Debreu Economy



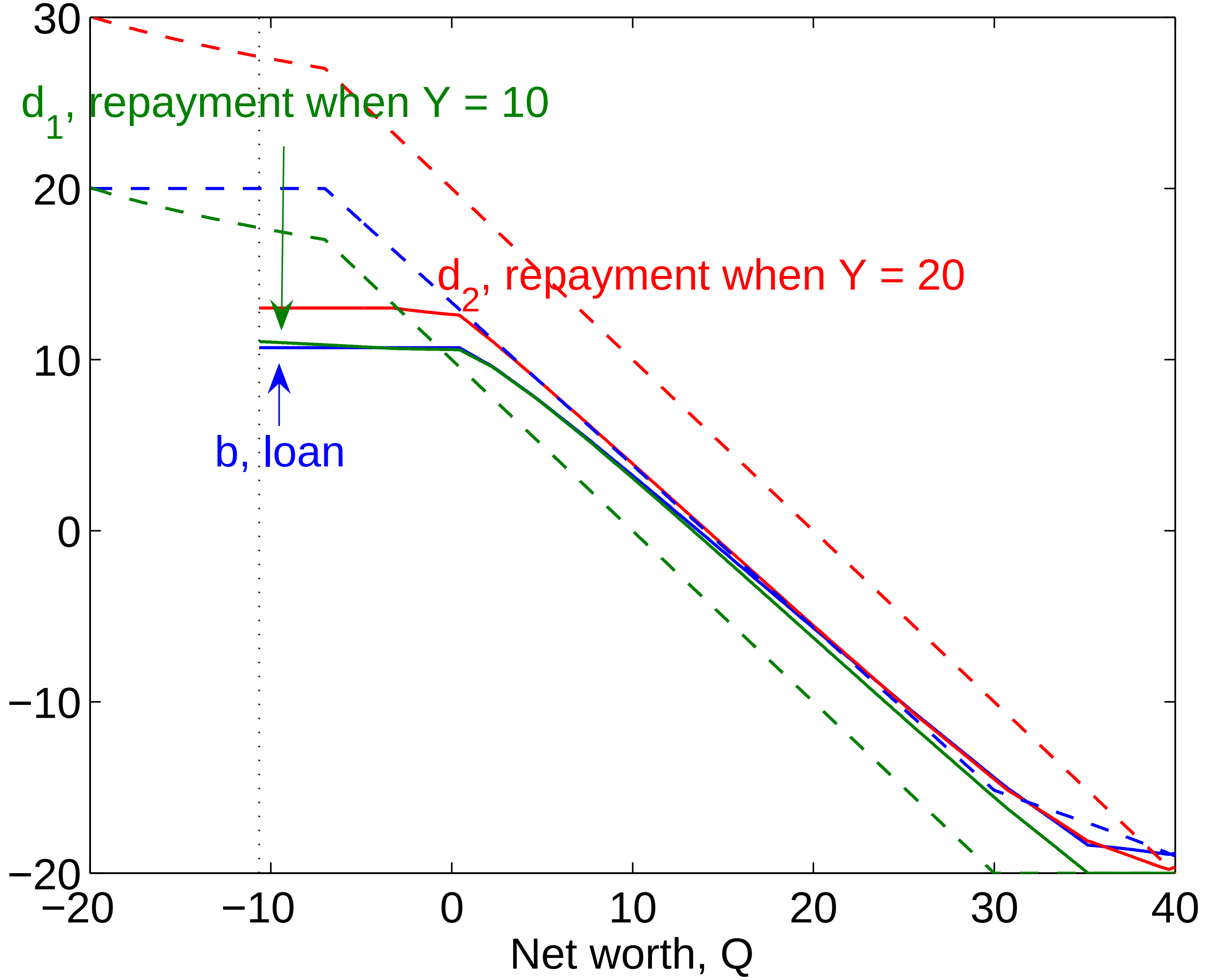
Contract in Moral Hazard Economy



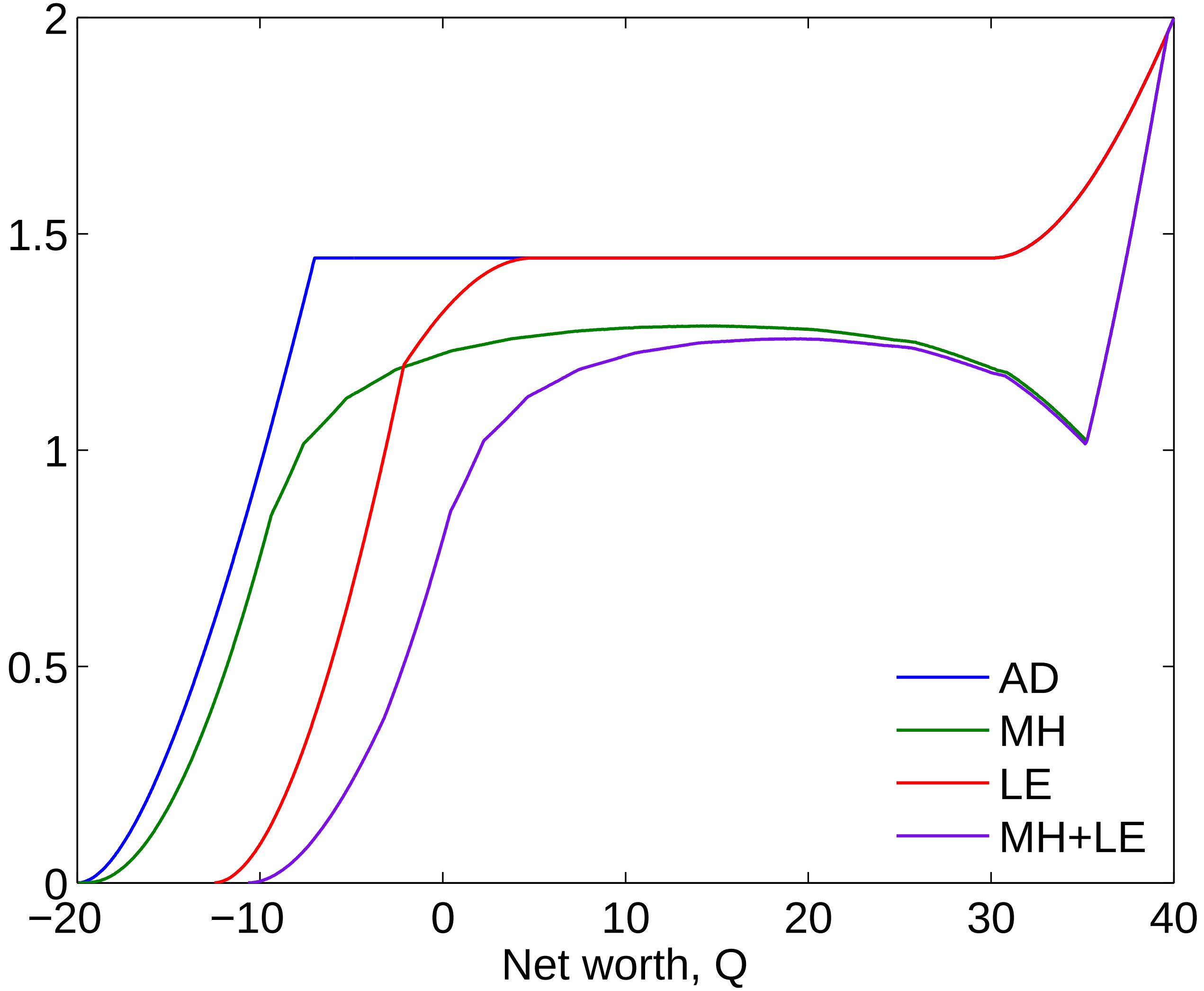
Contract in Limited Enforcement Economy



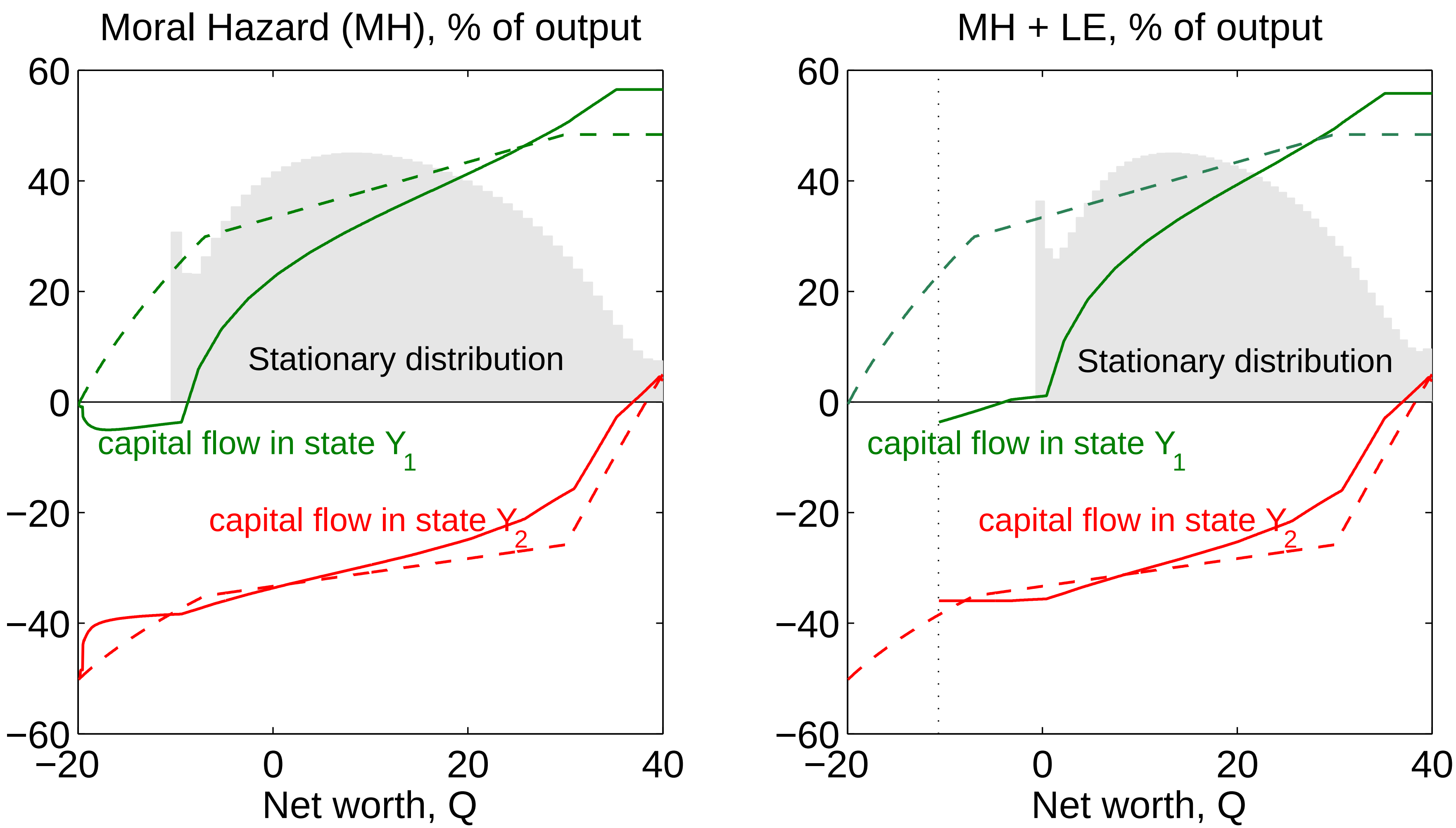
Contract in Moral Hazard + Limited Enforcement Economy



Investment



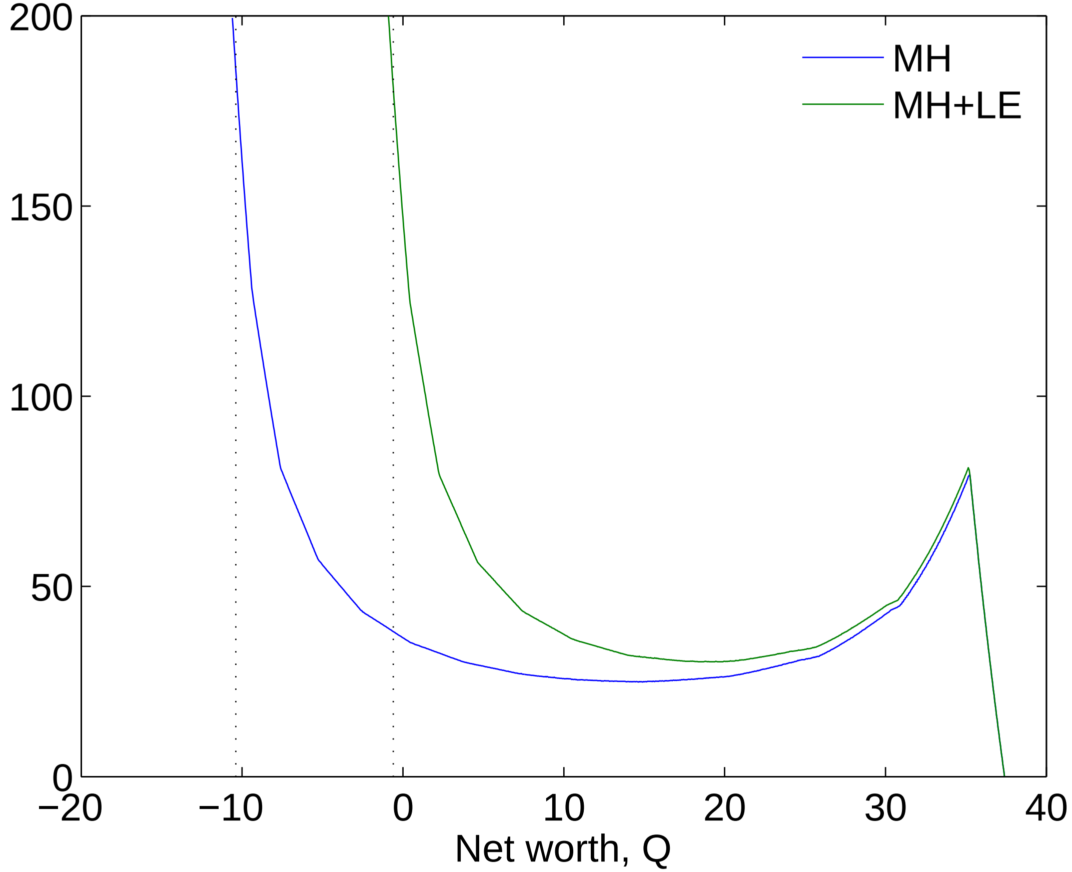
Capital Flows



MH: $p($*capital outflow in state* $Y = 10) = 3.4\%$

Asymmetric Information and Limited Enforcement Premia

$$\text{Internal rate of return} := \lambda'(I)\sum_{i=1}^{n}\Delta g_i Y_i$$



Model moments

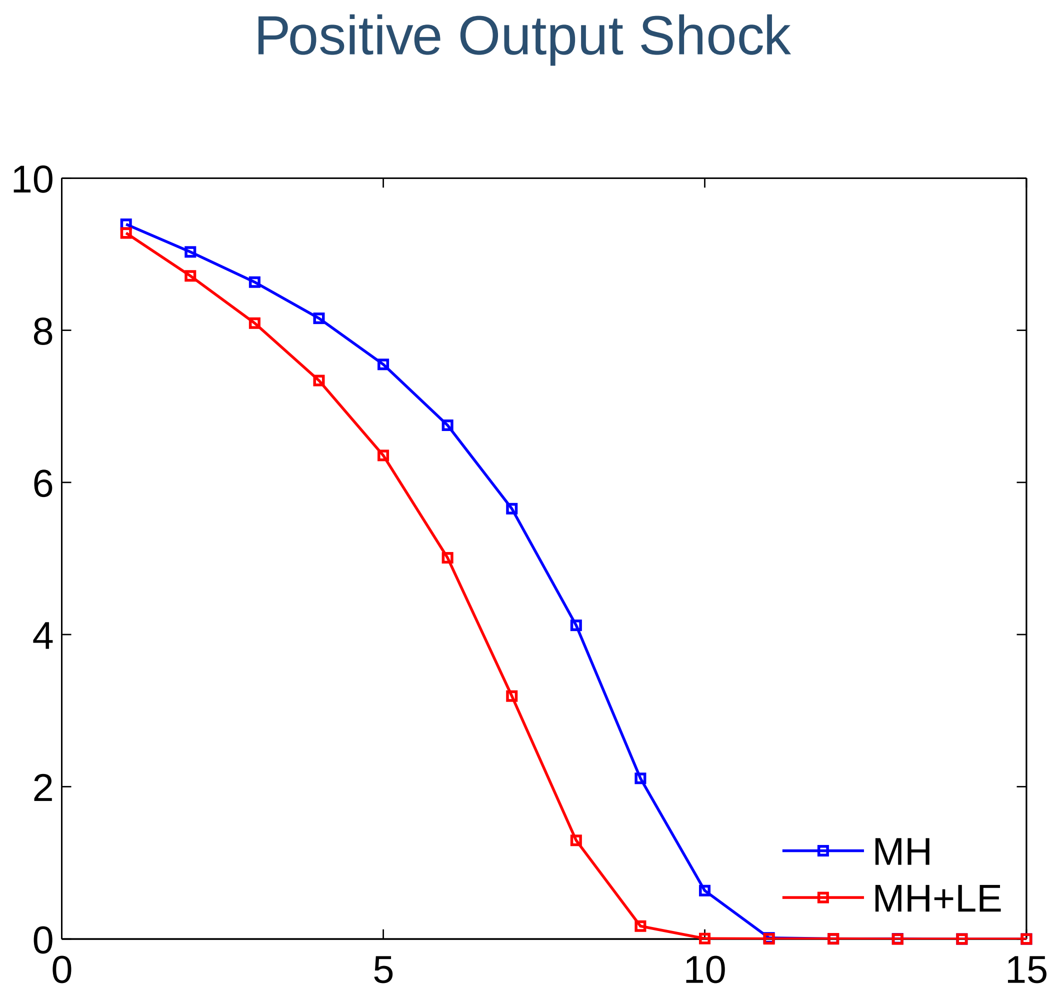
Country	$\sigma(c)/\sigma(y)$	$\rho(c,y)$	$E(r)$	$\sigma(r)$	$\rho(r,y)$	$\rho(ca,y)$
Argentina, ’93-’02	1.1135	0.9710	8.0142	4.7683	-0.5635	-0.6520
Moral hazard econ.	0.3557	0.3950	39.9033	33.3876	-0.2131	-0.9225
MH + LE econ.	0.3614	0.4610	47.5930	36.7624	-0.2581	-0.9195

Country’s debt servicing

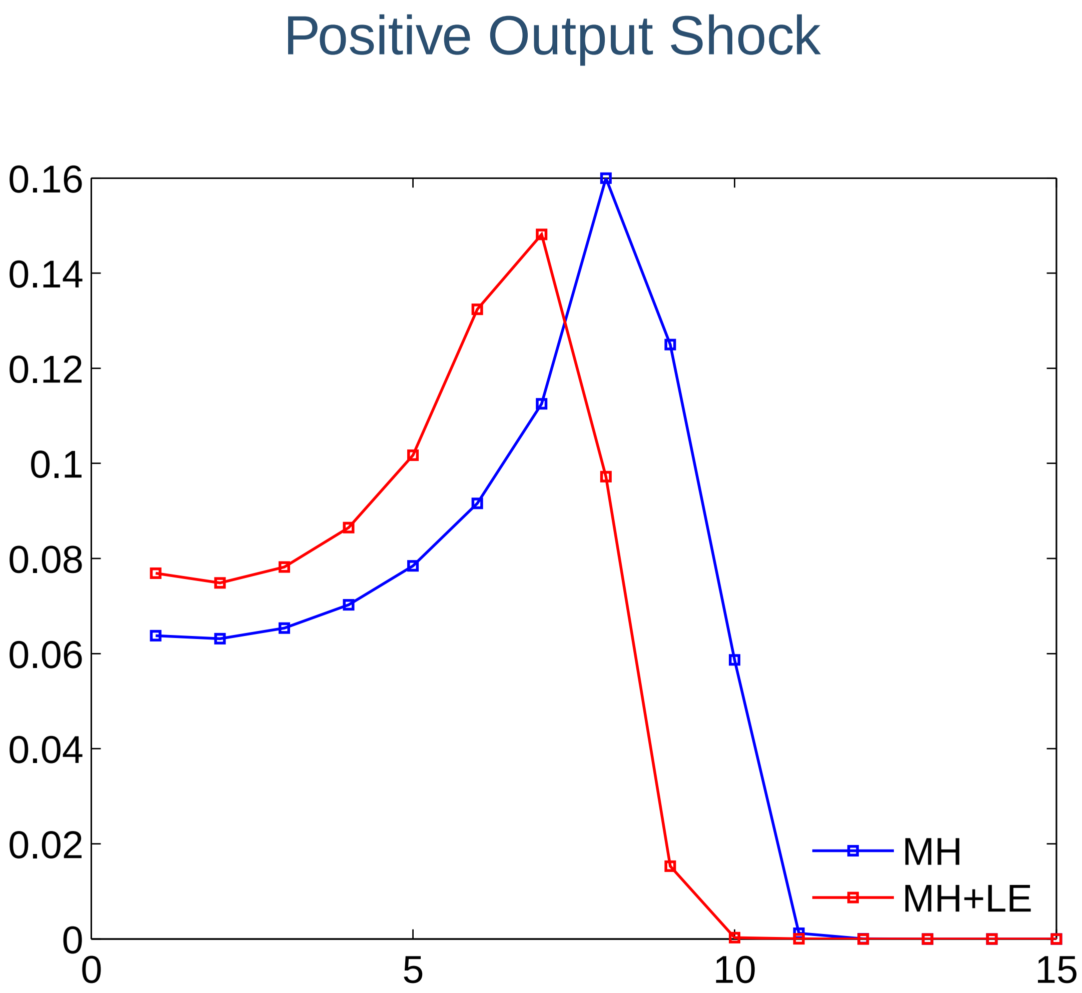
Country	debt service/output
Argentina	9.5%
Moral hazard econ.	19.5%
MH + LE econ.	-21.95%

Impulse Responses

Impulse Response of Net Worth

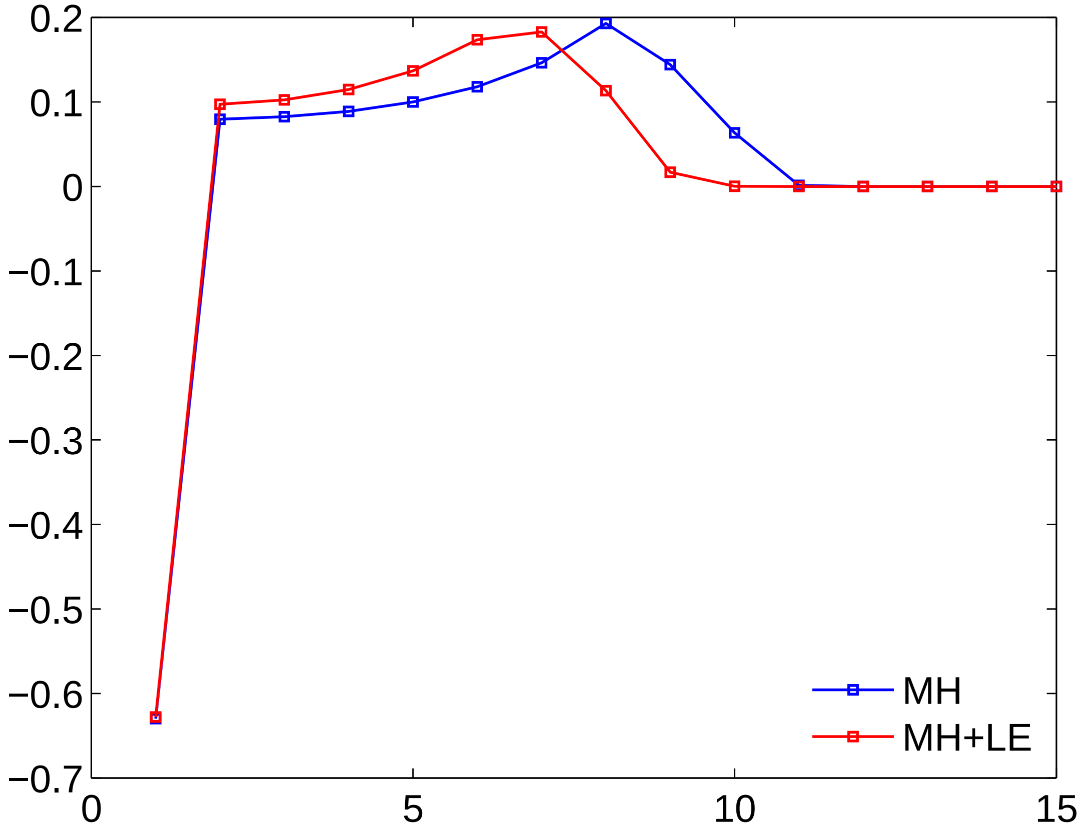


Impulse Response of Consumption

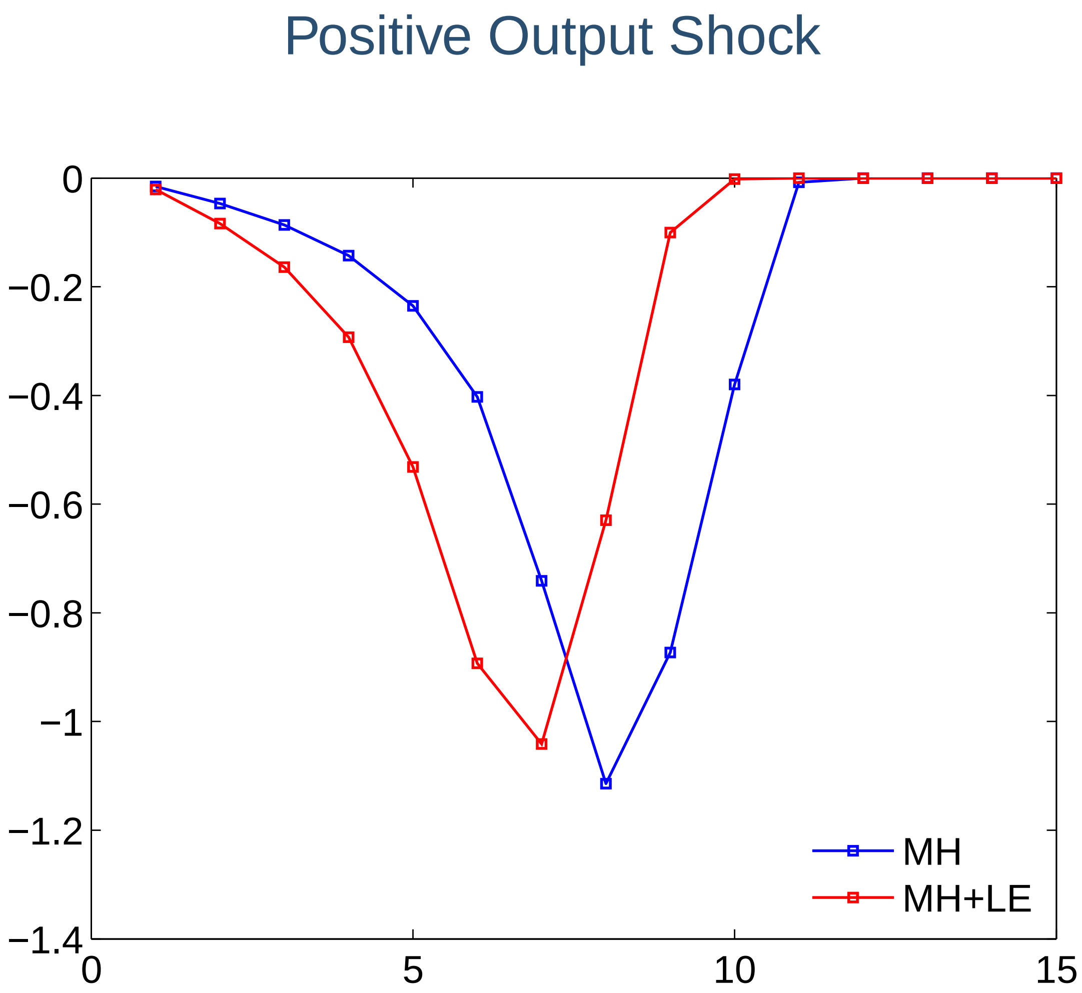


Impulse Response of Current Account

Positive Output Shock



Impulse Response of Finance Premium



Two-good economy

1. Consumption aggregate

$$C = F(C_T, C_N), F \text{ is a CRS aggregator;}$$

2. Exogenous endowment of non-tradable good E_N;
3. Price of tradable good C_T is normalized to 1;
4. Contract (b, d) is specified in terms of tradeable good;

5. Price of non-tradable

$$p_N = \frac{F_N(C_T, C_N)}{F_T(C_T, C_N)} = f(C_T/C_N), f' > 0;$$

6. Real exchange rate = price of cons. composite

$$e = \min_{C_T, C_N} \{C_T + p_N C_N\} \text{ s.t. } g(C_T, C_N) = 1$$

By envelope theorem $de/dp_N = C_N^* > 0$

Two-good economy

Let $V(Q, E_N)$ be optimal value to borrower under optimal contract when his net worth is Q and endowment of non-tradable good E_N.

$$V(Q, E_N) = \max_{c_T, c_N, b, d} u(F(C_T, C_N)) + \beta \int_{E'_N} \sum_{i=1}^{n} g(Y_i|I) V(Y_i - d_i, E'_N) dF(E'_N)$$

subject to

$$\begin{aligned} C_T + I &\leqslant Q + b && \text{resource constraint for T} \\ C_N &\leqslant E_N && \text{resource constraint for NT} \\ b &\leqslant \beta \textstyle\sum_{i=1}^{n} g(Y_i|I) d_i && \text{creditor's IR} \\ V_{\text{aut}}(Y_i) &\leqslant V(Y_i - d_i) && \text{participation con.} \\ & && \text{borrower's IR} \end{aligned}$$

$$I = \arg \max_{I \in [0, Q+b]} u(C_T, C_N) + \beta \int_{E'_N} \sum_{i=1}^{n} g(Y_i|I) V(Y_i - d_i, E'_N) dF(E'_N)$$

A simple example

Let consumption aggregate be

$$C = (\omega C_T^{1-1/\varepsilon} + (1-\omega)C_N^{1-1/\varepsilon})^{\varepsilon/(\varepsilon-1)}, \varepsilon > 0$$

Then

$$p_N = \frac{1-\omega}{\omega}\left(\frac{C_T}{C_N}\right)^{1/\varepsilon}$$

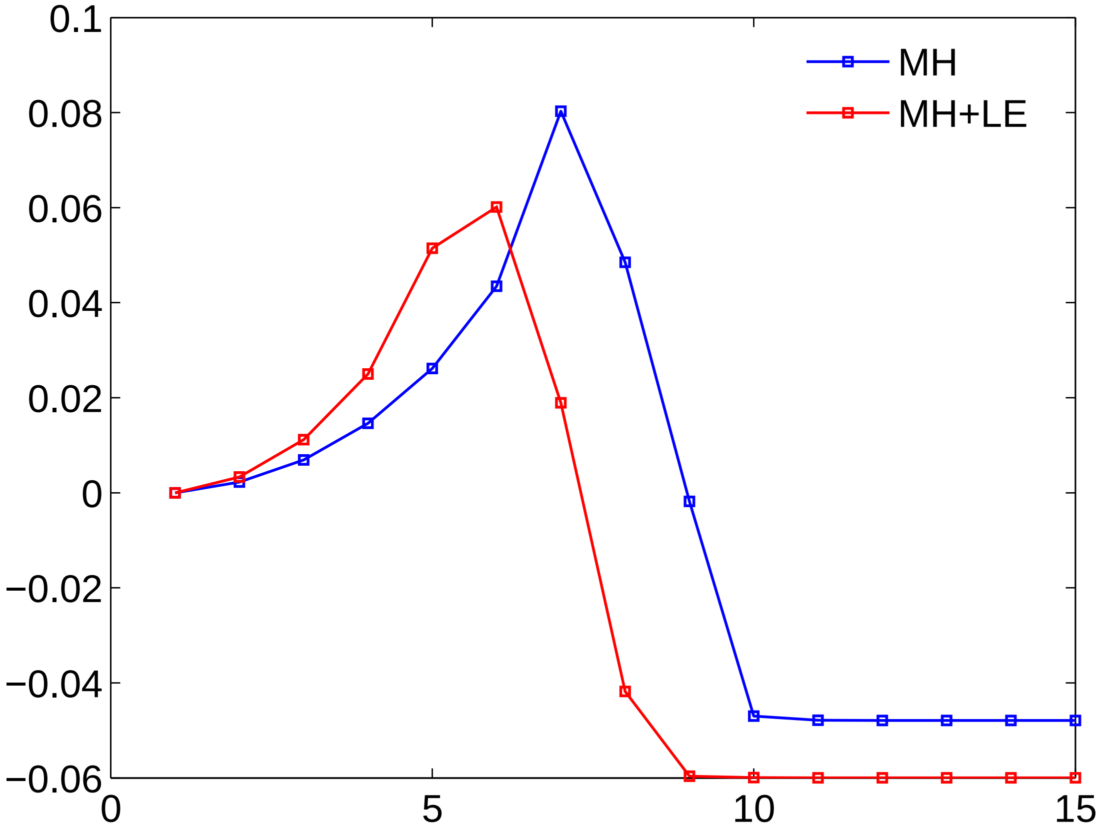
$$e = \left(\omega^\varepsilon + (1-\omega)^\varepsilon p_N^{1-\varepsilon}\right)^{1/(1-\varepsilon)}$$

Assumption: endowment of C_N is constant.

low output $\rightarrow$ cap. outflows $\rightarrow C_T/C_N \downarrow \rightarrow p_N \downarrow \rightarrow e \downarrow$

Impulse Response of Exchange Rate

Positive Output Shock



Heterogenous discount factors

Consider the case when borrower is less patient $\beta_b < \beta_c$.

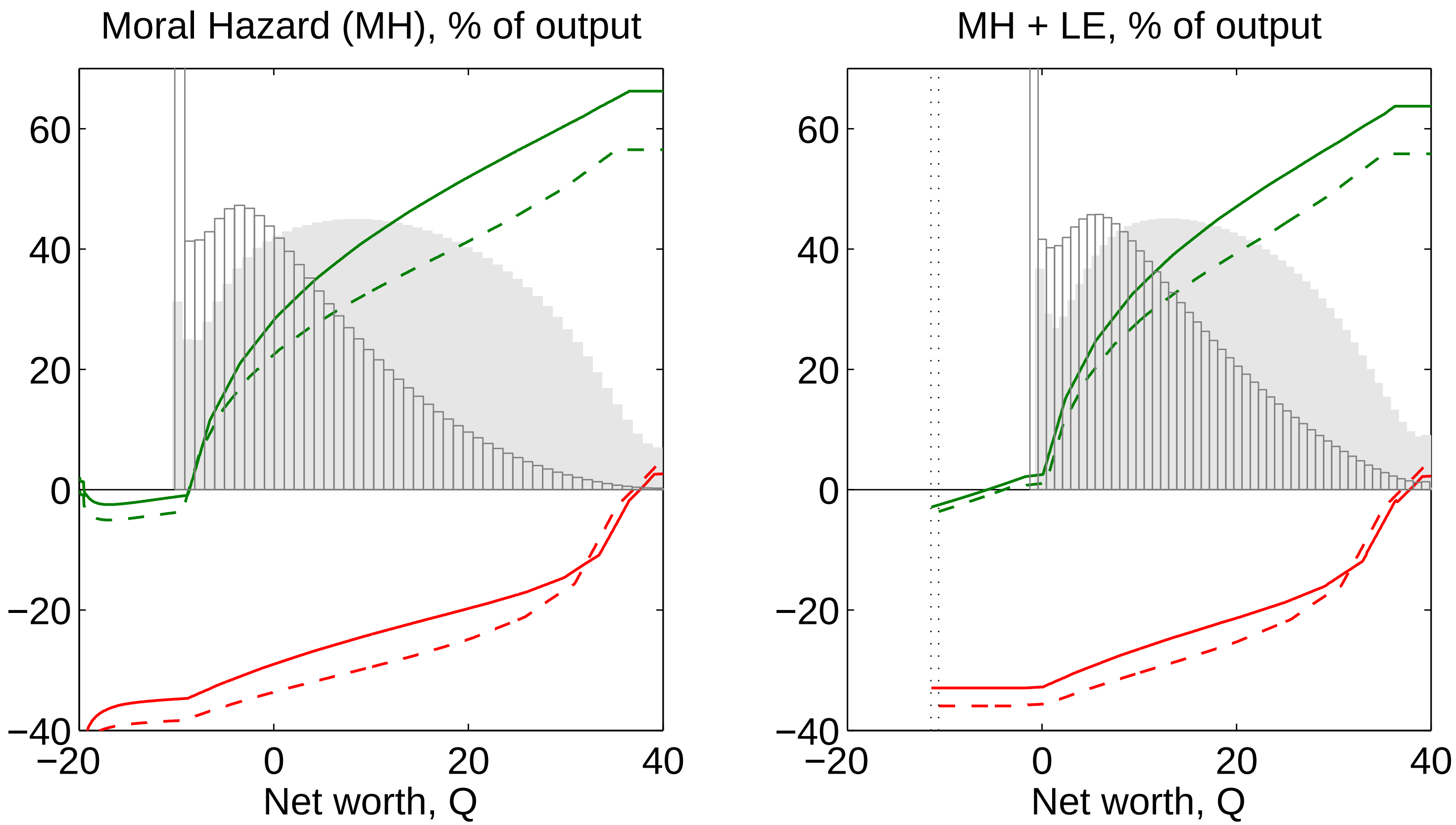
First-order optimality condition now is

$$V'(Q) = \frac{\beta_b}{\beta_c} V'(Y_i - d_i) \left(1 + \gamma_i + \mu \frac{\lambda'(I) \Delta g_i}{g(Y_i | I)} \right).$$

Relative to the case with $\beta_b = \beta_c$ repayment must increase. That is low levels of net worth receive more weight in the stationary distribution.

Increase in Creditor's Discount Factor

$$\beta_c = 0.96 > 0.95 = \beta_b$$



MH: $p($*capital outflow in state* $Y = 10) = 10.9\%$ (before was 3.4%)

Model moments

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$\beta_c = 0.96 > 0.95 = \beta_b$						
Moral hazard econ.	0.3868	0.5120	69.3468	61.0546	-0.4585	-0.8814
MH + LE econ.	0.3824	0.5265	69.8126	59.4894	-0.4622	-0.8862
$\beta_c = 0.96 > 0.95 = \beta_b$ and $\nu = 0.6$						
Moral hazard econ.	0.4075	0.5035	63.9936	55.2871	-0.4401	-0.8598
MH + LE econ.	0.4014	0.5238	64.7872	53.3901	-0.4435	-0.8670

Conclusions

In this work I compare 3 models:

1. Moral hazard economy
2. Limited enforcement economy
3. Moral hazard + limited enforcement economy

Theoretical results

1. Monotonicity of repayment schedule
2. Capital outflows in distress states

Numerical results

1. Moral hazard economy fits data best
2. Counter-cyclical current account
3. Significant (counter-cyclical) asymmetric information premia
4. Very persistent net worth and consumption