Optimal Contracts Under Costly State Falsification

J. Lacker and J. Weinberg, JPE 1989

Presented by Viktor Tsyrennikov

Motivation

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   (as opposed to debt-like contracts in existing models of imperfect information, e.g. costly state verification)

3. Optimal (non-falsification) contracts are linear

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Costly state verification model of Townsend (1979)

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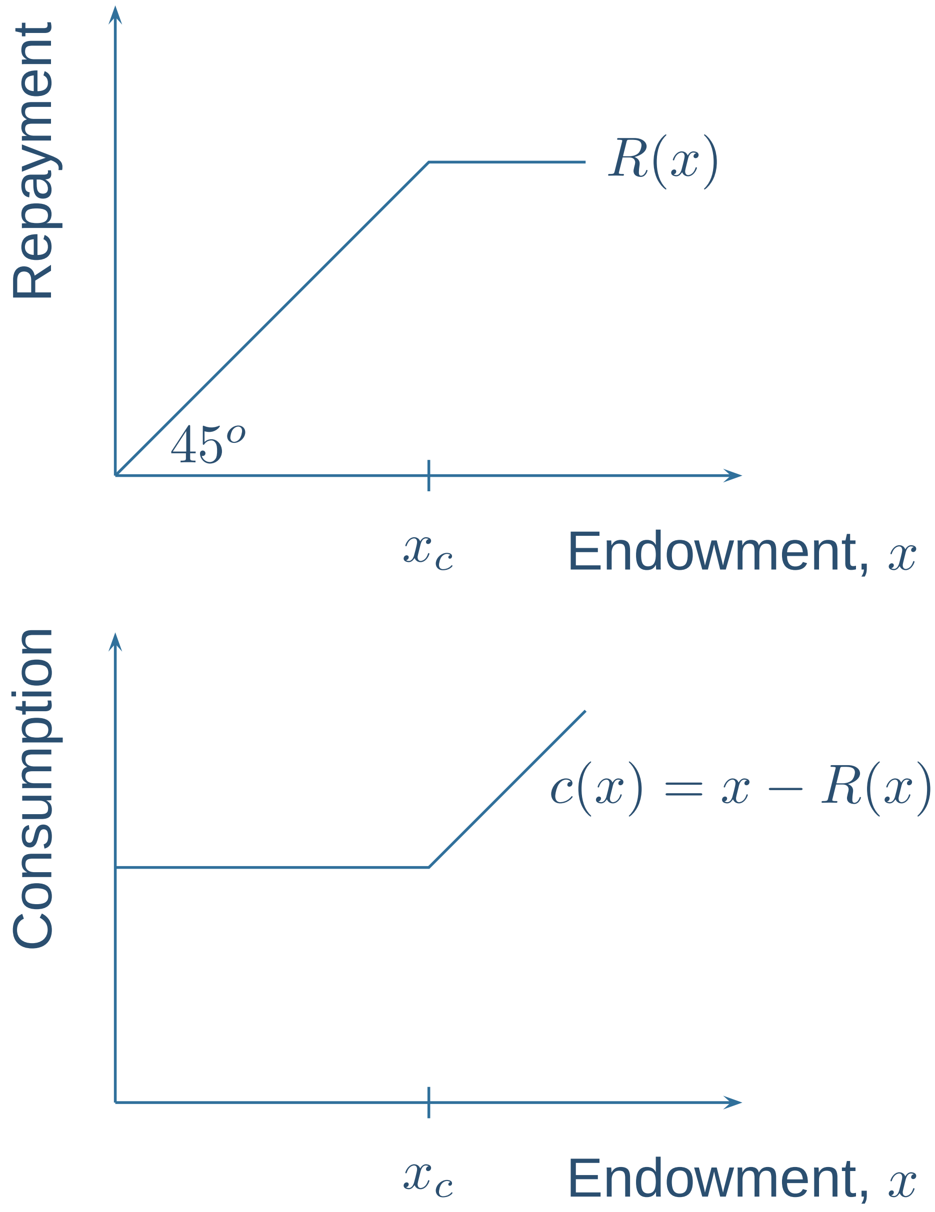
3. When verification is deterministic then the optimal contract resembles debt

Debt-like Contract



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4. Information

   Agent 2 can report y instead of x at cost $g(x - y)$

   $g' > 0, g(0) = 0$

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- Transfer function $R : M \times X \to R$
- Communication is superfluous
  Suppose the same report is chosen but different messages are sent:

$$m(x) = m \neq m(x') = m'$$
$$y(x) = y = y(x')$$

But by incentive-compatibility

$$x - R(m, y) - g(y - x) \geqslant x - R(m', y) - g(x - y)$$
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So $R(m, y) = R(m', y)$:
only $R(m, y) = R(y)$ *need to be considered.*

Allocation

- Consumption schedules are

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- Incentive compatibility

  (R, y, c_1, c_2) are incentive-compatible if

$$c_2(x) = \max_y \Big\{x - R(y) - g(x - y)\Big\}$$
$$y(x) = \arg\max_y \Big\{x - R(y) - g(x - y)\Big\}$$

Optimal Contract

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$$\int_0^{\bar{x}} u(c_2(x))dF(x)$$

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Subject to

$$\int_0^{\bar{x}} (w + R(y(x)))dF(x) \geqslant K \geqslant w$$

and $c_i \geqslant 0$ and (R, y, c_1, c_2) being incentive-compatible.

Optimal Non-Falsification Contract

Want: $y(x) = x$.

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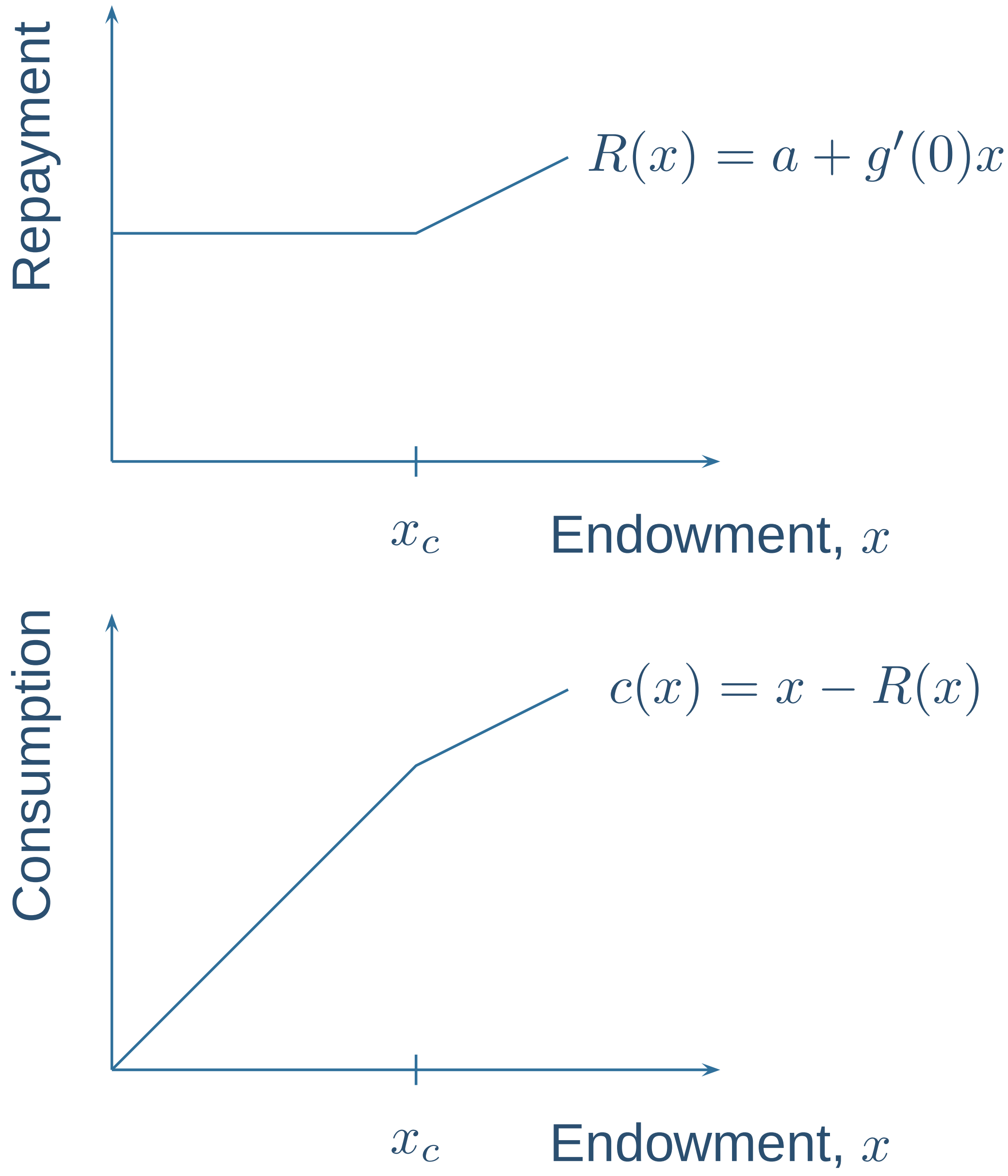
- Proposition 1. Suppose that

  a. $g'(0) \in [0, 1]$,

  b. $g(h) \geqslant g'(0)h, \quad \forall h \geqslant 0$.

  Then optimal non-falsification contract is linear.

Equity-like Non-Falsification Contract



Can Falsification Contract Do Better?

Consider $g(h) = \gamma_1 h + \gamma_2 h^2/2$ and $R(x) = x - c$.

- Agent 2 shades $h = min[(1-\gamma_1)/\gamma_2, x]$
- Agent 2 consumes

$$c_2(x) = \begin{cases} c + x - g(x), & x \leqslant h \\ c + h - g(h), & x \geqslant h \end{cases}.$$

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So, for large γ_2 falsification contract dominates the optimal non-falsification contract.

Optimal Falsification Contract

Proposition 2. Suppose that

a. $g(h)$ is non-decreasing in h for $h \geqslant 0$,

b. $g'(0) \in [0, 1]$.

Then $\exists a \geqslant 0$ such that

a. $R'(x) \leqslant g'(0), \forall x$

b. $R(x) = -w, x \in [0, a]$

c. $R'(x) > 0, x \in (a, \bar{x}]$.

So, globally optimal contract may not be linear.

When Non-Falsification Contracts Are Optimal?

Proposition 3. Suppose that

a. $g'(0) \in [0,1]$,

b. $g(h) \geqslant g'(0)h, \quad \forall h \geqslant 0$

c. $g'' > 0, g''' \leqslant 0$

d.

$$\frac{u'(0)}{u'(w+\bar{x})} - 1 \leqslant \frac{g'(0)f(x)}{g''(x)F(x)} \qquad (*).$$

Then optimal contract specifies no falsification.

$(*)$ is satisfied when

1. Agent is not ‘too’ risk averse
2. Falsification cost is not ‘too’ convex.

Conclusions

In costly state falsification environment:

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Other thoughts

- What happens in repeated/dynamic environment?
  (A: Is linearity preserved?)
- What happens when coupled with other frictions, e.g. limited enforcement/liability?
  (A: Can default happen in equilibrium?)