Efficient Allocations with Moral Hazard and Hidden Borrowing and Lending

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Working Paper, 2005.

Presented by Tomek Piskorski

Objective of the Paper

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- The purpose of the paper is to provide a tractable recursive framework to study an efficient allocation in the dynamic moral hazard model with hidden borrowing and lending and to characterize its properties.

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$$U_t^T(\tau, \sigma; h^t) \geq U_t^T(\tau, \tilde{\sigma}; h^t)$$

where $\tilde{\sigma}$ is any other plan of action after node h_t that satisfies the agent's borrowing and budget constraints.

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The planner can always provide additional insurance with respect to self insurance, at least in the last two periods of the program.

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- Fernandes and Phelan (2000) show that this adverse selection problem can be resolved by using one state variable for each agent type.
- With a continuum of possible asset levels, the number of types explodes and the relevant state becomes a function, i.e. an infinite dimensional object.

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- Verify (numerically) ex post the validity of the FOA by allowing the agent to re-maximize his lifetime utility taking the proposed optimal contract as given.
- If the agents value under re-maximization coincides with that implied by the optimal contract, the FOA is validated.

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The FOA: Replace IC with the two FOC conditions:

$$e : -u_e'(c_t(h^t),e_t(h^t)) = \beta \sum_i p_i'(e_t(h^t)) U_{t+1}(\mathcal{W};(h^t,y^i))$$

$$b : u_c'(c_t(h^t),e_t(h^t)) \geq \beta(1+r) \sum_i p_i(e_t(h^t)) u_c'(c_{t+1}(h^t,y^i),e_{t+1}(h^t,y^i))$$

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$$V^*_{FOA}(U_0) = \sup_{(U^i,x^i)_{i=1}^N \in M^*} \sum_i p_i^0 V(y^i, U^i, x^i) \text{ s.t. } \sum_i p_i^0 U^i \geq U^0$$

where V solves

$$V(y,U,x) = \sup_{(U^i,x^i)_{i=1}^N \in M^*, c \geq 0, e \in E} y - c + \tfrac{1}{1+r} \sum_i p_i(e) V(y^i, U^i, x^i) \text{ s.t.}$$

$$U = u(c,e) + \beta \sum_i p_i(e) U^i$$

$$-u_e'(c,e) = \beta \sum_i p_i'(e) U^i \quad u_c'(c,e) \geq \beta(1+r) \sum_i p_i(e) x^i \quad x = u_c'(c,e)$$

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Proposition 2: Assume M^* is compact, V is bounded and p^0 is degenerate at y. Let $U^*(U_0)$ be the expected discounted lifetime utility level obtained by the agent under the FOA efficient contract with the transfer scheme τ. The FOA is justified if and only if $U_0^R(\tau, y) = U^*(U_0)$.

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Challenge: In numerical application $U_0^R(\tau, y)$ is likely to differ from $U^*(U_0)$. How to distinguish if the difference is due to approximate errors or due to invalidity of the FOA?

Numerical Example:
Calibration

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$$Y = \{y_L = 0.1, y_H = 100\},$$

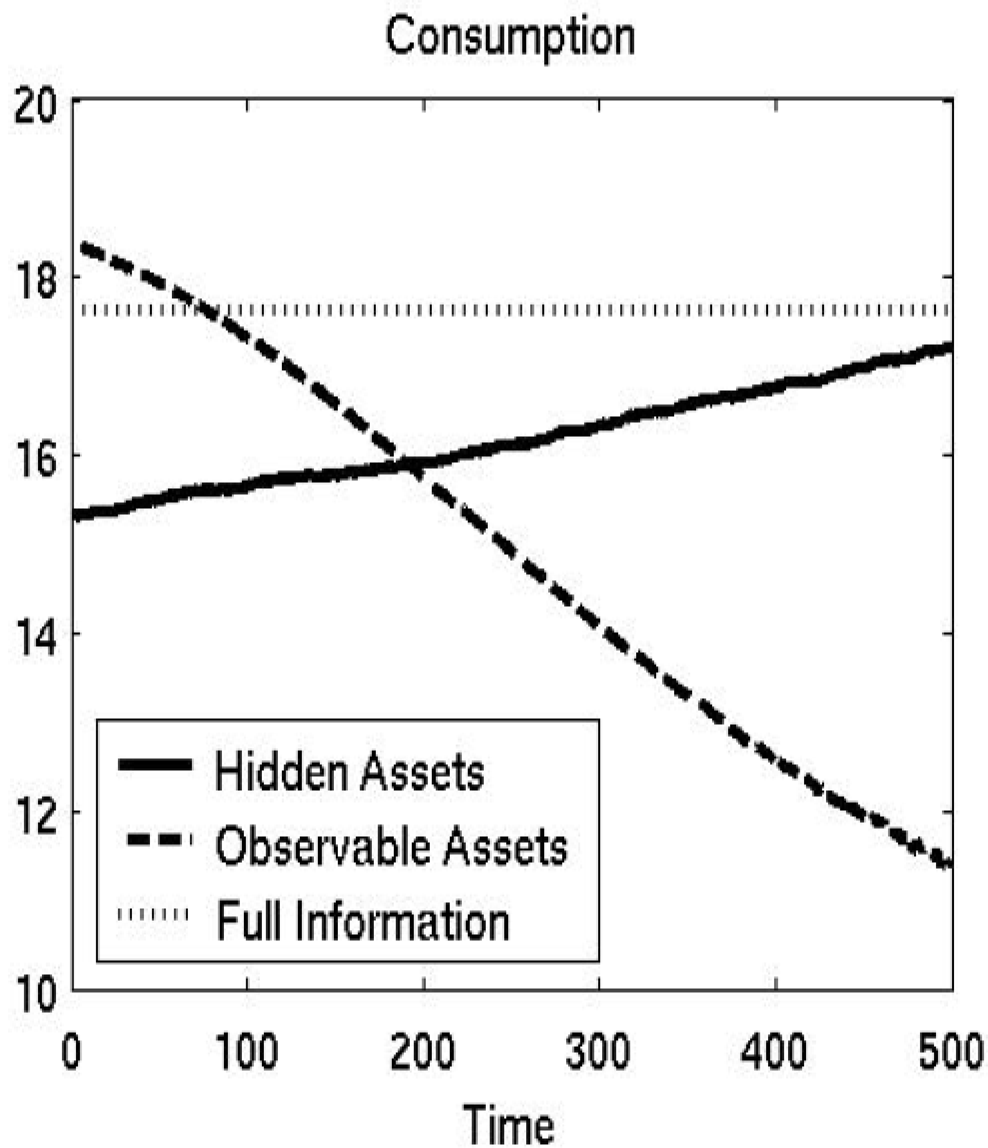
$$y_1 = y_L,$$
$$p[y = y_H|e] = 1 - e^{\rho e} \quad \rho = 0.1,$$
$$p[y = y_L|e] = 1 - p[y = y_H|e],$$

$$u(c,e) = \frac{c^{1-\sigma}}{1-\sigma} - \eta e^2 \quad \sigma = 2 \quad \eta = 0.0067,$$

$$r = 4.1\% \qquad (1+r)\beta = 1,$$

U_0 such that the expected principal's profit at efficient allocation is zero.

Numerical Example: Efficient Allocation
Average path for the first 500 periods



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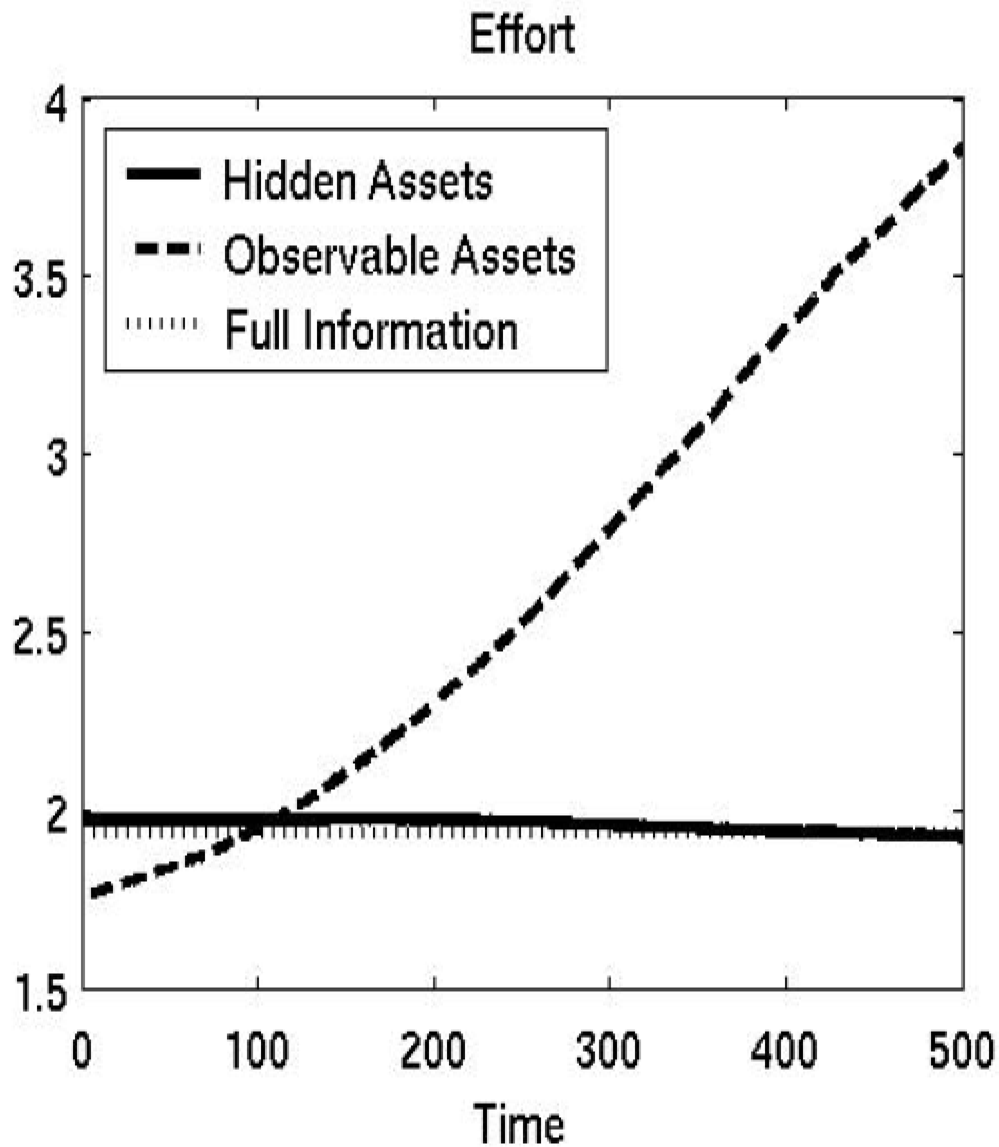

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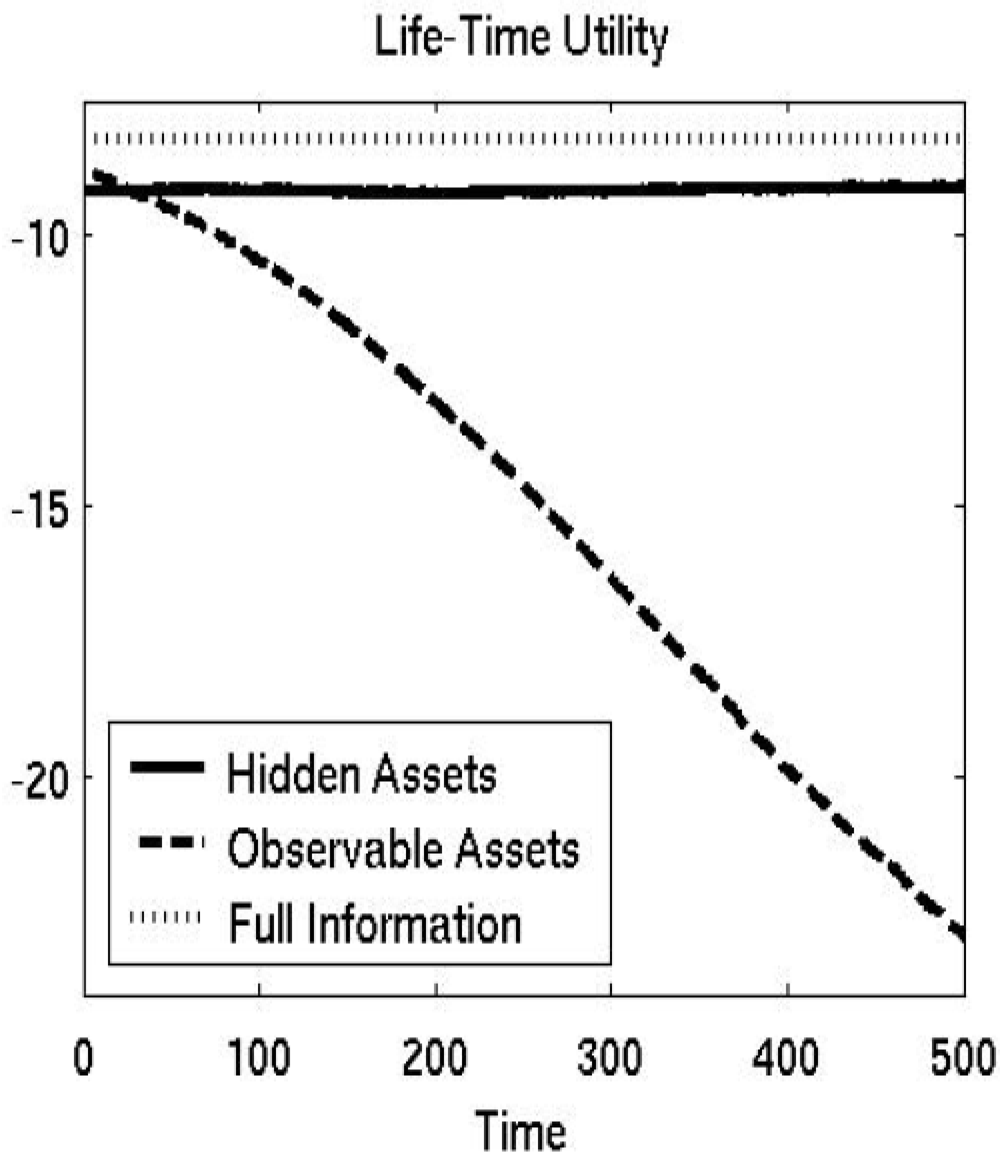

Numerical Examples: Efficient Allocations
Efficiency Gains

	Bond Economy	Full Information	Pure Moral Hazard	Moral Hazard Hidden Assets
High risk aversion (σ=2)				
$\beta(1+r)=1$ ($r=4.1\%$)				
Agent's life-time utility (%)	100	117.9	112.1	108.3
Consumption increase (%)	0	29.8	19.2	12.9
Welfare gains with higher initial assets $\beta(1+r)=1$ ($r=4.1\%$)				
Agent's life-time utility (%)	100	103.4	102.5	101.8
Consumption increase (%)	0	4.9	3.5	2.6
Low risk aversion (σ=0.5)				
$\beta(1+r)=1$ ($r=4.1\%$)				
Agent's life-time utility (%)	100	104.3	103.9	103.7
Consumption increase (%)	0	6.6	6.0	5.7