Asset Trading Volume with Dynamically Complete Markets and Heterogenous agents.

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Motivation

- Partial Equilibrium Analysis suggests that portfolio rebalancing is causing large trading volume.
- Purpose of the paper: Explore the trading volume in GE with Dynamically Complete Markets
- Main Result: Asset Trading Volume zero.
- We have to look somewhere else...

Economy: Lucas Asset pricing model with heterogenous agents

- discrete and infinite time.
- state:y_t that takes values in $Y = \{1, .., S\}$, with transition matrix Π
- $\sigma_t = (y_0, ..., y_t)$: history up to t, generic history $\sigma \in \sum \equiv \cup_t \sum_t$
- $h = 1, ..., H$ consumers, consume a single good c and transfer wealth with assets.

- Preferences: State dependent utility

$$U_h(c) = E\sum_{t=0}^{\infty}\beta^t u_h(c_t, y_t)$$

- Complete markets: $J = S$ linearly independent assets
  - $J^l \geq 0$ long lived assets that pay every period dividiend $d^j(y), j = 1, .., J^l$ in positive net supply
  - $S - J^l$ short-lived that pay once $d^j(y), j = J^l + 1, ..., S$, in zero net supply

- Portfolio holdings $\theta_t^h = (\theta_t^{hl}, \theta_t^{hs})$.Initial position $\theta_{-1}^h = (\theta_{-1}^{hl}, 0)$.

- Each consumer receives state dependent endowment $e^h(y)$.

- Endowment and dividend process

$$e_{H\times S} = \begin{pmatrix} e^1(1) & e^1(S) \\ \vdots & \vdots \\ e^H(1) & e^H(S) \end{pmatrix} \quad d_{S\times S} = \begin{pmatrix} d^1(1) & d^1(S) \\ \vdots & \vdots \\ d^S(1) & d^S(S) \end{pmatrix}$$

- Let

$$\omega_h(y) \equiv e^h(y) + \theta_{-1}^{hl} d^l(y)$$

Arrow-Debreu equilibrium.

- *Def 1: AD equilibrium* is prices $p(\sigma)$ and consumtion plan $\left\{c^h\right\}_{h,\sigma}$ such that:

1. Each agent solves

$$\max_{c^h} U_h(c^h)$$

s.t.

$$\sum_{\sigma\in\boldsymbol{\Sigma}} p(\sigma)c^h(\sigma) = \sum_{\sigma\in\boldsymbol{\Sigma}} p(\sigma)\omega^h(\sigma)$$

2. Markets clear

$$\sum_h c^h(\sigma) = \sum_h \omega^h(y(\sigma)), \forall\sigma \in \boldsymbol{\Sigma}$$

Financial Markets Equlibrium

- Def 2. *Financial Markets Equilibrium*: portfolio holdings θ and prices of assets q_t such that

1.
$$\max_{c^h, \theta^h} U_h(c^h)$$
s.t.
$$c_t^h + \theta_t^h q_t = e^h(y_t) + \theta_{t-1}^{hs} d^s(y_t) + \theta_{t-1}^{hl}(q_t^l + d_t^l(y_t))$$

2. Asset markets clear
$$\sum_h \theta_t^h(\sigma) = \sum_h \theta_{-1}^h, \text{ all } \sigma$$

Computation of the FME

- First and Second Welfare Theorems hold. Use Negishi Method with weights $\lambda = (1, \lambda_2, ..., \lambda_H)$

- STEP 1: Calculate Consumptions and Negishi Weights

- STEP 2: Calculate Asset prices

- STEP 3: Calculate portfolios and derive Trading Volume

STEP 1: Consumption allocation and Negishi Weights

- *Lemma:* Since $e^h(\sigma_t) = e^h(y), d(\sigma_t) = d(y)$, we have $c^h(\sigma_t) = c_y^h$

- We want to calculate $H \times S$ consumption points and $H - 1$ λ^h.

- Full risk-sharing

$$u_1'(c_y^1, y) = \lambda^h u_h'(c_y^h, y), y = 1, .., S, h = 2, ..., H \tag{1}$$

  So we have $(H - 1) \times S$ equations

- Define (scaled) A-D prices

$$p_y = u_1'(c_y^1, y), \text{all } y$$

Derive appropriate restrictions from Budget constraints (zero Transfers)

- PV of consumption of consumer h

$$V_y^h = p_y c_y^h + \beta E(V_{y+}^h | y)$$

- In vector notation

$$V^h = p \circ c^h + \beta \Pi V^h$$

where $\circ$ represents element-wise multiplication.Thus

$$V^h = (I_S - \beta \Pi)^{-1}(p \circ c^h)$$

- PV of endowments and Portfolio dividends

$$W^h = (I_S - \beta \Pi)^{-1}(p \circ \omega^h)$$

- At $t = 0$, at state y_0 we have $V_{y0}^{h} = W_{y0}^{h}, h = 1, .., H$.Thus

$$(I_S - \beta \Pi)^{-1}[p \circ (c^h - \omega^h)]_{y_0} = 0, h = 1, .., H \tag{2}$$

- So $(H - 1)$ (by Walras Law) equations

- S equations from market Clearing

$$\sum_{h=1}^{H} c_y^h = \sum_{h=1}^{H} \omega_y^h, \forall y = 1, .., S \tag{3}$$

- Therefore # (equations)=$(H-1) \times S + (H-1) + S = HS + H - 1 =$ #unknowns

Step 2: Calculate Asset prices

- Long lived Assets

$$q_y^j p_y = \beta E p_{y+} (q_{y+}^j + d_{y+}^j | y)$$

or

$$q^j \circ p = (I_S - \beta \Pi)^{-1} \beta \Pi (p \circ d^j)$$

- Short-lived assets

$$q_y^j = \frac{\beta \Pi_y (p \circ d^j)}{p_y}$$

Step 3: Calculate Portfolios

- *Lemma:* Portfolio Holdings θ are state contingent

- Pf: Equivalent recursive represenation

$$V(w,y) = \max_{\theta,c}\{u(c_y) + \beta EV(w^+, y^+)\}$$

s.t.

$$\begin{aligned} c_y + \theta \cdot q_y &= e_y + w \\ w_s^+ &= \theta^l(q_s^l + d_s^l) + \theta^s d_s^s \end{aligned}$$

- Solution: $\theta^* = \theta(w,y)$. But w=PV $(c-e) = w(y) \Rightarrow \theta_y = \theta(w_y, y)$ ■

Basic Result

- Assume that Π $\gg$ 0, i.e. every state is reachable in one transition.

- The dynamic budget of agent h at state y reads

$$\underbrace{c_y^h + \theta_y^h q_y - e_y^h}_{\text{amount to be financed}} \quad = \theta_z^{hl}(q_y^l + d_y^l) + \theta_z^{hs} d_y^s \ \forall y, z \in Y$$

- Fix LHS $\Rightarrow$*RHS has to finance LHS for every* z (crucial $\pi(y|z) > 0$).

- Thus

$$(\theta_y^{hl} - \theta_z^{hl})(q_s^l + d_s^l) + (\theta_y^{hs} - \theta_z^{hs}) d_s^s = 0, \forall s \text{ for any } y, z$$

or in vector notation

$$(\theta_y^h - \theta_z^h)D_1 = 0$$

where $D_1 \equiv (q^1 + d^1, .., d^S)^T$ and D_1 :full rank

- Therefore

$$\theta_y^h = \theta_z^h = \theta^h \forall y \in Y$$

- *Thm:* If $\Pi \gg 0$, portfolio holdings θ remain constant over states of the world for each agent h. No Trade after an initial round of trade.

- How to find θ^h:

$$\theta^{hl} d^l_y + \theta^{hs}(d^s_y - q^s_y) = c^h_y - e^h_y$$

or

$$\theta^h D_2 = c^h - e^h$$

where $D_2 \equiv (d_1, ..., d^{J^l}, d^{J^L+1} - q^{J^L+1}, ..., d^S - q^S)^T$. Thus

$$\theta^h = (c^h - e^h) D_2^{-1}$$

- *Thm (only short-lived assets): Let* Π ≫ 0 *and let the Economy start at* y_0*.Assume we have only S-1 short-lived assets, which have a dividend of* 0 *at* y_0*. Then the market is complete.*

- Pf: Assume we had S assets. We will show that the last asset is redundant. Since $\Pi \gg 0$, $\theta^h_y = \theta^h$. At $t = 0$ we have

$$c^h_{y0} + \theta^h q_{y_0} = e_{y_0}$$

- At any other period at state y_0 we have

$$c^h_{y0} + \theta^h q_{y_0} = e_{y_0} + \theta^h d_{y0}$$

Thus

$$\theta^h d_{y0} = 0$$

- Assume that asset S has a positive dividend $d^S_{y0} > 0$, whereas for all others it is zero. Then we get $\theta^{hS} = 0 \;\forall$ state. So the asset is not traded in any state.

Conclusion

- Trading Volume has to come from somewhere else and not from portfolio rebalancing.

- Incomplete markets, heterogenous beliefs, non-separable utility, etc.