Six anomalies looking for a model: A consumption based explanation of international finance puzzles

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Six Puzzles

- Backus-Smith anomaly

$$\begin{aligned} model: &\quad \Delta e_{t+1} = m^*_{t+1} - m_{t+1} \\ data: &\quad corr(\Delta c - \Delta c^*, \Delta e) = 0.072 \end{aligned}$$

- Brandt, Cochrane and Stanta-Clara puzzle

$$\begin{aligned} model: &\quad var(\Delta e_{t+1}) = var(m^*_{t+1}) + var(m_{t+1}) - 2cov(m^*_{t+1}, m_{t+1}) \\ data: &\quad std(\Delta e) = 0.11, std(m^{UK}) = 0.39, std(m^{US}) = 0.37 \end{aligned}$$

- UIP puzzle

$$\begin{aligned} model: &\quad E_t[\Delta e_{t+1}] = r_t - r^*_t - \frac{1}{2} var_t(m^*_{t+1}) + \frac{1}{2} var_t(m_{t+1}) \\ data: &\quad \Delta e_{t+1} = \alpha + \beta(r_t - r^*_t) + \xi_{t+1} : \beta = 0 \end{aligned}$$

Six Puzzles

- equity premium puzzle (average Sharpe ratios 0.38 for G7)
- low correlation in fundamentals, but high correlation in stock returns, and
- higher but not perfect correlation in risk-free rates

$$\begin{aligned} corr(\Delta c, \Delta c^*) &= 0.125 \\ corr(\Delta d, \Delta d^*) &= 0.108 \\ corr(r_{stk}, r^*_{stk}) &= 0.489 \\ corr(r_{rf}, r^*_{rf}) &= 0.678 \end{aligned}$$

Model

- Complete markets
- Representative agents have risk-sensitive preferences:

$$U_t = (1-\delta)\log C_t + \delta\theta \log E_t exp\left(\frac{U_{t+1}}{\theta}\right)$$
$$U_t^* = (1-\delta)\log C_t^* + \delta\theta \log E_t exp\left(\frac{U_{t+1}^*}{\theta}\right)$$

with $\theta = \frac{1}{1-\gamma}$.

Model

- Long-run risk in consumption:

$$
\begin{aligned}
\log C_{t+1} - \log C_t &= \Delta c_{t+1} = \mu_c + \lambda_1 z_{1,t} + \lambda_2 z_{2,t} + \lambda_t^{1/2} \varepsilon_{t+1} \\
\log C_{t+1}^* - \log C_t^* &= \Delta c_{t+1}^* = \mu_c + \lambda_1^* z_{1,t} + \lambda_2^* z_{2,t} + \lambda_t^{*1/2} \varepsilon_{t+1}^* \\
z_{1,t} &= \rho_1 z_{1,t-1} + \lambda_{t-1}^{1/2} \varepsilon_{1,t} \\
z_{2,t} &= \rho_2 z_{2,t-1} + \lambda_{t-1}^{*1/2} \varepsilon_{2,t} \\
\lambda_t &= \sigma(1-\rho_\lambda) + \rho_\lambda \lambda_{t-1} + \phi_\lambda \lambda_{t-1} \varepsilon_{\lambda,t} \\
\lambda_t^* &= \sigma(1-\rho_\lambda) + \rho_\lambda \lambda_{t-1}^* + \phi_\lambda \lambda_{t-1}^* \varepsilon_{\lambda,t}^*
\end{aligned}
$$

- Dividend dynamics:

$$
\begin{aligned}
\Delta d_{t+1} &= \mu_d + \lambda_{d_1} z_{1,t} + \lambda_{d_2} z_{2,t} + \lambda_t^{1/2} \varepsilon_{d,t+1} \\
\Delta d_{t+1}^* &= \mu_d + \lambda_{d_1}^* z_{1,t} + \lambda_{d_2}^* z_{2,t} + \lambda_t^{*1/2} \varepsilon_{d,t+1}^*
\end{aligned}
$$

Baseline Calibration

Risk aversion	γ	8.000
Subjective discount factor	δ	0.991
Average consumption growth	μ_c	0.001
Average dividend growth	μ_d	0.001
Loading on $z_{1,t}$ in Δc_{t+1}	λ_1	5.500
Loading on $z_{2,t}$ in Δc_{t+1}	λ_2	1.000
Loading on $z_{1,t}$ in Δc^*_{t+1}	λ^*_1	1.000
Loading on $z_{2,t}$ in Δc^*_{t+1}	λ^*_2	5.500
Loading on $z_{1,t}$ in Δc_{t+1}	λ_{d_1}	24.000
Loading on $z_{2,t}$ in Δc_{t+1}	λ_{d_2}	2.000
Loading on $z_{1,t}$ in Δc^*_{t+1}	$\lambda^*_{d_1}$	2.000
Loading on $z_{2,t}$ in Δc^*_{t+1}	$\lambda^*_{d_2}$	24.000
Persistence of first predictive factor	ρ_1	0.987
Persistence of second predictive factor	ρ_2	0.987
Persistence of stochastic volatility	ρ_λ	0.950
Predictive factors to consumption growth standard error ratio	φ_e	8.80E-03
Dividend to consumption growth standard error ratio	φ_d	4.500
Stochastic volatility to consumption growth std error ratio	φ_λ	3.50E-04
Standard error of the shock to consumption growth	σ	0.006

Correlations		
Predictive factors	$\rho_{1,2}$	0.600
Predictive factors and consumption growth	$\rho_{1,c} = \rho_{2,c^*}$	0.060
Predictive factors and consumption growth	$\rho_{1,c^*} = \rho_{2,c}$	0.600
Predictive factors and stochastic volatility	$\rho_{1,\lambda} = \rho_{2,\lambda^*} = -\rho_{2,\lambda} = -\rho_{1,\lambda^*}$	-0.285
Consumption growth and stochastic volatility	$\rho_{c,\lambda} = \rho_{c^*,\lambda^*} = -\rho_{c,\lambda^*} = -\rho_{c^*,\lambda}$	0.470
Dividend growths	ρ_{d,d^*}	0.050
Consumption growths	ρ_{c,c^*}	0.100

Baseline calibration - long-run risk

$$
\begin{aligned}
\log C_{t+1} - \log C_t &= \Delta c_{t+1} = \mu_c + 5.5 z_{1,t} + z_{2,t} + \lambda_t^{1/2} \varepsilon_{t+1} \\
\log C^*_{t+1} - \log C^*_t &= \Delta c^*_{t+1} = \mu_c + z_{1,t} + 5.5 z_{2,t} + \lambda_t^{*1/2} \varepsilon^*_{t+1} \\
z_{1,t} &= 0.987 z_{1,t-1} + \lambda_{t-1}^{1/2} \varepsilon_{1,t} \\
z_{2,t} &= 0.987 z_{2,t-1} + \lambda_{t-1}^{*1/2} \varepsilon_{2,t} \\
\lambda_t &= 0.006(1-0.95) + 0.95 \lambda_{t-1} + 0.00035 \lambda_{t-1} \varepsilon_{\lambda,t} \\
\lambda^*_t &= 0.006(1-0.95) + 0.95 \lambda^*_{t-1} + 0.00035 \lambda^*_{t-1} \varepsilon^*_{\lambda,t}
\end{aligned}
$$

Benchmark Result

	Data	Model	95% CI
Volatility of exchange rate growth	10.696	10.929	(9.105,12.954)
UIP regression slope	-0.136	-0.062	(-0.609,0.458)
Sharpe ratio	37.781	40.564	(2.783,76.447)
Backus and Smith anomaly	0.072	-0.056	(-0.279,0.165)
Correlation of risk-free rates	0.678	0.793	(0.189,0.954)
Correlation of excess returns	0.489	0.422	(0.232,0.578)
Volatility of consumption growth	1.937	1.909	(1.547,2.388)
AC(1)	0.203	0.347	(0.081,0.598)
AC(2)	0.191	0.158	(-0.154,0.484)
AC(3)	0.158	0.140	(-0.161,0.473)
AC(4)	0.014	0.125	(-0.141,0.448)
Correlation of consumption growths	0.125	0.247	(-0.032,0.546)
Volatility of dividend growth	9.329	9.845	(8.115,12.045)
AC(1)	0.158	0.109	(-0.142,0.384)
AC(2)	0.093	0.088	(-0.161,0.376)
AC(3)	0.086	0.087	(-0.161,0.358)
AC(4)	0.182	0.075	(-0.164,0.356)
Correlation of dividend growths	0.108	0.135	(-0.113,0.380)

Solution of the model

Log discount rate can be solved as follows:

$$\begin{aligned} m_{t+1} &= -v_{0,m} - v_m s'_t + \frac{1}{\theta}\left(\tilde{v}_1 \lambda_t^{1/2} + v_2 \lambda_t^{*1/2}\right)\eta'_{t+1} \\ m^*_{t+1} &= -v_{0,m} - v^*_m s'_t + \frac{1}{\theta}\left(v^*_1 \lambda_t^{1/2} + \tilde{v}^*_2 \lambda_t^{*1/2}\right)\eta'_{t+1} \\ s_t &= \left[z_{1,t}, z_{2,t}, \lambda_t, \lambda^*_t, h_t\right] \\ h_t &= \lambda_t^{1/2}\lambda_t^{*1/2} \\ \eta_t &= \left[\varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon_{c,t}, \varepsilon_{c^*,t}, \varepsilon_{\lambda,t}, \varepsilon_{\lambda^*,t}\right] \end{aligned}$$

Solution method

Volatility of real exchange rate changes

- Complete market $\rightarrow$

$$\begin{aligned}\Delta e_{t+1} &= m^*_{t+1} - m_{t+1} \\ &= \left(v_m - v^*_m\right) s'_t + \frac{1}{\theta}\left((v^*_1 - \tilde{v}_1)\lambda_t^{1/2} + (\tilde{v}^*_2 - v_2)\lambda_t^{*1/2} \right) \eta'_{t+1}\end{aligned}$$

- Key: pick correlation matrix of the shocks carefully.

Correlation of risk free rates

- Let's shut down uncertainty shocks for now.
- Short rates are:

$$
\begin{aligned}
r_t &= \bar{r}_t + \lambda_1 z_{1,t} + \lambda_2 z_{2,t} \\
r_t^* &= \bar{r}_t^* + \lambda_1^* z_{1,t} + \lambda_2^* z_{2,t}
\end{aligned}
$$

- Key: two serially correlated shocks with proper correlation.

Correlation of stock market returns

- Use Campbell-Shiller decomposition:

$$\begin{aligned} r_{d,t+1} &= k_0 + \Delta d_{t+1} - k_1 dp_{t+1} + dp_t \\ dp_t &= \bar{dp} + \frac{\lambda_1 - \lambda_{d1}}{1 - k_1\rho_1} z_{1,t} + \frac{\lambda_2 - \lambda_{d2}}{1 - k_1\rho_2} z_{2,t} \\ dp_t^* &= \bar{dp}^* + \frac{\lambda_2 - \lambda_{d2}}{1 - k_1\rho_2} z_{1,t} + \frac{\lambda_1 - \lambda_{d1}}{1 - k_1\rho_1} z_{2,t} \end{aligned}$$

- Key: pick $corr(z_1, z_2)$ carefully.

Backus-Smith puzzle

- Keep uncertainty shocks shut down.
- Innovations are as follows

$$\begin{aligned}\Delta e &= \beta_1\varepsilon_1 - \beta_2\varepsilon_2 + \lambda\varepsilon - \lambda^*\varepsilon^* \\ \Delta c - \Delta c^* &= \lambda\varepsilon - \lambda^*\varepsilon^*\end{aligned}$$

- asymmetric loadings $\rightarrow \beta_1 \neq \beta_2$
- Key: correlation: $\rho_{1,c} = \rho_{2,c^*} = 0.06$, $\rho_{1,c^*} = \rho_{2,c} = 0.6$

UIP puzzle

- Let's turn on uncertainty shocks again.
- Solve for slope explicitly:

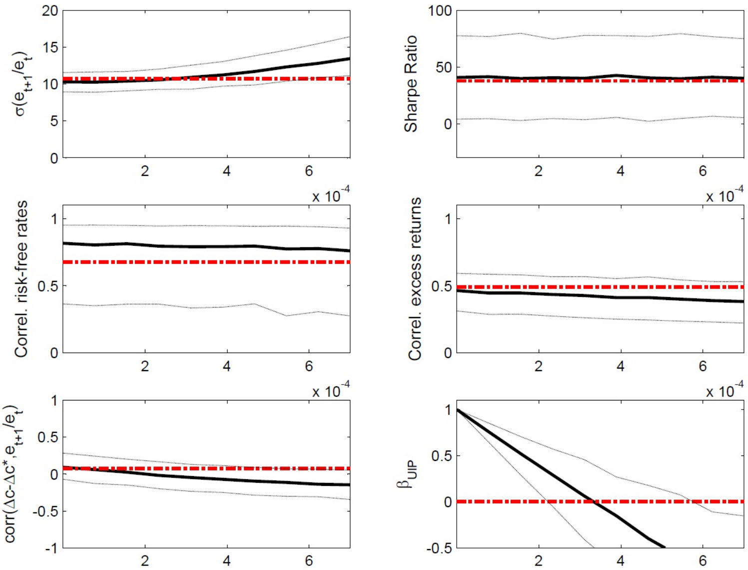
$$\begin{aligned} &\textit{UIP} \quad \textit{regression} && \Delta e_{t+1} = \alpha + \beta(r_t - r_t^*) + \xi_{t+1} \\ &\textit{Model} \quad \textit{counterpart} && m_{t+1}^* - m_{t+1} = \alpha + \beta\left(p_t + \frac{1}{2}q_t\right) + \xi_{t+1} \\ & && \hat{\beta} = \frac{Var(p_t) + \frac{1}{2}Cov(p_t, q_t)}{Var(p_t + \frac{1}{2}q_t)} \end{aligned}$$

where

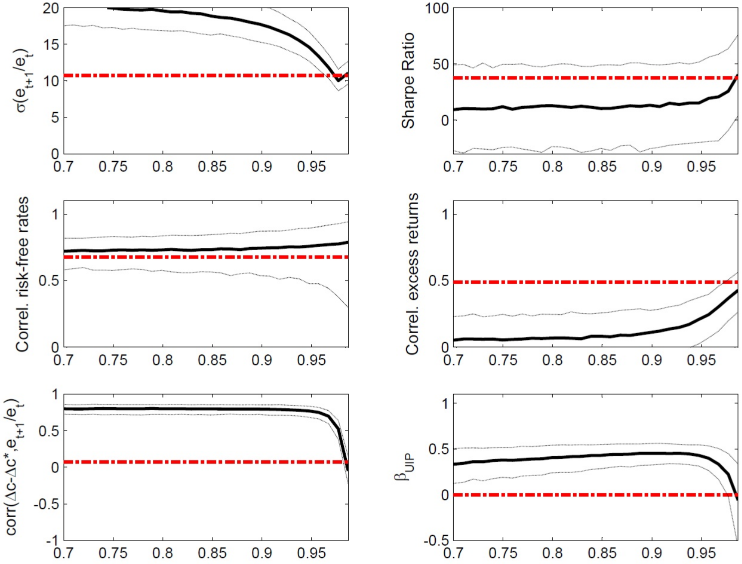
$$\begin{aligned} p_t &= E_t m_{t+1}^* - E_t m_{t+1} = A_1 z_{1,t} - A_2 z_{2,t} + B_1 \lambda_t - B_2 \lambda_t^* \\ q_t &= Var_t m_{t+1}^* - Var_t m_{t+1} = -B_1 \lambda_t + B_2 \lambda_t^* \end{aligned}$$

- Key: get $Cov(p_t, q_t)$ sufficiently negative.

Stochastic Volatility



Long-run risk



Conclusion

- It reverse-engineers a plausible consumption process, builds a complete market two-country model with risk-sensitive preferences and long run risk. It can explain the six puzzles.