Self-Fulfilling Debt Crises

by Harold Cole and Timothy Kehoe

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Introduction

- Mexico 1994/95: Investors lose confidence despite relatively sound fiscal policy (low debt/gdp ratio)
- Common explanation: Investor confidence is a "fundamental"
- Beliefs of investors can become self-fulfilling:
  - optimistic investors are willing to pay more for debt
  - rolling over the debt becomes less costly
  - government has less incentives to default

Introduction

The authors want to model this situation and ask

1. What are the states in which beliefs can be self-fulfilling?
2. What is the optimal debt policy if beliefs are "fundamentals" in the current state?
3. Are there measures that reduce the role of investor beliefs? (cost of default, maturity structure, government preferences)

The Model

Households:

$$\max_{c_t,\, k_{t+t}} E \sum_{t=0}^{\infty} \beta^t (c_t + v(g_t))$$

$$s.t.\ \ c_t + k_{t+1} \leq (1-\theta) a_t f(k_t)$$

- θ: exogenous tax rate
- $a_t = \begin{cases} \alpha < 1, & \textit{if the government has ever defaulted} \\ 1, & \textit{otherwise} \end{cases}$

The Model

Government:

- If $a_{t-1} = 1$: choose g_t, B_{t+1}, and a discrete repayment/default decision z_t s.t.

  $$g_t + z_t B_t \leq a_t \theta f(k_t) + q_t B_{t+1}$$

  to maximize HH utility
- Otherwise: $g_t = \alpha \theta f(k_t)$

The Model

Lenders:

- risk neutral
- competitive
- time preference β
- beliefs about the government's ability to repay may depend on sunspot $\zeta_t \sim U[0,1]$:
  optimistic if $\zeta_t \geq \pi$
  pessimistic if $\zeta_t < \pi$
- $s_t = (B_t, K_t, a_{t-1}, \zeta_t)$

The Model

Timing (within period t):

1. ζ_t is realized
2. Given the schedule $q(s_t, B_{t+1})$, the government chooses B_{t+1}
3. Given q_t, s_t, B_{t+1}, lenders decide whether to lend
   lend if $q_t \leq \beta E(z_{t+1} | s_t, B_{t+1})$
4. The government chooses g_t, z_t
5. Given a_t, consumers choose c_t, k_{t+1}

Recursive Formulation

Households:

$$
\begin{aligned}
V_c(k, s, B', g, z) = \max_{c, k'} & c + v(g) + \beta E V_c(k', s', B'(s'), g', z') \\
s.t.\ c + k' & \leq (1-\theta) a(s, z) f(k) \\
s' & = (B', K'(s, B', g, z), a(s, z), \zeta') \\
g' & = g(s', B'(s'), q(s', B'(s'))) \\
z' & = z(s', B'(s'), q(s', B'(s')))
\end{aligned}
$$

Recursive Formulation

Government:

$$
\begin{aligned}
V_g(s) = \max_{B'} \; & c(K, s, B', g, z) + v(g) + \beta E V_g(s') \\
s.t. \; g &= g(s, B', q(s, B')) \\
z &= z(s, B', q(s, B')) \\
s' &= (B', K'(s, B', g, z), a(s, z), \zeta')
\end{aligned}
$$

where $g()$ and $z()$ solve

$$
\begin{aligned}
\max_{g,z} \; c(K, s, B', g, z) + v(g) + \beta E V_g(s') \\
s.t. \; g + zB \leq \theta a(s, z) f(K) + qB' \\
z \in \{0, 1\} \\
g \geq 0 \\
s' = (B', K'(s, B', g, z), a(s, z), \zeta')
\end{aligned}
$$

Equilibrium

Definition

An equilibrium is a list of value functions V_c, V_g , policy functions c, k', B', g, z, a price function q, and a law of motion K' such that

1. Given B', g, and z, V_c is the value function for the consumer, and c and k' are the maximizing choices
2. Given q, c, K', g, and z, V_g is the value function for the government's first problem and B' is the maximizing choice
3. Given c, K', V_g, and B', g and z solve the government's second problem
4. $q(s,\ B') = \beta E\left\{z\left[s', B'(s')\right), q(s',\ B'(s'))\right]\right\}$
5. $K'(s, B', g, z) = k'(K, s, B', g, z)$

Crisis Zones

Assume that at time 0, $a_{-1} = 1$

- **No-crisis zone:** $B \leq \bar{b}(k)$
  if B is small, then $z = 1$ is always optimal, regardless of ζ
  $\frac{d\bar{b}(k)}{dk} > 0$
- **Default zone:** $B > \bar{B}(k, \pi)$
  if B is large, then $z = 0$ is optimal, regardless of ζ
- **Crisis zone:** $\bar{b}(k) < B < \bar{B}(k, \pi)$
  Beliefs matter: default if $\zeta < \pi$

Question: For what k and π is there a non-empty crisis zone?

Household behavior

Euler equation:

$$1 = \beta(1-\theta)E(a')f'(k')$$

$$k' = \begin{cases} k^n, & if \;\; E(a') = 1 \; (no - crisis\, zone) \\ k^\pi, & if \;\; E(a') = 1 - \pi + \alpha\pi \; (crisis\, zone) \\ k^d, & if \;\; E(a') = \alpha \; (default\, zone) \end{cases}$$

$\Rightarrow$for $t > 0$, $K_t \in \{k^n, k^\pi, k^d\}$, where $k^n > k^\pi > k^d$

Price of debt and policy outside the crisis zone

$$q = \begin{cases} \beta, & \textit{if } z = 1 \textit{ and } B' \leq \bar{b}(k^n) \\ \beta(1-\pi), & \textit{if } z = 1 \textit{ and } \bar{b}(k) < B' < \bar{B}(k,\pi) \\ 0, & \textit{otherwise} \end{cases}$$

- Default zone: $g_t = \theta\alpha f(K_t)$

$$\Rightarrow V_g^d(s, B', q) = c^d(K) + v(\theta\alpha f(K) + qB') + \beta\frac{c^d(k^d) + v(\theta\alpha f(k^d))}{1-\beta}$$

- No-crisis zone (assume $K_0 \geq k^n$):
  Euler equation $\Rightarrow g_t = g_{t+1} \Rightarrow B_t = B_1 \leq B_0 < \bar{b}(k^n)$, $\forall t$

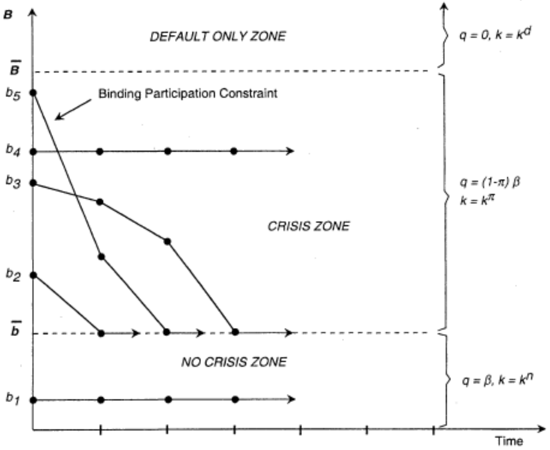
The government's trade-offs

1. Choice of z: low consumption today vs. permanent output loss
   Solution: $z = 1 \Leftrightarrow V_g^n(s, B', q) \geq V_g^d(s, B', q)$
   (= Participation constraint)

2. Given that $z = 1$, choice of B':
   Either smooth consumption and keep B constant or reduce B' and
   - sacrifice more current consumption (if B' is high)
   - exit the crisis zone (if B' is low enough)
     - in that case, q will be permanently higher
     - a and k will be permanently higher

Government's problem in the crisis zone: Resolve the second trade-off optimally, but satisfy the participation constraint!
Solution: exit crisis zone in T periods, where $T \in \{1, ..., \infty\}$

Optimal trajectories



Good policies?

Can the government shrink or move the crisis zone?

- Lower α to raise the cost of default (e.g., by relying more on trade)
  - this raises $\bar{B}()$ and $\bar{b}()$.
  - the default zone becomes smaller
  - the crisis zone can shrink
  - but if a crisis occurs, the cost is higher

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  - the crisis zone can shrink
  - but if a crisis occurs, the cost is higher
- Maturity structure:
  - Suppose the government starts in the crisis zone but can convert B one-period bonds into N equal quantities B_N of bonds with maturities $1, \ldots, N$
  - If N is sufficiently large, then the crisis zone vanishes (amount to be rolled over is small)

Extension: Domestically initiated crises

- So far: if $B_t < \bar{b}(K_t)$, then $K_{t+1} = k^n$ so that $B_{t+1} < \bar{b}(K_{t+1})$.
  - Whenever the economy is in the no-crisis zone, consumers expect that $K' = k^n$ and set $k' = k^n$, confirming the expectations.
  - The economy never leaves the no-crisis zone

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  - Again, if consumers expect that $K' = k^n$ they set $k' = k^n$
  - But if they believe that $K' = k^\pi$ so that $B' > \bar{b}(K')$, then $k' = k^\pi$ is optimal

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  - But if they believe that $K' = k^\pi$ so that $B' > \bar{b}(K')$, then $k' = k^\pi$ is optimal
- We can introduce a second sunspot ξ that coordinates the consumers' beliefs. The economy can now move in and out of the crises zone.
- This gives the government an incentive to choose $B' < \bar{b}(k^\pi)$

Conclusion

- Changes in foreign lenders' beliefs can lead to crises.
- The conditions for a crisis to occur may depend on domestic private sector beliefs.
- To avoid crises, governments should reduce debt levels or spread out the maturity structure.
- Crises are the result of a coordination failure. A lender of last resort (the U.S., in the Mexican example) can enhance welfare.