

# Optimal Fiscal Policy With Redistribution

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# Motivation

- Standard **Ramsey problem**: lump-sum taxes arbitrarily ruled-out
  - Labor taxes are smoothed; Capital tax is zero
- Arbitrary second-best?
- **Werning problem**: heterogenous agents, linear taxes on revenues from labor and capital, lump-sum taxes:
  - Positive (distortionary) labor taxes emerge **endogenously** as a redistributive tool
  - Trade-off: efficiency versus redistribution
- Extension: with nonlinear taxes (Mirleesian model)?

# Environment

- **The agents:**

- + **Heterogenous workers:**  $\forall i \in I$  a mass  $\pi_i$  of type- $i$  agents:
  - Productivity  $\theta_i$ :  $U^i(c_t^i, L_t^i) = U(c_t^i, \frac{L_t^i}{\theta_i})$
- + **Government**
- + **Representative firm**

- **Aggregate uncertainty:**  $s^t = (s_0, s_1, \dots, s_t)$  w.p.  $\nu_t(s^t)$

- **Resource constraint**  $\forall t \forall s^t$ :

$$c_t(s^t) + K_{t+1}(s^t) + g_t(s^t) \leq F(L_t(s^t), K_t(s^{t-1}), s^t) + (1 - \delta)K_t(s^{t-1})$$

$$\text{with } \sum_i \pi_i c_t^i(s^t) = c_t(s^t) \text{ and } \sum_i \pi_i L_t^i(s^t) = L_t(s^t)$$

- **Complete Markets**

# Competitive Equilibrium I

**Policy:** Given an initial debt, **linear** labor and capital taxes, **lump-sum** taxes, **not indexed** by  $i$ :

$$\{\tau_t(s^t), \kappa_t(s^t), T_t(s^t)\}_{t,s^t}$$

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+ **Government:**

$$\begin{aligned} \sum_{i \in I} \pi_i B^i(s_0) + \sum_{t,s^t} p_t(s^t) g_t(s^t) &\leq T + \dots \\ \dots + \sum_{t,s^t} p_t(s^t) (\tau(s^t) w_t(s^t) L_t(s^t) + \kappa_t(s^t) (r_t(s^t) - \delta) K_t(s^{t-1})) & \end{aligned}$$

where  $T = \sum_{t,s^t} p_t(s^t) T_t(s^t)$ .

+ **Representative firm:** Static maximization.

$$w_t = F_L(L_t, K_t, s_t) \quad \text{and} \quad r_t = F_K(L_t, K_t, s_t)$$

# Competitive Equilibrium II

+ **Households:**

$$\max_{\{c_t^i(s^t), L_t^i(s^t)\}} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \nu_t(s^t) U^i(c_t^i(s^t), L_t^i(s^t))$$

such that

$$\sum_{t, s^t} p_t(s^t) [c_t^i(s^t) - w_t(s^t)(1 - \tau_t(s^t))L_t^i(s^t) + T_t(s^t)] \leq \Omega_0^i$$

where  $\Omega_0^i \equiv (1 + (1 - \kappa_0)(r_0 - \delta))k_0^i + B^i(s_0)$ .

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First-order conditions:

$$p(s^t) = \beta \nu_t(s^t) \frac{U_c^i(s^t)}{U_c^i(s_0)}; \quad w_t(s^t)(1 - \tau_t^l(s^t)) = -\frac{U_L^i(s^t)}{U_c^i(s^t)}$$

Standard **implementability constraints** for each agent ( $\forall i \in I$ ):

$$\sum_{t, s^t} \beta^t \nu_t(s^t) [U_c^i(s^t)c_t^i(s^t) + U_L^i(s^t)L_t^i(s^t)] \leq U_c^i(s_0)(\Omega_0^i - T)$$

# Allocating Aggregates I

$\forall$  C.E.  $\exists$  **market weights**  $\phi \equiv \{\phi^i\}$ , with  $\phi^i \geq 0$  and  $\sum \phi^i \pi^i = 1$ , so that the **individual assignments** solve the static subproblem:

$$U^m(c, L; \phi) \equiv \max_{\{c^i, L^i\}} \sum_{i \in I} \phi^i \pi^i U^i(c^i, L^i)$$

$$\text{s.t. } \sum_{i \in I} c^i \pi^i = c \text{ and } \sum_{i \in I} L^i \pi^i = L$$

## Notation:

Let  $\hat{c}_t^i(c_t(s^t), L_t(s^t); \phi)$ ,  $\hat{L}_t^i(c_t(s^t), L_t(s^t); \phi)$ , the solution for  $i$ .

Let  $\hat{U}^i(c_t(s^t), L_t(s^t); \phi) = U^i(\hat{c}_t^i(c_t(s^t), L_t(s^t); \phi), \hat{L}_t^i(c_t(s^t), L_t(s^t); \phi))$ .

Then,  $U^m(c, L; \phi) \equiv \sum_{i \in I} \phi^i \pi^i \hat{U}^i(c, L; \phi)$ .

## Envelope Conditions:

$$U_c^m(c, L; \phi) = \phi^i U_c^i(\hat{c}^i, \hat{L}^i; \phi) \text{ and } U_L^m(c, L; \phi) = \phi^i U_L^i(\hat{c}^i, \hat{L}^i; \phi)$$



# Allocating Aggregates II

+ **FOC** if market-representative-agent:

$$w_t(1 - \tau_t) = \frac{-U_L^m(c_t, L_t; \phi)}{U_C^m(c_t, L_t; \phi)} = \frac{-\phi^i U_L^i(\hat{c}_t^i, \hat{L}_t^i; \phi)}{\phi^i U_C^i(\hat{c}_t^i, \hat{L}_t^i; \phi)}$$

+ **Implementability constraints** can be rewritten  $\forall i \in I$ :

$$\sum_{t, s^t} \beta^t \nu_t(s^t) [U_C^m(c_t(s^t), L_t(s^t); \phi) \hat{c}_t^i(c_t(s^t), L_t(s^t); \phi) + \dots \\ \dots + U_L^m(c_t(s^t), L_t(s^t); \phi) \hat{L}_t^i(c_t(s^t), L_t(s^t); \phi)] \leq U_C^m(c_0, L_0; \phi)(\Omega_0^i - T)$$

**Proposition:** Given  $\{\Omega_0^i\}$ , an aggregate allocation  $\{c_t(s^t), L_t(s^t), K_{t+1}(s^t)\}$  can be supported by a C.E.  $\Leftrightarrow$  the resource constraint holds and  $\exists$  market weights  $\phi$  and a lump-sum tax  $T$  such that the new implementability conditions hold  $\forall i \in I$ .

# The Planner Problem

Let  $\lambda \equiv \lambda_i$ , where  $\lambda_i$  is the Pareto weight of agent  $i$ .

$$\max_{\{c_t(s^t), L_t(s^t), K_t(s^t); T; \phi\}} \sum_i \lambda_i \pi_i \sum_{t, s^t} \nu_t(s^t) \hat{U}^i(c_t(s^t), L_t(s^t); \phi)$$

such that:

- Resource constraints hold  $\forall t \forall s^t$
- New Implementability constraints hold  $\forall i \in I$

# Two Examples: Optimal Wedges

+ **Separable Isoelastic Utility:**  $U^i(c, L) = \frac{1}{1-\sigma} c^{1-\sigma} - \frac{\alpha}{\gamma} (L/\theta^i)^\gamma$

-  $\tau$  is constant across time and states;  $\kappa = 0$

+ **Balanced Growth Preferences:**  $U^i(c, L) = \frac{(c^\alpha(1-L/\theta^i)^{1-\alpha})^{1-\sigma}}{1-\sigma}$

- Quantitatively:  $\tau$  is constant across time and states;  $\kappa$  is zero;

+ **A partial-equivalence with Ramsey:**

In these two cases, these two expressions are proportional for some  $\hat{\mu}$ :

$$\begin{cases} \sum_i \pi_i (\lambda_i \hat{U}^i(c, L) + \mu_i [U_c^m(c, L) \hat{c}^i + U_L^m(c, L) \hat{L}^i]) \\ \hat{U}^m(c, L) + \hat{\mu} [U_c^m(c, L) c + U_L^m(c, L) L] \end{cases}$$

# Distortionary Labor Taxes are Desirable (sometimes)

The **marginal cost from distortions** should be equated to the **marginal benefit from redistribution**.

- **RA:** the marginal benefit from redistribution is constant (zero).
- **Aggregate shocks but no shift in the distribution of types:** the marginal benefit from redistribution is basically constant.
- **Shocks to the distribution:**
  - Separable isoelastic utility function;
  - $g_t(s^t) = 0$ ,  $TFP(s_t) = 1$ ;  $\theta_t^H/\theta_t^L$  is a function of  $s_t$ .
  - Then,  $\tau(s^t) = \tau(s_t)$  and  $\kappa(s^t) = 0$ .

# Three Differences with a Ramsey Plan

- **Capital taxation and time-inconsistency:**
  - Ramsey: redistribution between private and public sector
  - Werning: redistribution between private agents
- **Debt management:**
  - Ramsey: debt as an insurance tool
  - Werning: a Ricardian Equivalence
- **State-contingent capital taxes:**
  - Werning: Cannot replicate a complete-market outcome

# A Mirleesian Economy with Aggregate Uncertainty

## Planner Problem:

$$\max \sum_{t, s^t} \beta^t \nu_t(s^t) \sum_{i \in I} \pi^i \lambda^i \left( u(c^i(s^t)) - v\left(\frac{L_t^i(s^t)}{\theta_t^i(s^t)}\right) \right) \text{ s.t.}$$

- Resource Constraint holds  $\forall t \forall s^t$ ;
- Incentive Constraint holds  $\forall i \in I$  and for all reports  $j \in I$

$$\sum_{t, s^t} \beta^t \nu_t(s^t) \left( [u(c^i(s^t)) - v\left(\frac{L_t^i(s^t)}{\theta_t^i(s^t)}\right)] - [u(c^j(s^t)) - v\left(\frac{L_t^j(s^t)}{\theta_t^j(s^t)}\right)] \right) \geq 0$$

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**Proposition:** *At any constrained-efficient allocation:*

- The intertemporal Euler equation holds.*
- The implicit marginal tax on labor  $\tau_t^i(s^t)$  depends only on the current skill distribution  $\{\theta_t^i(s^t)\}$ .*

# Conclusion/Next Steps

- A rationale for distortionary taxes
- A tractable model for Ramsey problems with heterogeneity
- Bridge between Ramsey and Mirlees