Optimal Fiscal Policy With Redistribution

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Motivation

- **Standard Ramsey problem**: lump-sum taxes arbitrarily ruled-out
  - Labor taxes are smoothed; Capital tax is zero

- Arbitrary second-best?

- **Werning problem**: heterogenous agents, linear taxes on revenues from labor and capital, **lump-sum taxes**:
  - Positive (distortionary) labor taxes emerge *endogenously* as a redistributive tool
  - Trade-off: efficiency versus redistribution

- Extension: with nonlinear taxes (Mirleesian model)?
Environment

- The agents:
  - Heterogenous workers: \( \forall i \in I \) a mass \( \pi_i \) of type-\( i \) agents:
    - Productivity \( \theta_i \):
      \[ U^i(c^i_t, L^i_t) = U(c^i_t, \frac{L^i_t}{\theta_i}) \]
  - Government
  - Representative firm

- Aggregate uncertainty:
  \( s^t = (s_0, s_1, \ldots, s_t) \) w.p. \( \nu_t(s^t) \)

- Resource constraint \( \forall t \forall s^t \):
  \[ c_t(s^t) + K_{t+1}(s^t) + g_t(s^t) \leq F(L_t(s^t), K_t(s^{t-1}), s^t) + (1 - \delta)K_t(s^{t-1}) \]
  with \( \sum_i \pi_i c^i_t(s^t) = c_t(s^t) \) and \( \sum_i \pi_i L^i_t(s^t) = L_t(s^t) \)

- Complete Markets
Competitive Equilibrium I

**Policy:** Given an initial debt, linear labor and capital taxes, lump-sum taxes, not indexed by $i$:

$$\{\tau_t(s^t), \kappa_t(s^t), T_t(s^t)\}_{t,s^t}$$
Competitive Equilibrium I

**Policy:** Given an initial debt, linear labor and capital taxes, lump-sum taxes, not indexed by $i$:

$$\{\tau_t(s^t), \kappa_t(s^t), T_t(s^t)\}_{t,s^t}$$

+ **Government:**

$$\sum_{i \in I} \pi_i B^i(s_0) + \sum_{t,s^t} p_t(s^t) g_t(s^t) \leq T + ...$$

$$... + \sum_{t,s^t} p_t(s^t)(\tau(s^t) w_t(s^t) L_t(s^t) + \kappa_t(s^t)(r_t(s^t) - \delta) K_t(s^{t-1}))$$

where $T = \sum_{t,s^t} p_t(s^t) T_t(s^t)$.

+ **Representative firm:** Static maximization.

$$w_t = F_L(L_t, K_t, s_t) \text{ and } r_t = F_K(L_t, K_t, s_t)$$
Competitive Equilibrium II

+ Households:

\[ \max \{ c_i^t(s^t), L_i^t(s^t) \} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \nu_t(s^t) U_i(c_i^t(s^t), L_i^t(s^t)) \]

such that

\[ \sum_{t,s^t} p_t(s^t) [c_i^t(s^t) - w_t(s^t)(1 - \tau_t(s^t))L_i^t(s^t) + T_t(s^t)] \leq \Omega_0^i \]

where \( \Omega_0^i \equiv (1 + (1 - \kappa_0)(r_0 - \delta))k_0^i + B^i(s_0) \).
Competitive Equilibrium II

+ Households:

$$\max_{\{c_t(s^t), L_t(s^t)\}} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \nu_t(s^t) U^i(c_t(s^t), L_t(s^t))$$

such that

$$\sum_{t,s^t} p_t(s^t) [c_t(s^t) - w_t(s^t)(1 - \tau_t(s^t))L_t(s^t) + T_t(s^t)] \leq \Omega_0^i$$

where $\Omega_0^i \equiv (1 + (1 - \kappa_0)(r_0 - \delta))k_0^i + B^i(s_0)$.

First-order conditions:

$$p(s^t) = \beta \nu_t(s^t) \frac{U^i_c(s^t)}{U^i(s_0)}; \quad w_t(s^t)(1 - \tau^i_t(s^t)) = -\frac{U^i_L(s^t)}{U^i_c(s^t)}$$

Standard implementability constraints for each agent ($\forall i \in I$):

$$\sum_{t,s^t} \beta^t \nu_t(s^t) [U^i_c(s^t)c_t(s^t) + U^i_L(s^t)L_t(s^t)] \leq U^i_c(s_0)(\Omega_0^i - T)$$
Allocating Aggregates I

∀ C.E. ∃ market weights φ ≡ {φ_i}, with φ_i ≥ 0 and ∑ φ_i π_i = 1, so that the individual assignments solve the static subproblem:

\[ U^m(c, L; \phi) \equiv \max_{\{c^i, L^i\}} \sum_{i \in I} \phi^i \pi^i U^i(c^i, L^i) \]

s.t. \( \sum_{i \in I} c^i \pi^i = c \) and \( \sum_{i \in I} L^i \pi^i = L \)

Notation:

Let \( \hat{c}^i_t(c_t(s^t), L_t(s^t); \phi), \hat{L}^i_t(c_t(s^t), L_t(s^t); \phi) \), the solution for \( i \).

Let \( \hat{U}^i(c_t(s^t), L_t(s^t); \phi) = U^i(\hat{c}^i_t(c_t(s^t), L_t(s^t); \phi), \hat{L}^i_t(c_t(s^t), L_t(s^t); \phi)) \).

Then, \( U^m(c, L; \phi) \equiv \sum_{i \in I} \phi^i \pi^i \hat{U}^i(c, L; \phi) \).

Envelope Conditions:

\( U^m_c(c, L; \phi) = \phi^i U^i_c(\hat{c}^i, \hat{L}^i; \phi) \) and \( U^m_L(c, L; \phi) = \phi^i U^i_L(\hat{c}^i, \hat{L}^i; \phi) \)
Allocating Aggregates II

+ **FOC** if market-representative-agent:

\[ w_t(1 - \tau_t) = \frac{-U_L^m(c_t, L_t; \phi)}{U_C^m(c_t, L_t; \phi)} = \frac{-\phi^j U_L^i(\hat{c}_t^i, \hat{L}_t^i; \phi)}{\phi^j U_C^i(\hat{c}_t^i, \hat{L}_t^i; \phi)} \]

+ **Implementability constraints** can be rewritten \( \forall i \in I \):

\[
\sum_{t,s^t} \beta^t \nu_t(s^t) \left[ U_C^m(c_t(s^t), L_t(s^t); \phi) \hat{c}_t^i(c_t(s^t), L_t(s^t); \phi) + \ldots + U_L^m(c_t(s^t), L_t(s^t); \phi) \hat{L}_t^i(c_t(s^t), L_t(s^t); \phi) \right] \leq U_C^m(c_0, L_0; \phi)(\Omega_0^i - T)
\]

**Proposition:** Given \( \{\Omega_0^i\} \), an aggregate allocation \( \{c_t(s^t), L_t(s^t), K_{t+1}(s^t)\} \) can be supported by a C.E. \( \iff \) the resource constraint holds and \( \exists \) market weights \( \phi \) and a lump-sum tax \( T \) such that the new implementability conditions hold \( \forall i \in I \).
The Planner Problem

Let $\lambda \equiv \lambda_i$, where $\lambda_i$ is the Pareto weight of agent $i$.

$$\max \{ c_t(s^t), L_t(s^t), K_t(s^t); \tau; \phi \} \sum_i \lambda_i \pi_i \sum_{t,s^t} \nu_t(s^t) \hat{U}^i(c_t(s^t), L_t(s^t); \phi)$$

such that:

- Resource constraints hold $\forall t \ \forall s^t$
- New Implementability constraints hold $\forall i \in I$
Two Examples: Optimal Wedges

+ **Separable Isoelastic Utility:** \( U^i(c, L) = \frac{1}{1-\sigma} c^{1-\sigma} - \frac{\alpha}{\gamma} (L/\theta^i)^\gamma \)
  - \( \tau \) is constant across time and states; \( \kappa = 0 \)

+ **Balanced Growth Preferences:** \( U^i(c, L) = \frac{(c^\alpha (1-L/\theta^i)^{1-\alpha})^{1-\sigma}}{1-\sigma} \)
  - Quantitatively: \( \tau \) is constant across time and states; \( \kappa \) is zero;

+ **A partial-equivalence with Ramsey:**

In these two cases, these two expressions are proportional for some \( \hat{\mu} \):

\[
\begin{align*}
\sum_i \pi_i (\lambda_i \hat{U}^i(c, L) + \mu_i[U^m_c(c, L)\hat{c}^i + U^m_L(c, L)\hat{L}^i]) \\
\hat{U}^m(c, L) + \hat{\mu}[U^m_c(c, L)c + U^m_L(c, L)L]
\end{align*}
\]
The marginal cost from distortions should be equated to the marginal benefit from redistribution.

- **RA:** the marginal benefit from redistribution is constant (zero).
- **Aggregate shocks but no shift in the distribution of types:** the marginal benefit from redistribution is basically constant.
- **Shocks to the distribution:**
  - Separable isoelastic utility function;
  - $g_t(s^t) = 0$, $TFP(s_t) = 1$; $\theta_t^H / \theta_t^L$ is a function of $s_t$.
  - Then, $\tau(s^t) = \tau(s_t)$ and $\kappa(s^t) = 0$. 

**Distortionary Labor Taxes are Desirable (sometimes)**
Three Differences with a Ramsey Plan

- **Capital taxation and time-inconsistency:**
  - Ramsey: redistribution between private and public sector
  - Werning: redistribution between private agents

- **Debt management:**
  - Ramsey: debt as an insurance tool
  - Werning: a Ricardian Equivalence

- **State-contingent capital taxes:**
  - Werning: Cannot replicate a complete-market outcome
A Mirleesian Economy with Aggregate Uncertainty

Planner Problem:

\[
\max \sum_{t,s^t} \beta^t \nu_t(s^t) \sum_{i \in I} \pi^i \lambda^i \left( u(c^i(s^t)) - v \left( \frac{L_i(s^t)}{\theta_i^t(s^t)} \right) \right) \text{ s.t.}
\]

- Resource Constraint holds \( \forall t \ \forall s^t \);
- Incentive Constraint holds \( \forall i \in I \) and for all reports \( j \in I \)

\[
\sum_{t,s^t} \beta^t \nu_t(s^t) \left( \left[ u(c^i(s^t)) - v \left( \frac{L_i(s^t)}{\theta_i^t(s^t)} \right) \right] - \left[ u(c^j(s^t)) - v \left( \frac{L_j(s^t)}{\theta_j^t(s^t)} \right) \right] \right) \geq 0
\]
A Mirleesian Economy with Aggregate Uncertainty

**Planner Problem:**

\[
\max \sum_{t,s^t} \beta^t \nu_t(s^t) \sum_{i \in I} \pi^i \lambda_i \left( u(c^i(s^t)) - v\left( \frac{L^i_t(s^t)}{\theta^i_t(s^t)} \right) \right) \]

s.t.

- Resource Constraint holds \( \forall t \ \forall s^t \);
- Incentive Constraint holds \( \forall i \in I \) and for all reports \( j \in I \)

\[
\sum_{t,s^t} \beta^t \nu_t(s^t) \left( \left[ u(c^i(s^t)) - v\left( \frac{L^i_t(s^t)}{\theta^i_t(s^t)} \right) \right] - \left[ u(c^j(s^t)) - v\left( \frac{L^j_t(s^t)}{\theta^j_t(s^t)} \right) \right] \right) \geq 0
\]

**Proposition:** At any constrained-efficient allocation:

(i) The intertemporal Euler equation holds.

(ii) The implicit marginal tax on labor \( \tau^i_t(s^t) \) depends only on the current skill distribution \( \{\theta^i_t(s^t)\} \).
Conclusion/Next Steps

- A rationale for distortionary taxes
- A tractable model for Ramsey problems with heterogeneity
- Bridge between Ramsey and Mirlees