Capital Taxation and Ownership When Markets Are Incomplete

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Motivation: Capital Taxes with Incomplete Markets

- **Chari et al. (1994)**: capital taxes with complete markets
  - Indeterminacy of capital taxes:

    The government can replicate complete markets with state-contingent capital taxes but non-contingent debt
  - Ex-ante capital tax is zero, but large variations across states
- **Farhi**: **capital taxes, incomplete markets**
  - Non-state-contingent capital taxes
  - Capital taxes: a hedging instrument?

Competitive Equilibrium

Environment:

- Uncertainty: $s_t \in \mathbb{S}$ (Markov process)
- Resource constraint $\forall t\ \forall s^t$:

$$c_t(s^t) + g_t(s^t) + k_{t+1}(s^t) \leq F(k_t(s^{t-1}), l_t(s^t), s_t) + k_t(s^{t-1})$$

- **Allocation:** $\{c_t(s^t), l_t(s^t), k_{t+1}(s^t)\}$, *given* k_0.
- **Government policy**: $\{\tau_t^l(s^t), \tau_{t+1}^k(s^t), b_{t+1}(s^t)\}$, *given* τ_0^k, b_0.
- **Prices**: $\{w_t(s^t), r_t^k(s^t), r_{t+1}(s^t)\}$, *given* r_0.

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\+ **Government**:

$$\begin{cases} g_t + (1+r_t)b_t \leq \tau_t^l l_t w_t + \tau_t^k r_t^k k_t + b_{t+1} \\ \underline{M}(k_{t+1}, u_c(c_t, l_t), s_t) \leq u_c(c_t, l_t) b_{t+1} \leq \overline{M}(k_{t+1}, u_c(c_t, l_t), s_t) \end{cases}$$

Competitive Equilibrium

+ **Household**: maximizes $\mathbb{E}_{-1}\sum_{t=0}^{\infty}\beta^t u(c_t, l_t)$ such that:

$$c_t + b_{t+1} + k_{t+1} \leq (1-\tau_t^l)w_t l_t + (1-\tau_t^k)r_t^k k_t + k_t + (1+r_t)b_t$$

First-order conditions:

$$\begin{cases} u_{c,t} = \beta(1+r_{t+1})\mathbb{E}_t u_{c,t+1} \\ u_{c,t} = \beta\mathbb{E}_t u_{c,t+1}[1+(1-\tau_{t+1}^k)r_{t+1}^k] \\ \tau_t^l = 1 + \frac{u_l}{w_t u_{c,t}} \end{cases}$$

+ **Firm**: static maximization:

$$\begin{cases} w_t = F_l(k_t, l_t, s_t) \\ r_t^k = F_k(k_t, l_t, s_t) \end{cases}$$

Ramsey Problem - A Recursive Formulation

State variables: $(k, \tilde{b}, \theta, s_-)$, where $\theta \equiv u_c(c_-, l_-)$ and $\tilde{b} \equiv \theta b$

Recursive Ramsey Problem:

$$V(k, \tilde{b}, \theta, s_-) = \max_{\{\tau^k, \{c_s, l_s, k'_s, \tilde{b}'_s\}_{s \in S}\}} \mathbb{E}\{u_s + \beta V(k'_s, \tilde{b}'_s, u_{c,s}, s) \mid s_-\}$$

subject to

$$\mathbb{E}[\beta u_{c,s}[1 + (1 - \tau^k)F_{k,s}] \mid s_-] = \theta \qquad (\mu)$$

$$\tilde{b}\frac{u_{c,s}}{\beta \mathbb{E}\{u_{c,s} \mid s_-\}} + g_s u_{c,s} \leq l_s F_{l,s} u_{c,s} + l_s u_{l,s} + \tau^k k F_{k,s} u_{c,s} + \tilde{b}'_s \qquad (\nu_s)$$

$$c_s + g_s + k'_s = F_s + k \qquad (\phi_s)$$

$$\underline{M}(k'_s, u_{c,s}, s) \leq \tilde{b}'_s \leq \bar{M}(k'_s, u_{c,s}, s) \qquad (\nu_{2,s};\ \nu_{1,s})$$

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First-Order Conditions: Away from debt limits, ν is a risk-adjusted martingale:

$$\nu_{s_-} = \frac{\mathbb{E}\{\nu_s u_{c,s} \mid s_-\}}{\mathbb{E}\{u_{c,s} \mid s_-\}} + (\nu_{1,s_-} - \nu_{2,s_-})$$

Numerical Simulation

- Utility function: $u(c, l) = (1-\gamma)\log c + \gamma \log(1-l)$
- Production function: $F(k, l, z, t) = k^{\alpha}(\exp(\rho t + s_1)l)^{(1-\alpha)} - \delta k$
- Government expenditures: $g_t = G\exp(\rho t + s_2)$
- Productivity and government spending shocks are assumed independent

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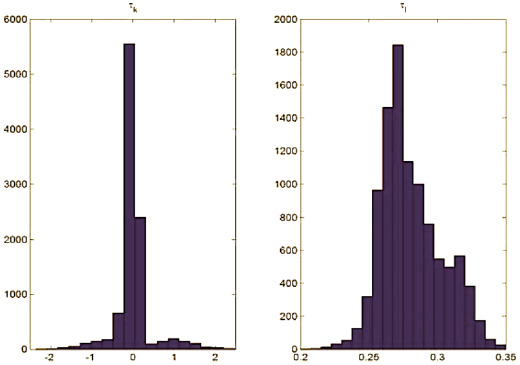
Simulation - One-year period length

	Mean	St. Dev.	Autocorr.
τ^L	28.4%	1.8%	0.85
τ^K	3.7%	53.8%	0.04

Numerical Simulation



Frequency distributions of capital and labor taxes in the model with a period length of one year (simulation: 10 000 periods).

Capital Taxes: the Hedging Term

$$\tau^k = T^h(k, \tilde{b}, \theta, s_-) + T^i(k, \tilde{b}, \theta, s_-) + T^b(k, \tilde{b}, \theta, s_-) \tag{1}$$

- Hedging : to smooth needs for funds across states
- **Proposition**: $T^h = 0$ *with complete markets.*
- **Proposition**: $T^h = 0$ *if* F *is Cobb-Douglas.*

$$T^h(k, \tilde{b}, \theta, s_-) = \frac{\mathbb{E}(-k(1-\tau^k)F_{kk,s}u_{c,s} \mid s_-)}{\mathbb{E}(F_{k,s}\psi_s \mid s_-) + \mathbb{E}(\nu_s F_{k,s}u_{c,s} \mid s_-)} * \dots$$

$$\dots \left[\frac{\text{cov}(kF_{k,s}u_{c,s}, \nu_s \mid s_-)}{\mathbb{E}(kF_{k,s}u_{c,s} \mid s_-)} - \frac{\text{cov}(kF_{kk,s}u_{c,s}, \nu_s \mid s_-)}{\mathbb{E}(kF_{kk,s}u_{c,s} \mid s_-)} \right]$$

Capital Taxes: the Intertemporal Term

$$\tau^k = T^h(k, \tilde{b}, \theta, s_-) + T^i(k, \tilde{b}, \theta, s_-) + T^b(k, \tilde{b}, \theta, s_-) \tag{2}$$

- Intertemporal: to manipulate interest rates, to help absorb the variations in present and future government surpluses
- **Proposition** *(Chari et al.): In complete markets, $T^h = 0$ if the utility function is CRRA and separable.*
- **Proposition**: $T^h = 0$ *with quasilinear utility function.*

$$T^h(k, \tilde{b}, \theta, s_-) = -\frac{\mathbb{E}([1 + (1 - \tau^k)F_{k,s}]u_{c,s}(\frac{\psi_s}{u_{c,s}} - \frac{\psi_{s_-}}{\theta}) \mid s_-)}{\beta \mathbb{E}(F_{k,s}u_{c,s}(\nu_s + \frac{\psi_s}{u_{c,s}}) \mid s_-)}$$

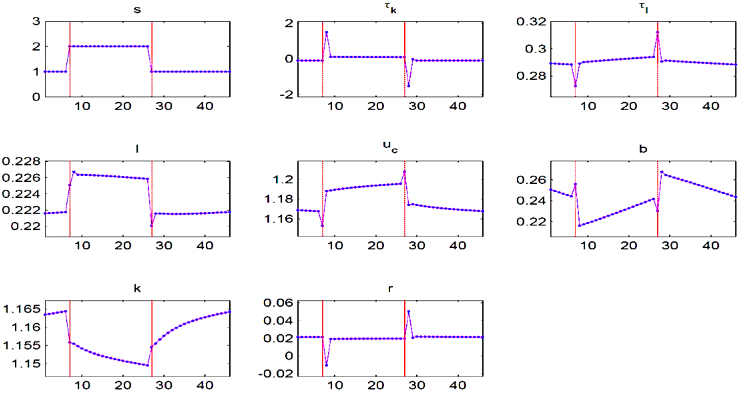
Capital Taxes: the Debt-Limits Term

$$\tau^k = T^h(k, \tilde{b}, \theta, s_-) + T^i(k, \tilde{b}, \theta, s_-) + T^b(k, \tilde{b}, \theta, s_-) \quad (3)$$

- Debt-Limits Term: to relax the borrowing limits
- **Proposition**: $T^b = 0$ *unless a debt limit is binding in the previous period.*
- **Proposition**: $T^b = 0$ *if the imposed debt limits do not depend on capital.*

$$T^b(k, \tilde{b}, \theta, s_-) = \frac{\nu_{2,s}\underline{M}_{k,s_-} - \nu_{1,s}\overline{M}_{k,s_-}}{\beta\mathbb{E}(F_{k,s}u_{c,s}(\nu_s + \frac{\psi_s}{u_{c,s}}) \mid s_-)}$$

Capital Taxes: Numerical Simulation



Simulated observations from the model with government expenditure shocks only and a one-year period length. The high (low) government expenditure state is $s = 2$ $(s = 1)$.

Capital Ownership

- Allowing the government to trade capital...
  - ... Enables the government to **hedge** its budget against aggregate shocks...
  - ... **Without the distortionary cost** to distorting capital accumulation.

- A long position to hedge against government expenditure shocks
- A short position to hedge against productivity shocks

- Numerical simulation: the government can replicate the **complete-markets** Ramsey outcome - with **extreme positions**