Capital Taxation and Ownership When Markets Are Incomplete

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Motivation: Capital Taxes with Incomplete Markets

- **Chari et al. (1994)**: capital taxes with complete markets
  - Indeterminacy of capital taxes:
    - The government can **replicate complete markets** with state-contingent capital taxes but non-contingent debt
  - **Ex-ante** capital tax is zero, but large variations **across states**

- **Farhi**: capital taxes, incomplete markets
  - **Non-state-contingent** capital taxes
  - Capital taxes: a hedging instrument?
Competitive Equilibrium

Environment:

- Uncertainty: $s_t \in S$ (Markov process)
- Resource constraint \( \forall t \forall s^t: \)

\[
c_t(s^t) + g_t(s^t) + k_{t+1}(s^t) \leq F(k_t(s^{t-1}), l_t(s^t), s_t) + k_t(s^{t-1})
\]

- Allocation: \( \{c_t(s^t), l_t(s^t), k_{t+1}(s^t)\} \), given \( k_0 \).
- Government policy: \( \{\tau^l_t(s^t), \tau^k_{t+1}(s^t), b_{t+1}(s^t)\} \), given \( \tau^k_0, b_0 \).
- Prices: \( \{w_t(s^t), r^k_t(s^t), r_{t+1}(s^t)\} \), given \( r_0 \).
Competitive Equilibrium

Environment:

- Uncertainty: $s_t \in \mathbb{S}$ (Markov process)
- Resource constraint $\forall t \forall s^t$:
  
  $$c_t(s^t) + g_t(s^t) + k_{t+1}(s^t) \leq F(k_t(s^{t-1}), l_t(s^t), s_t) + k_t(s^{t-1})$$

- **Allocation**: $\{c_t(s^t), l_t(s^t), k_{t+1}(s^t)\}$, given $k_0$.
- **Government policy**: $\{\tau^l_t(s^t), \tau^k_t(s^t), b_{t+1}(s^t)\}$, given $\tau^k_0$, $b_0$.
- **Prices**: $\{w_t(s^t), r^k_t(s^t), r_{t+1}(s^t)\}$, given $r_0$.

**+ Government**:

$$\begin{cases} 
  g_t + (1 + r_t)b_t \leq \tau^l_t l_t w_t + \tau^k_t r^k_t k_t + b_{t+1} \\
  \underline{M}(k_{t+1}, u_c(c_t, l_t), s_t) \leq u_c(c_t, l_t)b_{t+1} \leq \overline{M}(k_{t+1}, u_c(c_t, l_t), s_t)
\end{cases}$$
Competitive Equilibrium

+ **Household**: maximizes $E_{-1} \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$ such that:

$$c_t + b_{t+1} + k_{t+1} \leq (1 - \tau^l_t)w_t l_t + (1 - \tau^k_t)r^k_t k_t + k_t + (1 + r_t)b_t$$

First-order conditions:

$$
\begin{cases}
  u_{c,t} = \beta(1 + r_{t+1})E_t u_{c,t+1} \\
  u_{c,t} = \beta E_t u_{c,t+1}[1 + (1 - \tau^k_{t+1})r^k_{t+1}] \\
  \tau^l_t = 1 + \frac{u_l}{w_t u_{c,t}}
\end{cases}
$$

+ **Firm**: static maximization:

$$
\begin{cases}
  w_t = F_l(k_t, l_t, s_t) \\
  r^k_t = F_k(k_t, l_t, s_t)
\end{cases}
$$
Ramsey Problem - A Recursive Formulation

State variables: \((k, \tilde{b}, \theta, s_-)\), where \(\theta \equiv u_c(c-, l-)\) and \(\tilde{b} \equiv \theta b\)

Recursive Ramsey Problem:

\[
V(k, \tilde{b}, \theta, s_-) = \max_{\{\tau^k, \{c_s, l_s, k'_s, \tilde{b}'_s\}\}_{s \in S}} \mathbb{E}\{u + \beta V(k'_s, \tilde{b}'_s, u_{c,s}, s) \mid s_-\}
\]

subject to

\[
\mathbb{E}[\beta u_{c,s}[1 + (1 - \tau^k)F_{k,s}] \mid s_-] = \theta \quad \text{(\(\mu\))}
\]

\[
\tilde{b} \frac{u_{c,s}}{\beta \mathbb{E}\{u_{c,s} \mid s_-\}} + g_s u_{c,s} \leq l_s F_{l,s} u_{c,s} + l_s u_{l,s} + \tau^k k F_{k,s} u_{c,s} + \tilde{b}'_s \quad \text{(\(\nu_s\))}
\]

\[
c_s + g_s + k'_s = F_s + k \quad \text{(\(\phi_s\))}
\]

\[
\underline{M}(k'_s, u_{c,s}, s) \leq \tilde{b}'_s \leq \bar{M}(k'_s, u_{c,s}, s) \quad \text{(\(\nu_{2,s} ; \nu_{1,s}\))}
\]
Ramsey Problem - A Recursive Formulation

State variables: \((k, \tilde{b}, \theta, s_-)\), where \(\theta \equiv u_c(c-, l_-)\) and \(\tilde{b} \equiv \theta b\)

Recursive Ramsey Problem:

\[
V(k, \tilde{b}, \theta, s_-) = \max_{\tau^k, \{c_s, l_s, k'_s, \tilde{b}'_s\}_{s \in S}} \mathbb{E}\{u_s + \beta V(k'_s, \tilde{b}'_s, u_{c,s}, s) \mid s_-\}
\]

subject to

\[
\mathbb{E}[\beta u_{c,s}[1 + (1 - \tau^k)F_{k,s}] \mid s_-] = \theta \\
\tilde{b} \frac{u_{c,s}}{\beta \mathbb{E}\{u_{c,s} \mid s_-\}} + g_s u_{c,s} \leq l_s F_{l,s} u_{c,s} + l_s u_{l,s} + \tau^k kF_{k,s} u_{c,s} + \tilde{b}'_s \\
c_s + g_s + k'_s = F_s + k \\
\underline{M}(k'_s, u_{c,s}, s) \leq \tilde{b}'_s \leq \bar{M}(k'_s, u_{c,s}, s)
\]

First-Order Conditions: Away from debt limits, \(\nu\) is a risk-adjusted martingale:

\[
\nu_{s_-} = \frac{\mathbb{E}\{\nu_{s} u_{c,s} \mid s_-\}}{\mathbb{E}\{u_{c,s} \mid s_-\}} + (\nu_{1,s_-} - \nu_{2,s_-})
\]
Numerical Simulation

- Utility function: $u(c, l) = (1 - \gamma) \log c + \gamma \log(1 - l)$
- Production function: $F(k, l, z, t) = k^\alpha (\exp(\rho t + s_1)l^{1-\alpha} - \delta k$
- Government expenditures: $g_t = G \exp(\rho t + s_2)$
- Productivity and government spending shocks are assumed independent
Numerical Simulation

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- Production function: \( F(k, l, z, t) = k^\alpha \exp(\rho t + s_1)l^{(1-\alpha)} - \delta k \)
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Simulation - One-year period length

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>St. Dev.</th>
<th>Autocorr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau^L )</td>
<td>28.4%</td>
<td>1.8%</td>
<td>0.85</td>
</tr>
<tr>
<td>( \tau^K )</td>
<td>3.7%</td>
<td>53.8%</td>
<td>0.04</td>
</tr>
</tbody>
</table>
Numerical Simulation

Frequency distributions of capital and labor taxes in the model with a period length of one year (simulation: 10 000 periods).
Capital Taxes: the Hedging Term

\[ \tau^k = T^h(k, \tilde{b}, \theta, s-) + T^i(k, \tilde{b}, \theta, s-) + T^b(k, \tilde{b}, \theta, s-) \]  

- Hedging: to smooth needs for funds across states
- Proposition: \( T^h = 0 \) with complete markets.
- Proposition: \( T^h = 0 \) if \( F \) is Cobb-Douglas.

\[ T^h(k, \tilde{b}, \theta, s-) = \frac{\mathbb{E}(-k(1 - \tau^k)F_{kk,s}u_{c,s} \mid s-)}{\mathbb{E}(F_{k,s}\psi_s \mid s-) + \mathbb{E}(\nu_sF_{k,s}u_{c,s} \mid s-)} \ast \ldots \]

\[ \ldots \left[ \frac{\text{cov}(\nu_sF_{k,s}u_{c,s} \mid s-)}{\mathbb{E}(kF_{k,s}u_{c,s} \mid s-)} - \frac{\text{cov}(kF_{kk,s}u_{c,s} \mid s-)}{\mathbb{E}(kF_{kk,s}u_{c,s} \mid s-)} \right] \]
Capital Taxes: the Intertemporal Term

\[
\tau^k = T^h(k, \tilde{b}, \theta, s_-) + T^i(k, \tilde{b}, \theta, s_-) + T^b(k, \tilde{b}, \theta, s_-) \tag{2}
\]

- Intertemporal: to manipulate interest rates, to help absorb the variations in present and future government surpluses

- **Proposition** (Chari et al.): In complete markets, \( T^h = 0 \) if the utility function is CRRA and separable.

- **Proposition**: \( T^h = 0 \) with quasilinear utility function.

\[
T^h(k, \tilde{b}, \theta, s_-) = -\frac{\mathbb{E}([1 + (1 - \tau^k)F_{k,s}]u_{c,s}(\frac{\psi_s}{u_{c,s}} - \frac{\psi_{s-}}{\theta}) | s_-)}{\beta \mathbb{E}(F_{k,s}u_{c,s}(\nu_s + \frac{\psi_s}{u_{c,s}}) | s_-)}
\]
Capital Taxes: the Debt-Limits Term

\[ \tau^k = T^h(k, \tilde{b}, \theta, s_-) + T^i(k, \tilde{b}, \theta, s_-) + T^b(k, \tilde{b}, \theta, s_-) \]  \hspace{1cm} (3)

- **Debt-Limits Term:** to relax the borrowing limits
- **Proposition:** \( T^b = 0 \) unless a debt limit is binding in the previous period.
- **Proposition:** \( T^b = 0 \) if the imposed debt limits do not depend on capital.

\[ T^b(k, \tilde{b}, \theta, s_-) = \frac{\nu_2,s M_{k,s_-} - \nu_1,s \bar{M}_{k,s_-}}{\beta \mathbb{E}(F_{k,s} u_{c,s}(\nu_{s} + \psi_{s} u_{c,s}) \mid s_-)} \]
Simulated observations from the model with government expenditure shocks only and a one-year period length. The high (low) government expenditure state is $s = 2$ ($s = 1$).
Capital Ownership

- Allowing the government to trade capital...
  - ... Enables the government to **hedge** its budget against aggregate shocks...
  - ... **Without the distortionary cost** to distorting capital accumulation.

- A long position to hedge against government expenditure shocks
- A short position to hedge against productivity shocks

- Numerical simulation: the government can replicate the **complete-markets** Ramsey outcome - with **extreme positions**