Interbank Lending and Systemic Risk

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April 2, 2012

Overview

- Theory of decentralized interbank lending based on peer monitoring
- Illustrate commitment problem for central bank in decision to bail out and discuss Too big to fail argument
- Study the possibility of (efficient) systemic risk

Model

- $t = 0, 1, 2$
- n banks and a continuum of investors all with risk-neutral preferences and no discounting
- investors have deep pockets banks have initial endowment A_i
- Technology: convex costs borne at $t = 0$, $C(I)$; linear returns at $t = 2$ RI if successful.
- Liquidity shock: at $t = 1$ random amount ρI is needed to continue, i.i.d across across banks with distribution F
- Moral hazard: after ρ realizes bank privately chooses probability of success $p \in \{p_h, p_l\}$; choosing p_l yields private benefit BI

Timing

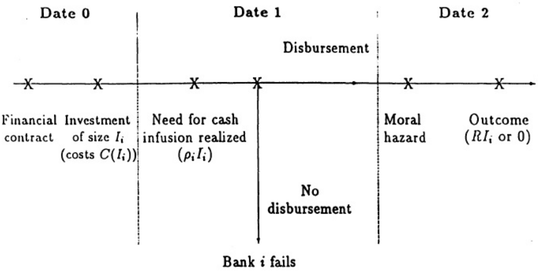

FIG. 1. The Timing of Events in the Model

Optimal Allocation

- Under risk neutrality bank i is paid an amount R_i iff its project succeeds
- Let $x_i : supp(\rho_i) \to \{0, 1\}$ denote a continuation rule; $U_i(\rho_i) = x_i(\rho_i)p_h R_i$ denote the interim expected utility of bank i; the interim expectded utility for investors $V_i(\rho_i)$ satisfies

$$V_i(\rho_i) + U_i(\rho_i) = x_i(\rho_i)(p_h R - \rho_i)I_i$$

- The incentive constraints are for $i = 1, ..., n$

$$(p_h - p_l)x_i(\rho_i)R_i \geq x_i(\rho_i)BI_i$$

$$\Delta p U_i(\rho_i) \geq x_i(\rho_i)(p_h BI_i)$$

Optimal allocation

- Given Pareto weights ν_i and λ for bank i and depositors, Pareto optima are obtained by choosing $\{I_i, U_i(\rho_i), x_i(\rho_i)\}_{i=1}^{n}$ to maximize

$$L = \sum_i E\left[(\nu_i - \lambda)U_i(\rho_i) + \lambda x_i(\rho_i)I_i(p_h R - \rho_i) + \lambda(A_i - C(I_i))\right]$$

$$s.t.$$

$$\Delta p U_i(\rho_i) \geq x_i(\rho_i)(p_h B I_i)$$

Optimal allocation

- We must have $\nu_i < \lambda$ for all i
- Incetive constraints will be binding

$$U_i(\rho_i) = \begin{cases} p_h \frac{BI_i}{\Delta p} & \text{if } x_i(\rho_i) = 1; \\ 0 & \text{if } x_i(\rho_i) = 0. \end{cases}$$

Optimal allocation

- Given I.C. are binding I_i and $x_i()$ are found by maximizing

$$\frac{L}{\lambda} = \sum_i I_i E\left\{\left[(\frac{\nu_i}{\lambda} - 1)\frac{p_h B}{\Delta p} + (p_h R - \rho_i)\right] x_i(\rho_i)\right\} - \sum_i [C(I_i) - A_i]$$

- Optimal continuation

$$x_i(\rho_i) = \begin{cases} 1 & \text{if } \rho_i \leq \rho_i^A; \\ 0 & \text{if } \rho_i > \rho_i^A. \end{cases}$$

$$\rho_i^A \equiv p_h(R - \frac{B}{\Delta p}) + \frac{\nu_i}{\lambda}\frac{p_h B}{\Delta p}$$

- Optimal threshold satisfies

$$\rho_i^A > p_h(R - \frac{B}{\Delta p}) \equiv \rho_0$$

- Optimal investment level

$$C'(I_i^A) = \int_0^{\rho_i^A} (\rho_i^A - \rho_i) f(\rho_i) d\rho_i$$

Implementation

- Liquidity requirement: Borrow $C(I_i) - A_i + \rho_i^A I_i^A$ invest $\rho_i^A I_i^A$ in Treasuries and commit not to dilute existing claims
- Credit line: Borrow $C(I_i) - A_i$ and obtain credit line.

Interbank monitoring (Date 0)

- 2 banks; bank 2 monitors bank 1's short term management
- At private and unobservable cost cI_1 bank 2 ensures that ρ_1 is distributed according to F_1
- If not monitored bank 1 can enjoy a private benefit SI_1 and induce a distribution $\widetilde{F}_1$
- The densities associated to the distributions satisfy MLRP:

$$l(\rho_1) \equiv \frac{f_1(\rho_1) - \widetilde{f}_1(\rho_1)}{f_1(\rho_1)}$$

  is decreasing in ρ_1
- Parameters are such that it is always efficient to monitor

Optimal allocation

- Pareto optima are obtained by choosing $\{I_i, U_i(\rho), x_i(\rho)\}_{i=1}^{2}$ to maximize

$$L = \sum_{i}^{2} E\left[(\nu_i - \lambda)U_i(\rho) + \lambda x_i(\rho) I_i(p_h R - \rho_i) + \lambda(A_i - C(I_i)) - \nu_2 c I_1\right]$$

$$s.t.$$

$$\Delta p U_i(\rho) \geq x_i(\rho)(p_h B I_i)$$

$$E\left\{U_2(\rho) I(\rho_1)\right\} \geq c I_1$$

$$x_i(\rho) = 1 for \rho_i < \rho_0$$

Optimal allocaation (monitoree)

- Bank 1 closure policy is the same as under autarky

$$x_1(\rho) = 1 \Leftrightarrow \rho_1 \leq \rho_1^* = \rho_1^A$$

- optimal ivnestment size is

$$C'(I_1^*) = \int_0^{\rho_1^*} (\rho_1^* - \rho_1) f(\rho_1) d\rho_1 + c(\frac{\nu_2 + \mu}{\lambda})$$

$$C'(I_1^*) = \int_0^{\rho_1^*} F_1(\rho_1) d\rho_1 + c(\frac{\nu_2 + \mu}{\lambda})$$

where μ is the multiplier on the incentive constraint of the monitor.

Monitoree

Theorem

Under optimal interbank lending:

- *The continuation decision and the welfare of the monitoree (borrowing bank) do not depend on the liquidity shock facing the monitor (lending bank)*
- *The monitoree is closed less often than under autarky*
- *When the unit cost of monitoring c is small, the monitoree invests more than under autarky*

Optimal allocation (monitor)

- Bank 2's closure policy is

$$x_1(\rho) = 1 \Leftrightarrow \rho_1 \leq \rho_2^*(\rho_1) = \rho_0 + \frac{\max\{0, \nu_2 + \mu I(\rho_1)\}}{\lambda} \frac{p_h B}{\Delta p}$$

- optimal ivnestment size is

$$C'(I_2^*) = E \int_0^{\rho_2^*(\rho_1)} F_2(\rho_2) d\rho_2$$

Monitor

Theorem

Under optimal interbank lending:

- *The continuation decision of the monitor (lending bank) depends on the liquidity shock facing the monitoree and in particular, when the credibility constraint is not binding*

$$x_2(\rho) = 1 \Leftrightarrow \rho_2 + \rho_{21}(\rho_1) \geq \rho_2^A$$

$$\rho_{21}(\rho_1) = -\frac{\mu}{\lambda} I(\rho_1) \frac{p_h B}{\Delta p}$$

  If instead ρ_1 is so high that the credibility constraint binds the threshold is the per unit pledgeable income

- *When the credibility constraint is not binding, the monitor invests more than under autarky*

Implementation

- Qualitative feature are clear: the monitor's liquidity should depend on the monitoree's shock through some form af interbank credit
- Specific form depends on the likelihood ratio
- Example: bank 1(monitoree) has liquidity $L_1 = (\rho_1 - \rho_0)I_1$ and option to dilute; bank 2 has liquidity L_2 and debt issued by 2 with face value $\beta \frac{\rho_0}{p_h} I_1$ that matures at $t = 2$ and option to dilute.
- At $t = 1$, if bank 1 survives, the value of the loan $V_{21}(\rho_1) = \beta \rho_1 I_1$ if $L_1 \geq \rho_1 I_1$, if babnk 1 needs to dilute

$$V_{21}(\rho_1) = \beta \left[\rho_1 I_1 - (\rho_1 I_1 - L_1)\right]$$

- This loan implements the optimum if and only if

$$L_2 + V_{21}(\rho_1) = \left[\rho_2^*(\rho_1) - \rho_0\right] I_2$$

Soft Budget Constraint and Too big to fail

- Soft Budget Constraint

$$\rho_2^*(\rho_1) < \rho_2 < \rho_0$$

- In these cases the borrowing bank is in troubles and causes the lending bank to be in distress as well even if the latter would be solvent provided one ignores interbank activities
- Should one assist the borrowing bank, TBTF, or should one just let the borrowing bank fail and assist the failing bank?

Central Bank Bailouts

Theorem

- *The central bank's inability to commit not to rescue a bank that incurs losses on the interbank market but is otherwise solvent may lead to the prohibition of interbank lending in cases where it would be allowed if the central bank's commitment were credible*
- *The SBC differs from the TBTF as the borrowing bank's closure decision is unrelated to the fragility of the lending bank. Central bank assistance to a soolvent but failing bank operates through a direct assistance to that bank rather than through a bail out of the borrowing bank*

Symmetric Date 1 Monitoring

- n banks located on a circle
- after ρ realizes bank i can monitor bank $i-1$ at a cost cI_{i-1}as long as the latter has not been shut down.
- monitornig costs c and reduces the benefit of shirking to the monitoring from BI to bI
- it is assumed that it is always optimal to monitor

Incentive constraints

- if $x_i(\rho) = 0$ and $x_{i-1}(\rho) = 1$

$$(p_h - p_l)U_i(\rho) \geq p_h c l_{i-1}$$

- if $x_i(\rho) = 1$ and $x_{i-1}(\rho) = 0$

$$(p_h - p_l)U_i(\rho) \geq p_h b l_i$$

- if $x_i(\rho) = 1$ and $x_{i-1}(\rho) = 1$

$$(p_h^2 - p_l^2)U_i(\rho) \geq p_h^2(b l_i + c l_{i-1})$$

- Behind these we have R^i_{11},R^i_{10} and R^i_{01} and some timing and informational assumptions

Incentive Constraints economies of scope

- $$U_i(\rho) = \frac{p_h^2}{p_h^2 - p_h^2}(bI_i + cI_{i-1}) = \frac{\gamma p_h}{\Delta p}(bI_i + cI_{i-1})$$
- $$U_i(\rho) = \frac{p_h}{\Delta p}\left[\gamma(bI_i + cI_{i-1})x_i x_{i-1} + cI_{i-1}x_{i-1}(1 - x_i) + bI_i x_i(1 - x_{i-1})\right]$$
- Taking expectations and rearranging

$$E\left[U_i\right] \geq \frac{p_h}{\Delta p}\left[bI_i E(x_i - (1-\gamma)x_i x_{i-1}) + cI_{i-1}E(x_{i-1} - (1-\gamma)x_i x_{i-1})\right]$$

- The pareto optima are obtained by maximizing the weighte sum of expected payoffs subject to the above incentive constraints for each i

Optimal contract

- The optimal closure decision is obtained by maximizing

$$H(x,\rho) = \sum_{i}^{n}(\nu_i - \lambda)\frac{p_h}{\Delta p}\left[bI_i E(x_i - (1-\gamma)x_i x_{i-1}) + cI_{i-1}E(x_{i-1} - (1-\gamma)x_i x_{i-1})\right]$$

$$+\sum_{i}^{n}\lambda I_i x_i (p_h R - \rho_i)$$

- Or more compactly

$$H(x,\rho) = \sum_{i=1}^{n} u_i x_i + \sum_{i=1}^{n} w_i x_i x_{i-1}$$

$$u_i(\rho_i) \equiv I_i \left\{ \lambda(p_h R - \rho_i) - (\lambda - \nu_i)\frac{p_h b}{\Delta p} - (\lambda - \nu_{i+1})\frac{p_h c}{\Delta p} \right\}$$

$$w_i \equiv (1-\gamma)(\lambda - \nu_i)\frac{p_h}{\Delta p}(bI_i + cI_{i-1}) > 0$$

Local and Global dependency

- Bank i is shut down iff
$$\frac{\partial H}{\partial x_i} = u_i + w_i x_{i-1} + w_{i+1} x i + 1 \leq 0$$
- Function H is supermodular and satisfies the single crossing property
$$\frac{\partial^2 H}{\partial x_i \partial x_j} \geq 0$$
$$\frac{\partial^2 H}{\partial x_i \partial x_j} \geq 0$$
- By a theorem of Milgrom and Shannon we have that the function mapping ρ to the set of maximizers of H is monotone (w.r.t the partial rdering on ρ and the strong set order on the maximizers)

Systemic Risk

Theorem

For all i and j, bank i is more likely to be liquidated if the liquidity shock facing bank j increases. There exists a value of ρ such that H has exactly two maxima $x^* = (0, ..., 0)$ and $x^{**} = (1, ..., 1)$. Thus a small increase in a bank's liquidity shock may imply the closure of the entire banking system.