Interbank Lending and Systemic Risk

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Overview

- Theory of decentralized interbank lending based on peer monitoring
- Illustrate commitment problem for central bank in decision to bail out and discuss Too big to fail argument
- Study the possibility of (efficient) systemic risk
Model

- $t = 0, 1, 2$
- $n$ banks and a continuum of investors all with risk-neutral preferences and no discounting
- Investors have deep pockets; banks have initial endowment $A_i$
- Technology: convex costs borne at $t = 0$, $C(I)$; linear returns at $t = 2$, $Rl$ if successful.
- Liquidity shock: at $t = 1$ random amount $\rho l$ is needed to continue, i.i.d across banks with distribution $F$
- Moral hazard: after $\rho$ realizes bank privately chooses probability of success $p \in \{p_h, p_l\}$; choosing $p_l$ yields private benefit $Bl$
Timing

Fig. 1. The Timing of Events in the Model
Optimal Allocation

- Under risk neutrality bank $i$ is paid an amount $R_i$ iff its project succeeds.
- Let $x_i : \text{supp} (\rho_i) \rightarrow \{0, 1\}$ denote a continuation rule; $U_i(\rho_i) = x_i(\rho_i)p_h R_i$ denote the interim expected utility of bank $i$; the interim expected utility for investors $V_i(\rho_i)$ satisfies

$$V_i(\rho_i) + U_i(\rho_i) = x_i(\rho_i)(p_h R - \rho_i)l_i$$

- The incentive constraints are for $i = 1, \ldots, n$

$$ (p_h - p_l)x_i(\rho_i) R_i \geq x_i(\rho_i)Bl_i $$

$$ \Delta pU_i(\rho_i) \geq x_i(\rho_i)(p_hBl_i) $$
Optimal allocation

Given Pareto weights $\nu_i$ and $\lambda$ for bank $i$ and depositors, Pareto optima are obtained by choosing $\{I_i, U_i(\rho_i), x_i(\rho_i)\}_{i=1}^n$ to maximize

$$L = \sum_i E \left[ (\nu_i - \lambda)U_i(\rho_i) + \lambda x_i(\rho_i)I_i(p_hR - \rho_i) + \lambda(A_i - C(I_i)) \right]$$

s.t.

$$\Delta p U_i(\rho_i) \geq x_i(\rho_i)(p_h B I_i)$$
Optimal allocation

- We must have $\nu_i < \lambda$ for all $i$
- Incentive constraints will be binding

$$U_i(\rho_i) = \begin{cases} 
\rho_i \frac{Bl_i}{\Delta p} & \text{if } x_i(\rho_i) = 1; \\
0 & \text{if } x_i(\rho_i) = 0.
\end{cases}$$
Given I.C. are binding $l_i$ and $x_i()$ are found by maximizing

$$\frac{L}{\lambda} = \sum_i l_i E \left\{ \left[ \left( \frac{\nu_i}{\lambda} - 1 \right) \frac{p_h B}{\Delta p} + (p_h R - \rho_i) \right] x_i(\rho_i) \right\} - \sum_i [C(l_i) - A_i]$$

Optimal continuation

$$x_i(\rho_i) = \begin{cases} 1 & \text{if } \rho_i \leq \rho_i^A; \\ 0 & \text{if } \rho_i > \rho_i^A. \end{cases}$$

$$\rho_i^A \equiv p_h (R - \frac{B}{\Delta p}) + \frac{\nu_i p_h B}{\lambda \Delta p}$$

Optimal threshold satisfies

$$\rho_i^A > p_h (R - \frac{B}{\Delta p}) \equiv \rho_0$$

Optimal investment level

$$C'(l_i^A) = \int_0^{\rho_i^A} (\rho_i^A - \rho_i) f(\rho_i) d\rho_i$$
Implementation

- Liquidity requirement: Borrow \( C(l_i) - A_i + \rho_i^A l_i^A \) invest \( \rho_i^A l_i^A \) in Treasuries and commit not to dilute existing claims.

- Credit line: Borrow \( C(l_i) - A_i \) and obtain credit line.
Interbank monitoring (Date 0)

- 2 banks; bank 2 monitors bank 1’s short term management
- At private and unobservable cost $c l_1$ bank 2 ensures that $\rho_1$ is distributed according to $F_1$
- If not monitored bank 1 can enjoy a private benefit $S l_1$ and induce a distribution $\tilde{F}_1$
- The densities associated to the distributions satisfy MLRP:
  \[ l(\rho_1) \equiv \frac{f_1(\rho_1) - \tilde{f}_1(\rho_1)}{f_1(\rho_1)} \]
  is decreasing in $\rho_1$
- Parameters are such that it is always efficient to monitor
Optimal allocation

- Pareto optima are obtained by choosing 
  \( \{ I_i, U_i(\rho), x_i(\rho) \}_{i=1}^{2} \) to maximize

\[
L = \sum_{i} E \left[ (\nu_i - \lambda) U_i(\rho) + \lambda x_i(\rho) I_i(p_h R - \rho_i) + \lambda (A_i - C(I_i)) - \nu_2 c_l \right]
\]

s.t.

\[
\Delta p U_i(\rho) \geq x_i(\rho)(p_h B I_i)
\]

\[
E \left\{ U_2(\rho) I(\rho_1) \right\} \geq c_l
\]

\[
x_i(\rho) = 1 \text{ for } \rho_i < \rho_0
\]
Optimal allocation (monitoree)

- Bank 1 closure policy is the same as under autarky

\[ \chi_1(\rho) = 1 \iff \rho_1 \leq \rho_1^* = \rho_1^A \]

- Optimal investment size is

\[ C'(l_1^*) = \int_0^{\rho_1^*} (\rho_1^* - \rho_1) f(\rho_1) d\rho_1 + c(\frac{\nu_2 + \mu}{\lambda}) \]

\[ C'(l_1^*) = \int_0^{\rho_1^*} F_1(\rho_1) d\rho_1 + c(\frac{\nu_2 + \mu}{\lambda}) \]

where \( \mu \) is the multiplier on the incentive constraint of the monitor.
Monitoree

Theorem

Under optimal interbank lending:

- The continuation decision and the welfare of the monitoree (borrowing bank) do not depend on the liquidity shock facing the monitor (lending bank).
- The monitoree is closed less often than under autarky.
- When the unit cost of monitoring $c$ is small, the monitoree invests more than under autarky.
Interbank Lending and Systemic Risk

Optimal allocation (monitor)

- Bank 2’s closure policy is

\[ x_1(\rho) = 1 \iff \rho_1 \leq \rho_2^*(\rho_1) = \rho_0 + \frac{\max\{0, \nu_2 + \mu l(\rho_1)\}}{\lambda} \frac{\rho_h B}{\Delta p} \]

- Optimal investment size is

\[ C'(l_2^*) = E \int_0^{\rho_2^*(\rho_1)} F_2(\rho_2) d\rho_2 \]
Under optimal interbank lending:

- The continuation decision of the monitor (lending bank) depends on the liquidity shock facing the monitoree and in particular, when the credibility constraint is not binding:

\[ x_2(\rho) = 1 \iff \rho_2 + \rho_{21}(\rho_1) \geq \rho^A_2 \]

\[ \rho_{21}(\rho_1) = -\frac{\mu}{\lambda} f(\rho_1) \frac{p_h B}{\Delta p} \]

If instead \( \rho_1 \) is so high that the credibility constraint binds the threshold is the per unit pledgeable income.

- When the credibility constraint is not binding, the monitor invests more than under autarky.
Implementation

- Qualitative feature are clear: the monitor’s liquidity should depend on the monitoree’s shock through some form af interbank credit.
- Specific form depends on the likelihood ratio.
- Example: bank 1 (monitoree) has liquidity $L_1 = (\rho_1 - \rho_0)I_1$ and option to dilute; bank 2 has liquidity $L_2$ and debt issued by 2 with face value $\beta \frac{\rho_0}{\rho_1} I_1$ that matures at $t = 2$ and option to dilute.
- At $t = 1$, if bank 1 survives, the value of the loan $V_{21}(\rho_1) = \beta \rho_1 I_1$ if $L_1 \geq \rho_1 I_1$, if bank 1 needs to dilute $V_{21}(\rho_1) = \beta [\rho_1 I_1 - (\rho_1 I_1 - L_1)]$
- This loan implements the optimum if and only if $L_2 + V_{21}(\rho_1) = [\rho^*_2(\rho_1) - \rho_0] I_2$.
Soft Budget Constraint and Too big to fail

- **Soft Budget Constraint**

\[ \rho^*_2(\rho_1) < \rho_2 < \rho_0 \]

- In these cases the borrowing bank is in troubles and causes the lending bank to be in distress as well even if the latter would be solvent provided one ignores interbank activities.

- Should one assist the borrowing bank, TBTF, or should one just let the borrowing bank fail and assist the failing bank?
Central Bank Bailouts

**Theorem**

- The central bank’s inability to commit not to rescue a bank that incurs losses on the interbank market but is otherwise solvent may lead to the prohibition of interbank lending in cases where it would be allowed if the central bank’s commitment were credible.

- The SBC differs from the TBTF as the borrowing bank’s closure decision is unrelated to the fragility of the lending bank. Central bank assistance to a solvent but failing bank operates through a direct assistance to that bank rather than through a bail out of the borrowing bank.
Symmetric Date 1 Monitoring

- $n$ banks located on a circle
- after $\rho$ realizes bank $i$ can monitor bank $i-1$ at a cost $c_{i-1}$ as long as the latter has not been shut down.
- monitoring costs $c$ and reduces the benefit of shirking to the monitoring from $B_I$ to $b_I$
- it is assumed that it is always optimal to monitor
Incentive constraints

- if \( x_i(\rho) = 0 \) and \( x_{i-1}(\rho) = 1 \)
  \[
  (p_h - p_l)U_i(\rho) \geq p_h c_{i-1}
  \]

- if \( x_i(\rho) = 1 \) and \( x_{i-1}(\rho) = 0 \)
  \[
  (p_h - p_l)U_i(\rho) \geq p_h b_{i-1}
  \]

- if \( x_i(\rho) = 1 \) and \( x_{i-1}(\rho) = 1 \)
  \[
  (p_h^2 - p_l^2)U_i(\rho) \geq p_h^2(b_{i-1} + c_{i-1})
  \]

Behind these we have \( R_{11}^i, R_{10}^i \) and \( R_{01}^i \) and some timing and informational assumptions
Incentive Constraints economies of scope

\[ U_i(\rho) = \frac{p^2}{p^2 - p_h^2} (bl_i + cl_{i-1}) = \frac{\gamma p_h}{\Delta p} (bl_i + cl_{i-1}) \]

\[ U_i(\rho) = \frac{p_h}{\Delta p} [\gamma (bl_i + cl_{i-1}) x_i x_{i-1} + cl_{i-1} x_{i-1} (1 - x_i) + bl_i x_i (1 - x_i - 1)] \]

Taking expectations and rearranging

\[ E[U_i] \geq \frac{p_h}{\Delta p} [bl_i E(x_i - (1 - \gamma) x_i x_{i-1}) + cl_{i-1} E(x_{i-1} - (1 - \gamma) x_i x_{i-1})] \]

The pareto optima are obtained by maximizing the weighte sum of expected payoffs subject to the above incentive constraints for each \( i \)
Optimal contract

The optimal closure decision is obtained by maximizing

\[ H(x, \rho) = \sum_{i=1}^{n} (\nu_i - \lambda) \frac{p_h}{\Delta p} [b_i E(x_i - (1 - \gamma)x_{i-1}) + c_{i-1} E(x_{i-1} - (1 - \gamma)x_ix_{i-1})] \]

\[ + \sum_{i=1}^{n} \lambda l_i x_i (p_h R - \rho_i) \]

Or more compactly

\[ H(x, \rho) = \sum_{i=1}^{n} u_i x_i + \sum_{i=1}^{n} w_i x_i x_{i-1} \]

\[ u_i(\rho_i) \equiv l_i \left\{ \lambda(p_h R - \rho_i) - (\lambda - \nu_i) \frac{p_h b}{\Delta p} - (\lambda - \nu_{i+1}) \frac{p_h c}{\Delta p} \right\} \]

\[ w_i \equiv (1 - \gamma)(\lambda - \nu_i) \frac{p_h}{\Delta p} (b_i + c_{i-1}) > 0 \]
Local and Global dependency

- Bank $i$ is shut down iff
  \[
  \frac{\partial H}{\partial x_i} = u_i + w_i x_{i-1} + w_{i+1} x_i + 1 \leq 0
  \]

- Function $H$ is supermodular and satisfies the single crossing property
  \[
  \frac{\partial^2 H}{\partial x_i \partial x_j} \geq 0
  \]
  \[
  \frac{\partial^2 H}{\partial x_i \partial x_j} \geq 0
  \]

- By a theorem of Milgrom and Shannon we have that the function mapping $\rho$ to the set of maximizers of $H$ is monotone (w.r.t the partial rdering on $\rho$ and the strong set order on the maximizers)
Systemic Risk

Theorem

For all $i$ and $j$, bank $i$ is more likely to be liquidated if the liquidity shock facing bank $j$ increases. There exists a value of $\rho$ such that $H$ has exactly two maxima $x^* = (0, ..., 0)$ and $x^{**} = (1, ..., 1)$. Thus a small increase in a bank’s liquidity shock may imply the closure of the entire banking system.