Optimal Fiscal and Monetary Policy: Equivalence Results

by Isabel Correia, Juan Pablo Nicolini, and Pedro Teles, JPE 2008
presented by Taisuke Nakata

March 1st, 2011
Introduction

Question: How does sticky price affect the conduct of fiscal/monetary policy?

- A model in which a fraction $\alpha$ of firms set prices one period in advance.
- One-period non-state-contingent debt.
- Labor income and consumption tax.

Answer: Not at all.

- Ramsey allocations are the same under sticky prices and under flexible prices.
- Policies that implement the Ramsey allocations are also the same.
Environment

- Discrete Time, Infinite Horizon.
- $s^t \equiv (s_0, s_1, s_2, ..., s_t)$ where $s_t \in S_t$

3 types of agents:
- A representative household.
- Government.
- A continuum of firms, indexed by $i \in [0, 1]$:
  - Firms $i \in [0, \alpha)$: sticky-price firms that set their prices one period in advance.
  - Firms $i \in [\alpha, 1]$: flexible-price firms that set their price contemporaneously.
- $y_{i,t}(s^t) = A(s^t)n_i(s^t)$

3 goods:
- $C_k(s^t) = \left[ \int_0^1 c_{k,i}(s^t)^{(\theta-1)/\theta} \, di \right]^{\theta/(\theta-1)}$, $(k = 1, 2)$
- $G(s^t) = \left[ \int_0^1 g_i(s^t)^{(\theta-1)/\theta} \, di \right]^{\theta/(\theta-1)}$
Household

$$\max_{\{c_1(s^t), c_2(s^t), N(s^t), M(s^t), \bar{B}(s^t), B(s^{t+1})\}_{t=0}^{\infty}, i \in [0,1]} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(C_1(s^t), C_2(s^t), N(s^t))$$

subject to

$$P^c(s^t) C_1(s^t) \leq M(s^t)$$
$$M(s^t) + \bar{B}(s^t) + \sum_{s^{t+1} \mid s^t} Q(s^{t+1} \mid s^t) B(s^{t+1}) \leq \mathcal{W}(s^t)$$

where

$$\mathcal{W}(s^{t+1}) \equiv M(s^t) + R(s^t) \bar{B}(s^t) + B(s^{t+1}) - \int_0^1 p^c_i(s^t) c_{1, i}(s^t) di$$
$$- \int_0^1 p^c_i(s^t) c_{2, i}(s^t) di + [1 - \tau^n(s^t)] w(s^t) N(s^t)$$

$$p^c_i(s^t) = [1 + \tau^c(s^t)] p_i(s^t)$$

$$P^c(s^t) = \left[ \int_0^1 [p^c_i(s^t)]^{(1-\theta)} di \right]^{1/(1-\theta)}$$

$$\mathcal{W}(s^0) = 0$$
Household FONCs

\[
\begin{align*}
\frac{c_{1,i}(s^t)}{C_1(s^t)} &= \left[ \frac{p_i^c(s^t)}{P_c(s^t)} \right]^{-\theta} & \frac{c_{2,i}(s^t)}{C_2(s^t)} &= \left[ \frac{p_i^c(s^t)}{P_c(s^t)} \right]^{-\theta} \\
u_{C,1}(s^t) &= R(s^t)(\geq 1) \\
u_{C,2}(s^t) &= \frac{P_c(s^t)}{[1 - \tau^n(s^t)]w(s^t)} \\
u_{C,1}(s^t) &= \beta R(s^t)E_t \left[ \frac{u_{C,1}(s^{t+1})}{P_c(s^{t+1})} \right] \\
Q(s^{t+1}|s^t) &= \beta \pi(s^{t+1}|s^t) \frac{u_{C,1}(s^{t+1})}{u_{C,1}(s^t)} \frac{P_c(s^t)}{P_c(s^{t+1})}
\end{align*}
\]
\[
\min_{g_i(s^t), i \in [0,1]} \int_0^1 p_i^c(s^t) g_i(s^t) \, di
\]

subject to \[
\left[ \int_0^1 g_i(s^t)^{(\theta-1)/\theta} \, di \right]^{\theta/(\theta-1)} = G(s^t)
\]

\[
\Rightarrow g_i(s^t) = G(s^t) \left[ \frac{p_i^c(s^t)}{P^c(s^t)} \right]^{-\theta}
\]

A government policy consists of \[\{g_i(s^t), \tau^c(s^t), \tau^n(s^t), R(s^t), M^g(s^t), \bar{B}^g(s^t)\}_{t=0}^{\infty}\]
Firms

Flexible Price Firms \((i \in [\alpha, 1])\):

\[
\max_{p_i(s^t)} \quad p_i(s^t)y_i(s^t) - w(s^t)n_i(s^t)
\]

subject to

\[
y_{i,t}(s^t) = \left[ \frac{p_i^{c}(s^t)}{P^c(s^t)} \right]^{-\theta} Y(s^t) \quad \& \quad y_{i,t}(s^t) = A(s^t)n_i(s^t)
\]

\[\Rightarrow p_i(s^t) = \frac{\theta}{\theta - 1} \frac{w(s^t)}{A(s^t)} \equiv p_f(s^t)\]

Sticky Price Firms \((i \in [0, \alpha])\):

\[
\max_{p_i(s^{t-1})} \quad \sum_{s^{t+1}|s^{t-1}} Q(s^{t+1}|s^{t-1})[p_i(s^{t-1})y_i(s^t) - w(s^t)n_i(s^t)]
\]

subject to the same constraints as above.

\[\Rightarrow p_i(s^{t-1}) = E_{t-1}[v(s^t)p_f(s^t)] \equiv p_s(s^{t-1})\]

At \(t=0\), they charge \(p_{-1}\) which is exogenously given.
Firms

Flexible Price Firms \((i \in [\alpha, 1])\):

\[
\max_{p_i(s^t)} \quad p_i(s^t)y_i(s^t) - w(s^t)n_i(s^t)
\]

subject to

\[
y_{i,t}(s^t) = \left[ \frac{p_i^c(s^t)}{P(s^t)} \right]^{-\theta} Y(s^t) \quad \& \quad y_{i,t}(s^t) = A(s^t)n_i(s^t)
\]

\[\Rightarrow p_i(s^t) = \frac{\theta}{\theta - 1} \frac{w(s^t)}{A(s^t)} \equiv p_f(s^t)\]

Sticky Price Firms \((i \in [0, \alpha])\):

\[
\max_{p_i(s^{t-1})} \quad \sum_{s^{t+1}|s^{t-1}} Q(s^{t+1}|s^{t-1})[p_i(s^{t-1})y_i(s^t) - w(s^t)n_i(s^t)]
\]

subject to the same constraints as above.

\[\Rightarrow p_i(s^{t-1}) = E_{t-1}[v(s^t)p_f(s^t)] \equiv p_s(s^{t-1})\]

At \(t=0\), they charge \(p_{-1}\) which is exogenously given.
Market-Clearing

\[ c_{1,i}(s^t) + c_{2,i}(s^t) + g_i(s^t) = A(s^t)n_i(s^t) \text{ for each } i. \]

\[ \Rightarrow \left[ C_1(s^t) + C_2(s^t) + G(s^t) \right] \int_0^1 \left[ \frac{p^c(s^t)}{P^c(s^t)} \right]^{-\theta} di = A(s^t)N(s^t) \]

- \( N(s^t) = \int_0^1 n_i(s^t) di \)
- \( M(s^t) = M^g(s^t) \)
- \( \bar{B}(s^t) = \bar{B}^g(s^t) \)
- \( B(s^{t+1}) = 0 \)
Competitive Equilibrium

A competitive equilibrium consists of

- **Allocations:** \( \{ C_1(s^t), C_2(s^t), N(s^t) \}_{t=0}^{\infty}, \{ c_{1,i}(s^t), c_{2,i}(s^t), n_i(s^t) \}_{t=0}^{\infty} \) for all \( i \), and \( \{ M(s^t), \bar{B}(s^t), B(s^{t+1}) \}_{t=0}^{\infty} \).
- **Prices/Policies:** \( \{ p_i(s^t), P^c(s^t), w(s^t), Q(s^{t+1}|s^t), g_i(s^t), G(s^t), \tau^c(s^t), \tau^n(s^t), M^g(s^t), \bar{B}^g(s^t), R(s^t) \}_{t=0}^{\infty} \)

such that

- the allocation solves the household’s problem given prices/policies.
- \( p_i(s^t) \) solves the firm’s problem given prices/policies.
- \( g_i(s^t) \) solves the government’s problem given \( p_i(s^t), P^c(s^t), \) and \( G(s^t) \).
- markets clear.
Two objects of interest

$\Omega_f$: The set of implementable allocations for goods and labor
$\{C_1(s^t), C_2(s^t), N(s^t)\}_{t=0}^{\infty}$ when $\alpha = 0$ (i.e., under flexible prices).

$\Omega_s(\alpha)$: The set of implementable allocations for goods and labor
$\{C_1(s^t), C_2(s^t), N(s^t)\}_{t=0}^{\infty}$ when $0 < \alpha < 1$ (i.e., under sticky prices.)
Logic: A Big Picture

Show $\Omega_f \subset \Omega_s(\alpha)$ for any $0 < \alpha < 1$

Define $\Omega_R$ ("relaxed set") such that $\Omega_f \subset \Omega_s(\alpha) \subset \Omega_R$ for any $0 < \alpha < 1$.

Show optimal allocation in $\Omega_R$ is in $\Omega_f$.

Therefore, optimal allocation in $\Omega_s(\alpha)$ is in $\Omega_f$ for any $0 < \alpha < 1$. 
Proposition 1 (Part 1): \( \Omega_f \) is characterized by

\[
E_0 \sum_{t=0}^{\infty} \beta^t \left[ u_{C,1}(s^t)C_1(s^t) + u_{C,2}(s^t)C_2(s^t) + u_N(s^t)N(s^t) \right] = 0
\]

\[
\beta^t \left[ u_{C,1}(s^t) \right] \geq u_{C,2}(s^t)
\]

\[
C_1(s^t) + C_2(s^t) + G(s^t) = A(s^t)N(s^t)
\]

Proposition 1 (Part 2): Given \( P^c(s_0) \), each allocation in \( \Omega_f \) is implemented with a unique path for \( \{ R(s^t), M^g(s^t), B^g(s^t), \frac{1 + \tau^c(s^t)}{1 - \tau^n(s^t)} \frac{w(s^t)}{p_f(s^t)}, [1 + \tau^c(s^t)]p_f(s^t) \} \)

Corollary 1: There are multiple combinations of \( \{ \tau^c(s^t), \tau^n(s^t), w(s^t), p_f(s^t) \} \) consistent with each implementable allocation. One of them supports constant \( p_f(s^t) = \bar{P} \) for any \( \bar{P} > 0 \).
Proposition 1 (Part 1): $\Omega_f$ is characterized by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ u_{C,1}(s^t) C_1(s^t) + u_{C,2}(s^t) C_2(s^t) + u_N(s^t) N(s^t) \right] = 0$$

$$u_{C,1}(s^t) \geq u_{C,2}(s^t)$$

$$C_1(s^t) + C_2(s^t) + G(s^t) = A(s^t) N(s^t)$$

Proposition 1 (Part 2): Given $P^c(s_0)$, each allocation in $\Omega_f$ is implemented with a unique path for $\{ R(s^t), M^g(s^t), \tilde{B}^g(s^t), \frac{1+\tau^c(s^t)}{1-\tau^n(s^t)}, \frac{w(s^t)}{p_f(s^t)}, [1 + \tau^c(s^t)] p_f(s^t) \}_{t=0}^{\infty}$.

Corollary 1: There are multiple combinations of $\{ \tau^c(s^t), \tau^n(s^t), w(s^t), p_f(s^t) \}$ consistent with each implementable allocation. One of them supports constant $p_f(s^t) = \bar{P}$ for any $\bar{P} > 0$. 
Proposition 2 (Part 1): $\Omega_f \subset \Omega_s(\alpha)$ for any $0 < \alpha < 1$.

Proof:
Let $p_f(s^t) = p_{-1}$. Then, $p_s(s^{t-1}) = p_f(s^t) = p_{-1}$. The equilibrium conditions of the sticky price model collapse to the ones under flexible prices, plus the restriction that $p_f(s^t) = p_{-1}$.

Proposition 2 (Part 2): Each allocation in $\Omega_f$ can be implemented with policies that are independent of $\alpha$. 
Proposition 2 (Part 1): $\Omega_f \subset \Omega_s(\alpha)$ for any $0 < \alpha < 1$.

Proof:
Let $p_f(s^t) = p_{-1}$. Then, $p_s(s^{t-1}) = p_f(s^t) = p_{-1}$. The equilibrium conditions of the sticky price model collapse to the ones under flexible prices, plus the restriction that $p_f(s^t) = p_{-1}$.

Proposition 2 (Part 2): Each allocation in $\Omega_f$ can be implemented with policies that are independent of $\alpha$. 
The relaxed set $\Omega_R$ is the set of aggregate allocations $\{C_1(s^t), C_2(s^t), N(s^t)\}_{t=0}^\infty$ such that there exist consumer prices $\{p^c_i(s^t), P^c(s^t)\}$ satisfying:

$$E_0 \sum_{t=0}^\infty \beta^t \left[ u_{C,1}(s^t)C_1(s^t) + u_{C,2}(s^t)C_2(s^t) + u_N(s^t)N(s^t) \right] = 0$$

$$u_{C,1}(s^t) \geq u_{C,2}(s^t)$$

$$\left[ C_1(s^t) + C_2(s^t) + G(s^t) \right] \int_0^1 \left[ \frac{p^c_i(s^t)}{P^c(s^t)} \right]^{-\theta} di = A(s^t)N(s^t)$$

Proposition 3 (Part 1): $\Omega_s(\alpha) \subset \Omega_R$ for any $0 < \alpha < 1$ (thus $\Omega_f \subset \Omega_R$)

Why? Any allocation in $\Omega_s(\alpha)$ satisfies the conditions above.
Proposition 3 (Part 2): Optimal allocation in $\Omega_R$ is in $\Omega_f$.

Proof: Optimal allocation is attained when $\int_0^1 \left[ \frac{p^c_i(s^t)}{P^c(s^t)} \right]^{-\theta} \, di$ is minimized. This distortion term is minimized when prices are the same across firms. When that happens, the conditions collapse to the ones under flexible prices.

Since $\Omega_f \subset \Omega_s(\alpha) \subset \Omega_R$, optimal allocation in $\Omega_s(\alpha)$ is in $\Omega_f$ for any $0 < \alpha < 1$.

Proposition 4: Policies/prices that implement the Ramsey allocation do not depend on $\alpha$. 

Proposition 3 (Part 2): Optimal allocation in $\Omega_R$ is in $\Omega_f$.

Proof: Optimal allocation is attained when
$$
\int_0^1 \left[ \frac{p_i^c(s^t)}{P^c(s^t)} \right]^{-\theta} \, di
$$
is minimized. This distortion term is minimized when prices are the same across firms. When that happens, the conditions collapse to the ones under flexible prices.

Since $\Omega_f \subset \Omega_s(\alpha) \subset \Omega_R$, optimal allocation in $\Omega_s(\alpha)$ is in $\Omega_f$ for any $0 < \alpha < 1$.

Proposition 4: Policies/prices that implement the Ramsey allocation do not depend on $\alpha$. 
Proposition 3 (Part 2): Optimal allocation in \( \Omega_R \) is in \( \Omega_f \).

Proof: Optimal allocation is attained when \( \int_0^1 \left[ \frac{p_i^c(s^t)}{P^c(s^t)} \right]^{-\theta} di \) is minimized. This distortion term is minimized when prices are the same across firms. When that happens, the conditions collapse to the ones under flexible prices.

Since \( \Omega_f \subset \Omega_s(\alpha) \subset \Omega_R \), optimal allocation in \( \Omega_s(\alpha) \) is in \( \Omega_f \) for any \( 0 < \alpha < 1 \).

Proposition 4: Policies/prices that implement the Ramsey allocation do not depend on \( \alpha \).