Optimal Fiscal and Monetary Policy: Equivalence Results

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Introduction

Question: How does sticky price affect the conduct of fiscal/monetary policy?

- A model in which a fraction α of firms set prices one period in advance.
- One-period non-state-contingent debt.
- Labor income and consumption tax.

Answer: Not at all.

- Ramsey allocations are the same under sticky prices and under flexible prices.
- Policies that implement the Ramsey allocations are also the same.

Environment

- Discrete Time, Infinite Horizon.
- $s^t \equiv (s_0, s_1, s_2, ..., s_t)$ where $s_t \in S_t$

3 types of agents:

- A representative household.
- Government.
- A continuum of firms, indexed by $i \in [0, 1]$:
  - Firms $i \in [0, \alpha)$: sticky-price firms that set their prices one period in advance.
  - Firms $i \in [\alpha, 1]$: flexible-price firms that set their price contemporaneously.
  - $y_{i,t}(s^t) = A(s^t) n_i(s^t)$

3 goods:

- $C_k(s^t) = \left[\int_0^1 c_{k,i}(s^t)^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)}, (k = 1, 2)$
- $G(s^t) = \left[\int_0^1 g_i(s^t)^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)}$

Household

$$\max_{\{c_{1,i}(s^t), c_{2,i}(s^t), N(s^t), M(s^t), \bar{B}(s^t), B(s^{t+1})\}_{t=0}^{\infty}, i \in [0,1]} \quad \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(C_1(s^t), C_2(s^t), N(s^t))$$

subject to

$$P^c(s^t) C_1(s^t) \leq M(s^t)$$
$$M(s^t) + \bar{B}(s^t) + \sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t) B(s^{t+1}) \leq \mathbf{W}(s^t)$$

where

$$\begin{aligned}
\mathbf{W}(s^{t+1}) &\equiv M(s^t) + R(s^t)\bar{B}(s^t) + B(s^{t+1}) - \int_0^1 p_i^c(s^t) c_{1,i}(s^t) di \\
&\quad - \int_0^1 p_i^c(s^t) c_{2,i}(s^t) di + [1 - \tau^n(s^t)] w(s^t) N(s^t) \\
p_i^c(s^t) &= [1 + \tau^c(s^t)] p_i(s^t) \\
P^c(s^t) &= \left[\int_0^1 [p_i^c(s^t)]^{(1-\theta)} di \right]^{1/(1-\theta)} \\
\mathbf{W}(s^0) &= 0
\end{aligned}$$

Household FONCs

$$
\begin{aligned}
\frac{c_{1,i}(s^t)}{C_1(s^t)} &= \Big[\frac{p_i^c(s^t)}{P^c(s^t)}\Big]^{-\theta} \quad \& \quad \frac{c_{2,i}(s^t)}{C_2(s^t)} = \Big[\frac{p_i^c(s^t)}{P^c(s^t)}\Big]^{-\theta} \\
\frac{u_{C,1}(s^t)}{u_{C,2}(s^t)} &= R(s^t)(\geq 1) \\
-\frac{u_{C,2}(s^t)}{u_N(s^t)} &= \frac{P^c(s^t)}{[1-\tau^n(s^t)]w(s^t)} \\
\frac{u_{C,1}(s^t)}{P^c(s^t)} &= \beta R(s^t) E_t\left[\frac{u_{C,1}(s^{t+1})}{P^c(s^{t+1})}\right] \\
Q(s^{t+1}|s^t) &= \beta\pi(s^{t+1}|s^t)\frac{u_{C,1}(s^{t+1})}{u_{C,1}(s^t)}\frac{P^c(s^t)}{P^c(s^{t+1})}
\end{aligned}
$$

Government

$$\min_{g_i(s^t), i\in[0,1]} \quad \int_0^1 p_i^c(s^t) g_i(s^t) di$$

$$\text{subject to } \left[\int_0^1 g_i(s^t)^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)} = G(s^t)$$

$$\Rightarrow g_i(s^t) = G(s^t)\left[\frac{p_i^c(s^t)}{P^c(s^t)}\right]^{-\theta}$$

A government policy consists of $\{g_i(s^t), \tau^c(s^t), \tau^n(s^t), R(s^t), M^g(s^t), \bar{B}^g(s^t)\}_{t=0}^{\infty}$

Firms

Flexible Price Firms ($i \in [\alpha, 1]$):

$$\max_{p_i(s^t)} \quad p_i(s^t)y_i(s^t) - w(s^t)n_i(s^t)$$

subject to

$$y_{i,t}(s^t) = \left[\frac{p_i^c(s^t)}{P^c(s^t)}\right]^{-\theta} Y(s^t) \quad \& \quad y_{i,t}(s^t) = A(s^t)n_i(s^t)$$

$$\Rightarrow p_i(s^t) = \frac{\theta}{\theta-1}\frac{w(s^t)}{A(s^t)} \equiv p_f(s^t)$$

Sticky Price Firms ($i \in [0, \alpha)$):

$$\max_{p_i(s^{t-1})} \quad \sum_{s^{t+1}|s^{t-1}} Q(s^{t+1}|s^{t-1})[p_i(s^{t-1})y_i(s^t) - w(s^t)n_i(s^t)]$$

subject to the same constraints as above.

$$\Rightarrow p_i(s^{t-1}) = E_{t-1}[v(s^t)p_f(s^t)] \equiv p_s(s^{t-1})$$

At t=0, they charge p_{-1} which is exogenously given.

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Market-Clearing

- $c_{1,i}(s^t) + c_{2,i}(s^t) + g_i(s^t) = A(s^t)n_i(s^t)$ for each i.
  $$\Rightarrow \left[C_1(s^t) + C_2(s^t) + G(s^t)\right] \int_0^1 \left[\frac{p_i^c(s^t)}{P^c(s^t)}\right]^{-\theta} di = A(s^t)N(s^t)$$
- $N(s^t) = \int_0^1 n_i(s^t)di$
- $M(s^t) = M^g(s^t)$
- $\bar{B}(s^t) = \bar{B}^g(s^t)$
- $B(s^{t+1}) = 0$

Competitive Equilibrium

A competitive equilibrium consists of

- Allocations: $\{C_1(s^t), C_2(s^t), N(s^t)\}_{t=0}^{\infty}$, $\{c_{1,i}(s^t), c_{2,i}(s^t), n_i(s^t)\}_{t=0}^{\infty}$ for all i, and $\{M(s^t), \bar{B}(s^t), B(s^{t+1})\}_{t=0}^{\infty}$.
- Prices/Policies: $\{p_i(s^t), P^c(s^t), w(s^t), Q(s^{t+1}|s^t),$ $g_i(s^t), G(s^t), \tau^c(s^t), \tau^n(s^t), M^g(s^t), \bar{B}^g(s^t), R(s^t)\}_{t=0}^{\infty}$

such that

- the allocation solves the household's problem given prices/policies.
- $p_i(s^t)$ solves the firm's problem given prices/policies.
- $g_i(s^t)$ solves the government's problem given $p_i(s^t), P^c(s^t)$, and $G(s^t)$.
- markets clear.

Two objects of interest

Ω_f: The set of implementable allocations for goods and labor $\{C_1(s^t), C_2(s^t), N(s^t)\}_{t=0}^{\infty}$ when $\alpha = 0$ (i.e., under flexible prices).

$\Omega_s(\alpha)$: The set of implementable allocations for goods and labor $\{C_1(s^t), C_2(s^t), N(s^t)\}_{t=0}^{\infty}$ when $0 < \alpha < 1$ (i.e., under sticky prices.)

Logic: A Big Picture

Show $\Omega_f \subset \Omega_s(\alpha)$ for any $0 < \alpha < 1$

Define Ω_R ("relaxed set") such that $\Omega_f \subset \Omega_s(\alpha) \subset \Omega_R$ for any $0 < \alpha < 1$.

Show optimal allocation in Ω_R is in Ω_f.

Therefore, optimal allocation in $\Omega_s(\alpha)$ is in Ω_f for any $0 < \alpha < 1$.

Ω_f

Proposition 1 (Part 1): Ω_f is characterized by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u_{C,1}(s^t) C_1(s^t) + u_{C,2}(s^t) C_2(s^t) + u_N(s^t) N(s^t) \right] = 0$$

$$u_{C,1}(s^t) \geq u_{C,2}(s^t)$$

$$C_1(s^t) + C_2(s^t) + G(s^t) = A(s^t) N(s^t)$$

Proposition 1 (Part 2): Given $P^c(s_0)$, each allocation in Ω_f is implemented with a unique path for $\{R(s^t), M^g(s^t), \bar{B}^g(s^t), \frac{1+\tau^c(s^t)}{1-\tau^n(s^t)}, \frac{w(s^t)}{p_f(s^t)}, [1+\tau^c(s^t)]p_f(s^t)\}_{t=0}^{\infty}$.

Corollary 1: There are multiple combinations of $\{\tau^c(s^t), \tau^n(s^t), w(s^t), p_f(s^t)\}$ consistent with each implementable allocation. One of them supports constant $p_f(s^t) = \bar{P}$ for any $\bar{P} > 0$.

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$\Omega_s(\alpha)$

Proposition 2 (Part 1): $\Omega_f \subset \Omega_s(\alpha)$ for any $0 < \alpha < 1$.

Proof:
Let $p_f(s^t) = p_{-1}$. Then, $p_s(s^{t-1}) = p_f(s^t) = p_{-1}$. The equilibrium conditions of the sticky price model collapse to the ones under flexible prices, plus the restriction that $p_f(s^t) = p_{-1}$.

Proposition 2 (Part 2): Each allocation in Ω_f can be implemented with policies that are independent of α.

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Ω_R

The relaxed set Ω_R is the set of aggregate allocations $\{C_1(s^t), C_2(s^t), N(s^t)\}_{t=0}^{\infty}$ such that there exist consumer prices $\{p_i^c(s^t), P^c(s^t)\}$ satisifying:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u_{C,1}(s^t) C_1(s^t) + u_{C,2}(s^t) C_2(s^t) + u_N(s^t) N(s^t) \right] = 0$$

$$u_{C,1}(s^t) \geq u_{C,2}(s^t)$$

$$\left[C_1(s^t) + C_2(s^t) + G(s^t) \right] \int_0^1 \left[\frac{p_i^c(s^t)}{P^c(s^t)} \right]^{-\theta} di = A(s^t) N(s^t)$$

Proposition 3 (Part 1): $\Omega_s(\alpha) \subset \Omega_R$ for any $0 < \alpha < 1$ (thus $\Omega_f \subset \Omega_R$)
Why? Any allocation in $\Omega_s(\alpha)$ satisfies the conditions above.

Ramsey allocations and policies

Proposition 3 (Part 2): Optimal allocation in Ω_R is in Ω_f.

Proof: Optimal allocation is attained when $\int_0^1 \left[\frac{p_i^c(s^t)}{P^c(s^t)}\right]^{-\theta} di$ is minimized. This distortion term is minimized when prices are the same across firms. When that happens, the conditions collapse to the ones under flexible prices.

Since $\Omega_f \subset \Omega_s(\alpha) \subset \Omega_R$, optimal allocation in $\Omega_s(\alpha)$ is in Ω_f for any $0 < \alpha < 1$.

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