Learning while Voting: Determinants of Collective Experimentation

Strulovici, Econometrica 2010
presented by Shengxing Zhang

April 2011

Motivation

- Reforms with consequences on
  - individuals in the society unknown and heterogeneous
  - social welfare unknown
- How do incentives evolve as preferences and heterogeneity get revealed?
- How do collective decision rule affect efficiency?

Setup

- continuous time
- choice of experimentation must be made collectively
- a two-armed bandit problem:
  - agent i's flow utility: $d\pi^i_j(t)$, $j \in \{R, S\}$
  - safe arm S: flow payoff $d\pi^i_S(t) = sdt$
  - risky arm R: can be of two types, individuals randomly assigned to B or G type
    - $d\pi^i_R(t) = hdZ^i(t)$, if $i \in G$, $h > 0$
    - $d\pi^i_R(t) = 0$, if $i \in B$
  - $\{Z^i\}_{1 \leq i \leq N}$ is a family of independent Poisson processes with intensity λ
  - $g = \lambda h > s > 0$
- belief p: probability that R is good
  - common prior p_0
  - belief updating: no news is bad news

$$\dot{p} = -\lambda p(1-p)$$

- news is public

plan of presentation

- single decision maker's choice
- social planner's choice
- social choice under majority rule
- designing voting procedures

Singe decision maker's choice – problem

Definition of collective decision rule $D = \{D_t\}_{t \geq 0}$:

- takes value in action space $\{R, S\}$
- adapted to the filtration generated by payoffs

Single decision maker's problem:

$$u(p_0) = \max_{D_t} E_0 \left[\int_0^\infty e^{-rt} d\pi_{D_t}(t) \right]$$

subject to:

$$\begin{aligned} dp_t = -\lambda p(1-p)dt \quad & \text{if} \quad D_t = R, d\pi_R(t) = 0 \\ dp_t = 1 - p_t \quad & \text{if} \quad D_t = R, d\pi_R(t) = h \\ dp_t = 0 \quad & \text{if} \quad D_t = S \end{aligned}$$

Single decision maker's choice – solution

- can be solved by Markov strategy
- state variable: belief p
- $D(p) = R$ iff $ru(p) = pg + \lambda p(g/r - u(p)) + \dot{p}u'(p) > s$
- cutoff belief: experiment iff $p > p^{SD}$
- value matching and smooth pasting at p^{SD}: $u(p^{SD}) = s/r, u'(p^{SD}) = 0$
- $p^{SD}g + \lambda p^{SD}(g - s)/r = s$, where $g = h\lambda$.
- value of the option of experimentation:

$$p^{SD} < p^{M} = s/g$$

Collective Choice

Definition of collective decision rule $C = \{C_t\}_{t \geq 0}$:

- takes value in action space $\{R, S\}$
- adapted to the filtration generated by payoffs

Individual's value function:

$$V_t^{i,C} = E_t \left[\int_t^{\infty} e^{-r(\tau - t)} d\pi^i_{C_\tau}(\tau) \right]$$

Control sharing leads to worse outcome than that of single decision makers:

- expected *loser trap* under C

$$L_t^{i,C} = E_t \left[\int_t^{\infty} e^{-r\tau - t} 1_{\{C_t = R, D_t^i = S\}} \cdot (d\pi^i_R(t) - sdt) \right]$$

- expected *winner frustration* under C

$$W_t^{i,C} = E_t \left[\int_t^{\infty} e^{-r\tau - t} 1_{\{C_t = S, D_t^i = R\}} \cdot (sdt - d\pi^i_R(t)) \right]$$

Collective choice: utilitarian rule / efficient social choice

- two groups: "sure winners" and "unsure voters"
- state variable: number of "sure winners", k, belief, p.
- $C(k,p) = R$ iff

$$\begin{aligned} rV^C(k,p) &= \left(\tfrac{k}{N}\right)g + \left(1-\tfrac{k}{N}\right)pg + (N-k)\lambda p\left[\tfrac{V^C(k+1,p)}{N} - \tfrac{s}{r}\right] \\ &+\dot{p}\tfrac{\partial}{\partial p}V^C(k,p) > s \end{aligned}$$

- cutoff belief conditional on k, $q(k)$, solves

$$\left(\frac{k}{N}\right)g + \left(1-\frac{k}{N}\right)pg + (N-k)\lambda p\left[\frac{V^C(k+1,p)}{N} - \frac{s}{r}\right] = s$$

- $q(k)$ is decreasing in k for all $k < \frac{s}{g}N$ and is equal to zero for $k \geq \frac{s}{g}N$
- k increases monotonically, we can use backward induction to solve the problem

Collective choice: voting rule

- voting rule: $C_t(\{d_t^i\}_i) \in \{R, S\}$, $d^i = \{d_t^i\}_{t\geq 0}$ is adapted to the filtration of individual's random payoff.
- consider undominated Markov strategy
- For "sure winners": always vote for R
- For "unsure voters":
  - state variable: belief p and the set of sure winners, K.
  - $d^i(K, p) = R$ iff

$$rV^{i,C}(p,K) = pg + \dot{p}\tfrac{\partial}{\partial p}V^{i,C}(p,K)$$
$$+\lambda p \textstyle\sum_{j\notin K}\left(V^{i,C}(K\bigcup\{j\},p) - V^{i,C}(K,p)\right) > s$$

  - optimal stopping rule $q(|K|)$,characterized by

$$pg + \lambda p \sum_{j\notin K}\left(V^{i,C}(K\bigcup\{j\},p) - V^{i,C}(K,p)\right) = s$$

Majoritarian voting rule – comparison

- cutoff belief: $p(k)$ for majoritarian, $q(k)$ for utilitarian, q^{SD} for single decision maker
- $p(k)g + (N-k)\lambda p(k) \left[\frac{\bar{w}(k+1,p(k))}{N-k} + \frac{N-k-1}{N-k}\bar{u}(k+1,p(k)) - \frac{s}{r}\right] = s$
- $\left(\frac{k}{N}\right)g + \left(1-\frac{k}{N}\right)q(k)g + (N-k)\lambda q(k)\left[\frac{V^C(k+1,q(k))}{N} - \frac{s}{r}\right] = s$
- $p^{SD}g + \lambda p^{SD}(g-s)/r = s$
- majoritarian vs utilitarian rules vs single-decision-maker rule:

$$p^{SD} < q(k) \leq p(k) < p^M$$

Majoritarian rule – equilibrium characterization

Let $k_N = (N-1)/2$, under simple majority rule, there exists a unique Markov equilibrium in undominated strategies such that

- $p(k)$ is nonincreasing in k for all $k < k_N$
- $\bar{u}(k,p)$ and $\bar{w}(k,p)$ are nondecreasing in p
- $\bar{w}(k,p)$ is nondecreasing in k for all p
- $\bar{u}(k,p)$ is increasing in k for all p, and $k < k_N$
- $\bar{u}(k_N+1,p) < u(k_N,p)$ for all p
- $\bar{u}(k,p) = pg/r$ and $\bar{w}(k,p) = g/r$ for all p and $k > k_N$

Design of voting procedures

- Suppose R is elected at t iff it gets v_t votes.
- There exists a deterministic quorum function v_t such that the unique Markov equilibrium in undominated strategies implements the utilitarian policy
- v_t is increasing and determined by initial belief p_0 and utilitarian cutoff $q(k)$.
- problem of commitment

Robust finding

- collective experimentation is shorter than single decision maker equivalent
- collective experimentation is shorter than utilitarian equilibrium
- there always exists some experimentation