Learning while Voting: 
Determinants of Collective Experimentation

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Motivation

- Reforms with consequences on
  - individuals in the society unknown and heterogeneous
  - social welfare unknown
- How do incentives evolve as preferences and heterogeneity get revealed?
- How do collective decision rule affect efficiency?
Setup

- continuous time
- choice of experimentation must be made collectively
- a two-armed bandit problem:
  - agent $i$’s flow utility: $d\pi^i_j(t), j \in \{R, S\}$
  - safe arm $S$: flow payoff $d\pi^i_S(t) = s dt$
  - risky arm $R$: can be of two types, individuals randomly assigned to $B$ or $G$ type
    - $d\pi^i_R(t) = h dZ^i(t)$, if $i \in G$, $h > 0$
    - $d\pi^i_R(t) = 0$, if $i \in B$
  - $\{Z^i\}_{1 \leq i \leq N}$ is a family of independent Poisson processes with intensity $\lambda$
    - $g = \lambda h > s > 0$
  - belief $p$: probability that $R$ is good
    - common prior $p_0$
    - belief updating: no news is bad news
      $$\dot{p} = -\lambda p(1 - p)$$
  - news is public
plan of presentation

- single decision maker’s choice
- social planner’s choice
- social choice under majority rule
- designing voting procedures
Singe decision maker’s choice – problem

Definition of collective decision rule $D = \{D_t\}_{t \geq 0}$:

- takes value in action space $\{R, S\}$
- adapted to the filtration generated by payoffs

Single decision maker’s problem:

$$u(p_0) = \max_{D_t} E_0 \left[ \int_0^\infty e^{-rt} d\pi_{D_t}(t) \right]$$

subject to:

$$dp_t = -\lambda p(1 - p) dt \quad \text{if} \quad D_t = R, \; d\pi_R(t) = 0$$
$$dp_t = 1 - p_t \quad \text{if} \quad D_t = R, \; d\pi_R(t) = h$$
$$dp_t = 0 \quad \text{if} \quad D_t = S$$
Single decision maker’s choice – solution

- can be solved by Markov strategy
- state variable: belief $p$
- $D(p) = R$ iff $ru(p) = pg + \lambda p(g/r - u(p)) + \dot{pu}'(p) > s$
- cutoff belief: experiment iff $p > p^{SD}$
- value matching and smooth pasting at $p^{SD}$: $u(p^{SD}) = s/r$, $u'(p^{SD}) = 0$
- $p^{SD}g + \lambda p^{SD}(g - s)/r = s$, where $g = h\lambda$.
- value of the option of experimentation:

$$p^{SD} < p^{M} = s/g$$
Collective Choice

Definition of collective decision rule $C = \{C_t\}_{t \geq 0}$:
• takes value in action space $\{R, S\}$
• adapted to the filtration generated by payoffs

Individual’s value function:

$$V_{t}^{i, C} = E_t \left[ \int_{t}^{\infty} e^{-r(\tau-t)} d\pi_t^i(\tau) \right]$$

Control sharing leads to worse outcome than that of single decision makers:
• expected loser trap under $C$

$$L_{t}^{i, C} = E_t \left[ \int_{t}^{\infty} e^{-r\tau-t} 1_{\{C_t=R, D_t=S\}} \cdot (d\pi_t^R(t) - sdt) \right]$$

• expected winner frustration under $C$

$$W_{t}^{i, C} = E_t \left[ \int_{t}^{\infty} e^{-r\tau-t} 1_{\{C_t=S, D_t=R\}} \cdot (sdt - d\pi_t^R(t)) \right]$$
Collective choice: utilitarian rule / efficient social choice

- two groups: "sure winners" and "unsure voters"
- state variable: number of "sure winners", $k$, belief, $p$.
- $C(k, p) = R$ iff

$$rV^C(k, p) = \left( \frac{k}{N} \right) g + \left( 1 - \frac{k}{N} \right) pg + (N - k)\lambda p \left[ \frac{V^C(k+1, p)}{N} - \frac{s}{r} \right]$$

$$+ \dot{p} \frac{\partial}{\partial p} V^C(k, p) > s$$

- cutoff belief conditional on $k$, $q(k)$, solves

$$\left( \frac{k}{N} \right) g + \left( 1 - \frac{k}{N} \right) pg + (N - k)\lambda p \left[ \frac{V^C(k+1, p)}{N} - \frac{s}{r} \right] = s$$

- $q(k)$ is decreasing in $k$ for all $k < \frac{s}{g} N$ and is equal to zero for $k \geq \frac{s}{g} N$
- $k$ increases monotonically, we can use backward induction to solve the problem
Collective choice: voting rule

- voting rule: $C_t(\{d^i_t\}_i) \in \{R, S\}$, $d^i = \{d^i_t\}_{t \geq 0}$ is adapted to the filtration of individual’s random payoff.
- consider undominated Markov strategy
- For ”sure winners”: always vote for $R$
- For ”unsure voters”:
  - state variable: belief $p$ and the set of sure winners, $K$.
  - $d^i(K, p) = R$ iff
    \[
    rV^{i, C}(p, K) = pg + \hat{p} \frac{\partial}{\partial p} V^{i, C}(p, K) \\
    + \lambda p \sum_{j \notin K} \left( V^{i, C}(K \cup \{j\}, p) - V^{i, C}(K, p) \right) > s
    \]
- optimal stopping rule $q(|K|)$, characterized by
  \[
  pg + \lambda p \sum_{j \notin K} \left( V^{i, C}(K \cup \{j\}, p) - V^{i, C}(K, p) \right) = s
  \]
Majoritarian voting rule – comparison

- cutoff belief: $p(k)$ for majoritarian, $q(k)$ for utilitarian, $q^{SD}$ for single decision maker
  
  \[ p(k)g + (N - k)\lambda p(k) \]

  \[ \left[ \frac{\bar{w}(k+1,p(k))}{N-k} + \frac{N-k-1}{N-k} \bar{u}(k + 1, p(k)) - \frac{s}{r} \right] = s \]

  \[ (\frac{k}{N})g + (1 - \frac{k}{N}) q(k)g + (N - k)\lambda q(k) \]

  \[ \left[ \frac{V^C(k+1,q(k))}{N} - \frac{s}{r} \right] = s \]

  \[ p^{SD}g + \lambda p^{SD}(g - s)/r = s \]

- majoritarian vs utilitarian rules vs single-decision-maker rule:
  \[ p^{SD} < q(k) \leq p(k) < p^M \]
Let $k_N = (N - 1)/2$, under simple majority rule, there exists a unique Markov equilibrium in undominated strategies such that

- $p(k)$ is nonincreasing in $k$ for all $k < k_N$
- $\bar{u}(k, p)$ and $\bar{w}(k, p)$ are nondecreasing in $p$
- $\bar{w}(k, p)$ is nondecreasing in $k$ for all $p$
- $\bar{u}(k, p)$ is increasing in $k$ for all $p$, and $k < k_N$
- $\bar{u}(k_N + 1, p) < u(k_N, p)$ for all $p$
- $\bar{u}(k, p) = pg / r$ and $\bar{w}(k, p) = g / r$ for all $p$ and $k > k_N$
Design of voting procedures

- Suppose $R$ is elected at $t$ iff it gets $v_t$ votes.
- There exists a deterministic quorum function $v_t$ such that the unique Markov equilibrium in undominated strategies implements the utilitarian policy.
- $v_t$ is increasing and determined by initial belief $p_0$ and utilitarian cutoff $q(k)$.
- Problem of commitment.
Robust finding

- collective experimentation is shorter than single decision maker equivalent
- collective experimentation is shorter than utilitarian equilibrium
- there always exists some experimentation