Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard

Prescott, Townsend, Econometrica 1984
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Questions

Two general questions which interest me in the paper:

- What are the mechanisms offered by competing principles?
- Do the equilibria achieve pareto optima?

Environment

This presentation focuses on a private information labor market.

- Timing: planning period and execution period ($t \in \{0, 1\}$)
- Players: consumers, manufacturing firms, and brokers.
- One consumption good.
- Production technology: $y = al$.
- Each broker and firm host infinitely many consumers.
- Consumer preference: $U_\theta(c, l)$ is concave in consumption c and labor supply l, θ is a preference shock at $t = 1$.
- Consumers have no initial endowment.

Timing

- Planning period ($t = 0$):
    - Consumers choose and sign contracts with a broker.
    - Brokers purchase contracts from firms, repackage them and then offer the package to consumers. They compete by offering favorable terms.
    - A competitive market for contingent contracts between firms and brokers.
- Execution period ($t = 1$):
    - Nature assigns independently type θ to consumers. $\theta \in \{1, 2\}$. Population of type θ agent is λ_θ.
    - production takes place.
    - contracts carried out.

Representation for allocations

To allow for random mechanisms, we express allocations in terms of probability distributions, The support of the distributions is discrete $(c, l) \in \{(c_1, l_1), (c_2, l_2), \ldots, (c_n, l_n)\} = L$. And for all $(c, l) \in L$

- Consumption: $x_\theta(c, l)$, type dependent.
- Endowment: $\xi(c, l)$, which assigns probability 1 to $(0, 0)$.
- production: $y(c, l)$.

Assume for now that there exist prices contingent on type θ, consumption c and labor supply l. Denote the price by $p_\theta(c, l)$.

Lotteries and Convexity of Feasible Contract Set

- $U_\theta(\tilde{c}_{\theta'}, \tilde{l}_{\theta'}) = \sum_{(c,l)} x_{\theta'}(c,l) U_\theta(c,l)$, a linear functional of $x_{\theta'}$.
- the set of feasible contracts are convexified.

$\forall x^1 = (x^1_\theta)_{\theta \in \{1,2\}}, x^2 = (x^2_\theta)_{\theta \in \{1,2\}} \in S_{IC}$,

$$U_\theta \cdot x^1_\theta \geq U_\theta \cdot x^1_{\theta'}$$

$$U_\theta \cdot x^2_\theta \geq U_\theta \cdot x^2_{\theta'}$$

$$\therefore U_\theta \cdot \left(\lambda x^1_\theta + (1-\lambda) x^2_\theta\right) \quad \geq \quad U_\theta \cdot \left(\lambda x^1_{\theta'} + (1-\lambda) x^2_{\theta'}\right)$$

Then $\lambda x^1 + (1-\lambda) x^2 \in S_{IC}$, $\forall \lambda \in (0,1)$

Pareto Optima

$$\max_{x_\theta(c,l)\geq 0} \sum_\theta \lambda_\theta x_\theta(c,l) U_\theta(c,l)$$

s.t.

$$\begin{aligned}
\textstyle\sum_{(c,l)} x_1(c,l)U_1(c,l) &\geq \textstyle\sum_{(c,l)} x_2(c,l)U_1(c,l) && \text{IC1} \\
\textstyle\sum_{(c,l)} x_2(c,l)U_2(c,l) &\geq \textstyle\sum_{(c,l)} x_1(c,l)U_2(c,l) && \text{IC2} \\
\textstyle\sum_\theta \lambda_\theta \sum_{(c,l)} x_\theta(c,l)al &\geq \textstyle\sum_\theta \lambda_\theta \sum_{(c,l)} x_\theta(c,l)c && \text{feasibility} \\
\textstyle\sum_{(c,l)} x_\theta(c,l) &= 1, \forall\theta \in \{1,2\}
\end{aligned}$$

- A linear programming problem with convex constraint.
- Shadow prices for contingent option, $x_\theta(c,l)$, can be solved.
- Pareto optima for other economies can be solved in a similar way using Negishi Algorithm.

Contract Structure

- Direct mechanism offered by a broker specifies:
  - report of information from agents
  - recommended actions based on agents' report
  - payoff that depends on reported information, recommended actions, actual information and actual actions.
- Recommendations on unverifiable actions and reports should be incentive compatible.

Contract Structure - Cont'd

- The only unverifiable information here is a preference shock at $t = 1$. So the contract
  - is shock contingent and contains two mutually exclusive options of allocation on consumption and labor supply
  - allows consumers to select the option contingent on their preference shock at $t = 1$
- The contract can be summarized by $\{(\tilde{c}_\theta, \tilde{l}_\theta)\}_{\theta \in \{1,2\}}$, where $(\tilde{c}_\theta, \tilde{l}_\theta)$ can be random variables.

Consumers' problem

$$\max_{x \in \bar{X}} \sum_{\theta} \lambda_{\theta} \sum_{(c,l)} x_{\theta}(c,l) U_{\theta}(c,l)$$

$$\bar{X} = \left\{ (x_{\theta}(\cdot,\cdot))_{\theta=1}^{2} : \sum_{\theta} \sum_{(c,l)} p_{\theta}(c,l) x_{\theta}(c,l) \leq \sum_{\theta} \sum_{(c,l)} p_{\theta}(c,l) \xi_{\theta}(c,l) \right\}$$

- Exclusiveness: ex post, contracts restrict agents to only consumption bundles $(x_1, 0)$,or $(0, x_2)$.

Brokers' problem

Brokers sell contracts with shock-contingent options to consumers and buy commitments from firms.

- Expenditure: $\sum_\theta \sum_{(c,l)} p_\theta(c,l) y(c,l)$
- Revenue: $\sum_\theta \sum_{(c,l)} p_\theta(c,l) \left(x(c,l) + \xi(c,l)\right)$

Brokers compete with each other, which implies:

- Pareto optimality
- Zero profit: Revenue = Expenditure.

Functions of brokers:

- Generate information for firms by correlating beliefs with securities provided.
- Make sure the exclusiveness condition is carried out.

Firms' problem

- Firms sell to brokers commitments to employing type θ consumer to produce al units of output and deliver c units of consumption.
- Firms' objective:

$$\max_{\{y_\theta(c,l)\}\in Y} \sum_\theta \sum_{(c,l)} y_\theta(c,l) p_\theta(c,l)$$

$$Y = \left\{ y = \{y_\theta(c,l)\} : \sum_\theta \lambda_\theta \sum_{(c,l)} y_\theta(c,l)(al - c) \geq 0 \right\}$$

- not hard to guess: $p_\theta(c,l) = \lambda_\theta(c - al)$.

Competitive Equilibrium

- A *competitive equilibrium* given endowment ξ, is allocation bundle $((x_\theta^*, y^*))$ and a price vector p^* for which
  - (i) given p^*, (x_θ^*) solves households' problem.
  - (ii) given p^*, y^* solves firms' problem.
  - (iii) given p^*, brokers' problem.
  - (iv) markets clear: $\sum_\theta \lambda_\theta x_\theta^* = y^* + \xi$.
- Brokers can be implicit in the equilibrium, as long as the assumption on exclusiveness is maintained. Condition (iii) can be omitted. Not in general true.

Decentralization under Adverse Selection at $t = 0$

- Similar to the situation at $t = 1$ in the current setup.
  At $t = 0$:

$$\sum_{\theta} \sum_{(c,l)} p_\theta(c,l) x_\theta(c,l) = \sum_{\theta} \sum_{(c,l)} p_\theta(c,l) \xi_\theta(c,l)$$

  At $t = 1$:

$$\sum_{(c,l)} p_\theta(c,l) x_\theta(c,l) = \sum_{(c,l)} p_\theta(c,l) \xi_\theta(c,l) + t(\theta), \forall \theta$$

$$\sum_{\theta} \lambda_\theta t(\theta) = 0$$

- In general, $t(\theta) \neq 0$. So CE solution here does not apply to the case with ex ante asymmetric information.
- One solution: Bisin Gottardi (2006) on efficient CE with adverse selection.

Are convexities and lotteries necessary?

- tools used in the proof for competitive equilibrium: separating hyperplane theorem.
- tools from cooperative game theory:
  - Core under complete information: Townsend (1978).
  - Cooperative game and core under asymmetric information for exchange economy: Forges, Mertens, Vohra, (2002).
- Green: repeated game approach, grim-trigger strategy...