Bubbles and Crashes

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Introduction

- The goal of this paper is to show that:
  - even in the presence of rational arbitrageurs a bubble may exists for a long time;
  - bubble may exists even though enough agents are aware of it and would be able to burst it if they act collectively;
  - lack common knowledge assumption is crucial for existence of bubbles;
  - in such environment, news may have disproportional impact on market compared to their information content.

Model (Price Process)

- Price process is assumed to be exogenous;
- At time $t = 0$ the price is $p_0 = 1$;
- From $t = 0$ onwards, price grows at rate g, $p_t = e^{gt}$;
- Until the time t_0 the price is justified by fundamentals;
- For any $t > t_0$, only a fraction $1 - \beta\left(t - t_0\right)$ of the price is justified by fundamentals;
- t_0 is a random variable with exponential distribution, $\Phi\left(t_0\right) = 1 - e^{-\lambda t_0}$;
- The price process is assumed to be driven by behavioral traders;

Model (Price Process)

- The bubble will burst for exogenous reasons at $t_0 + \overline{\tau}$;
  - in that case the price drops to $\left(1 - \overline{\beta}\right) p\left(t\right);$
- Or if the selling pressure exceeds level κ.
  - in this case the price drops to $\left(1 - \beta\left(t - t_0\right)\right) p\left(t\right);$

Model (Arbitrageurs)

- There is mass 1 of rational, risk-neutral agents (arbitrageurs);
- These arbitrageurs become sequentially aware of the bubble;
- At each $t_0 < t < t_0 + \eta$ a mass $\frac{1}{\eta}$ of arbitrageurs becomes aware of mispricing;
- Arbitrageurs do not observe t_0 and don't know how many of others observed signal;

Model (Arbitrageurs)

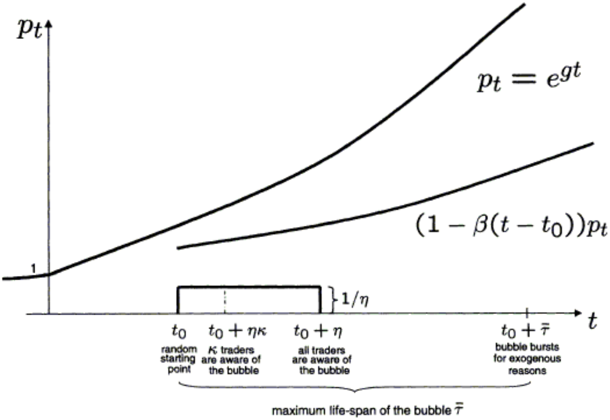
- Type of arbitrageur is $t_i \in [t_0, t_0 + \eta]$, the time he received the news;
- Arbitrageurs choose their holding of a stock in $[0, 1]$, where 1 is the maximum short position they can take;
- Every time an arbitrageur changes his position he incurs a cost ce^{rt};
- We refer to arbitrageurs position as his selling pressure.
- $\Pi(t|t_i)$ is the belief of type t_i that the bubble burst before time t.

Model (Strategies)

- Let $\sigma(t, t_i)$ denote the selling pressure of arbitrageur t_i at time t.
- A strategy of an arbitrageur i is given by $\sigma(\cdot, t_i) : [0, t_i + \overline{\tau}] \rightarrow [0, 1]$;
- The aggregate selling pressure is $s(t, t_0) = \int_{t_0}^{\min\{t, t_0 + \overline{\tau}\}} \sigma(t, t_i)\, dt_i$
- We say that there is a bubble if κ of arbitrageurs are aware of mispricing;
  - this happens at time $t_0 + \kappa\eta$;
- The time that bubble burst is then

$$T^*(t_0) = \inf\{t | s(t, t_0) \geq \kappa \text{ or } t = t_0 + \overline{\tau}\}$$

Summary of the model



Trading equilibrium and Preliminary Results

Definition

A trading equilibrium is a Perfect Bayesian Equilibrium such that if $\sigma(t, t_i) > 0$ arbitrageur t_i believes $\sigma(t, t_j) > 0$ for all $t_j < t_i$.

Lemma

1. $\sigma(t, t_i) \in \{0, 1\}$
2. $\sigma(t, t_i) = 1 \Rightarrow \sigma(t, t_j) = 1$ *for all* $t_j < t_i$ *and* $\sigma(t, t_i) = 0 \Rightarrow \sigma(t, t_j) = 0$ *for all* $t_j > t_i$
3. *Arbitrageurs use trigger strategies, once they go short (at* $T(t_i)$*)* $\forall t > T(t_i)$ *they keep their position* $\sigma(t, t_i) = 1$.

Sell Out Condition

- The expected payoff to arbitrageur t_i from selling out at time t is given by

$$\int_{t_i}^{t} e^{-rs}\left[1-\beta\left(s-T^{*-1}\left(s\right)\right)\right]p\left(s\right)\pi\left(s|t_i\right)ds+e^{-rt}p\left(t\right)\left(1-\Pi\left(t|t_i\right)\right.$$

- Arbitrageurs will keep maximum short position if

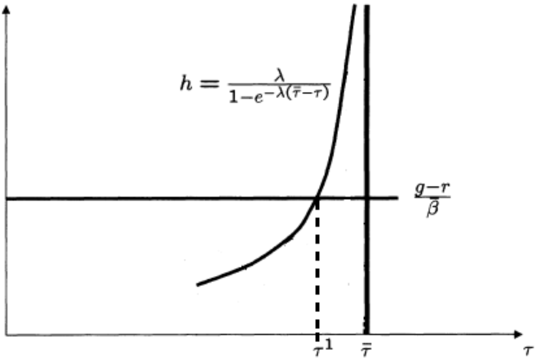
$$\frac{\pi\left(t|t_i\right)}{\left(1-\Pi\left(t|t_i\right)\right)}>\frac{g-r}{\beta\left(t-T^{*-1}\left(t\right)\right)}$$

- and maximum long position otherwise.

Persistence of Bubbles

- Under the common knowledge of bubble there is unique equilibrium in which bubble burst immediately.
- In this model existence of bubble is never a common knowledge:
  - arbitrageurs become sequentially informed.
  - they don't know their position in "line".
  - this breaks common knowledge in the model.
- If κ or η is large enough, then we can show that bubble will always persist for some time;
- That is there is no equilibrium in which all arbitrageurs sell their stocks at the moment they receive the news.

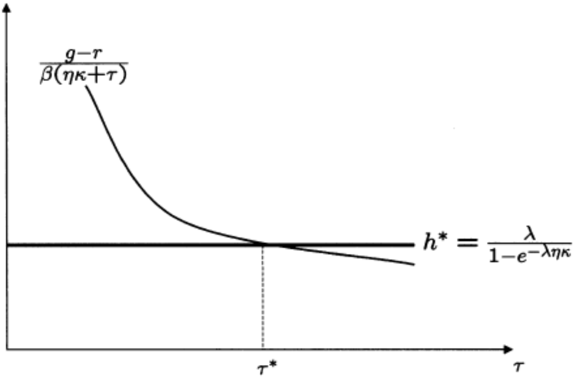
Persistent of Bubbles (exogenous crash)

- 

- This is true if

$$t_0 + \eta\kappa + \tau^1 > t_0 + \overline{\tau}$$

Persistent of Bubbles (endogenous crash)

- Otherwise, bubble will burst for endogenous reasons
- 

- If κ,η or $g - r$ are large enough then each arbitrageur waits for a strictly positive period of time
- Hence, the bubble burst at $\tau^* > 0$

Impact of news

- At the heart of the problem is the coordination problem between arbitrageurs;
- A public news may act like a coordination device;
- Hence even if the news have little informative content it may lead to a market crash.

Conclusions

- Presence of rational, fully informed arbitrageurs do not preclude existence of bubble;
- A bubble may last for a long time even if agents are aware of it;
- This is possible due to lack of common knowledge;
- News can have large impact on behavior of agents by acting like a coordination device

Appendix (News)

- News arrive with Poisson arrival rate θ;
- They are uninformative and serve only as a coordination device;
- They are observed only by traders who are aware of the bubble for time interval τ_e;
- "News" leads to multiplicity of equilibria;
- There is an equilibrium such that:
  - all arbitrageurs who observe the news sell out;
  - if the bubble burst they stay out of the market;
  - if the bubble doesn't burst they re-enter the market.