Bubbles and Crashes
Abreu and Brunnermeier

Discussion by Michal Szkup

NYU

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Introduction

The goal of this paper is to show that:

- even in the presence of rational arbitrageurs a bubble may exists for a long time;
- bubble may exists even though enough agents are aware of it and would be able to burst it if they act collectively;
- lack common knowledge assumption is crucial for existence of bubbles;
- in such environment, news may have disproportional impact on market compared to their information content.
Model (Price Process)

- Price process is assumed to be exogenous;
- At time $t = 0$ the price is $p_0 = 1$;
- From $t = 0$ onwards, price grows at rate $g$, $p_t = e^{gt}$;
- Until the time $t_0$ the price is justified by fundamentals;
- For any $t > t_0$, only a fraction $1 - \beta (t - t_0)$ of the price is justified by fundamentals;
- $t_0$ is a random variable with exponential distribution, $\Phi (t_0) = 1 - e^{-\lambda t_0}$;
- The price process is assumed to be driven by behavioral traders;
The bubble will burst for exogenous reasons at $t_0 + \tau$;
  ▶ in that case the price drops to $(1 - \beta) p(t)$;

Or if the selling pressure exceeds level $\kappa$.
  ▶ in this case the price drops to $(1 - \beta (t - t_0)) p(t)$;
Model (Arbitrageurs)

- There is mass 1 of rational, risk-neutral agents (arbitrageurs);
- These arbitrageurs become sequentially aware of the bubble;
- At each $t_0 < t < t_0 + \eta$ a mass $\frac{1}{\eta}$ of arbitrageurs becomes aware of mispricing;
- Arbitrageurs do not observe $t_0$ and don’t know how many of others observed signal;
Model (Arbitrageurs)

- Type of arbitrageur is $t_i \in [t_0, t_0 + \eta]$, the time he received the news;
- Arbitrageurs choose their holding of a stock in $[0, 1]$, where 1 is the maximum short position they can take;
- Every time an arbitrageur changes his position he incurs a cost $ce^{rt}$;
- We refer to arbitrageurs position as his selling pressure.
- $\Pi(t|t_i)$ is the belief of type $t_i$ that the bubble burst before time $t$. 

Abreu and Brunnermeier (2003) Bubbles and Crashes 04/11 6 / 16
Model (Strategies)

- Let $\sigma(t, t_i)$ denote the selling pressure of arbitrageur $t_i$ at time $t$.
- A strategy of an arbitrageur $i$ is given by $\sigma(\cdot, t_i): [0, t_i + \bar{\tau}] \to [0, 1]$;
  $$\min\{t, t_0 + \bar{\tau}\}$$
- The aggregate selling pressure is $s(t, t_0) = \int_{t_0}^{\min\{t, t_0 + \bar{\tau}\}} \sigma(t, t_i) \, dt_i$
- We say that there is a bubble if $\kappa$ of arbitrageurs are aware of mispricing;
  - this happens at time $t_0 + \kappa \eta$;
- The time that bubble burst is then
  $$T^*(t_0) = \inf \{t \mid s(t, t_0) \geq \kappa \text{ or } t = t_0 + \bar{\tau}\}$$
Summary of the model

\[ p_t = e^{gt} \]

\[ (1 - \beta(t - t_0))p_t \]

- \( t_0 \) to \( t_0 + \eta \): random starting point, \( K \) traders are aware of the bubble
- \( t_0 + \eta \) to \( t_0 + \bar{\eta} \): all traders are aware of the bubble
- \( t_0 + \bar{\eta} \): bubble bursts for exogenous reasons

maximum life-span of the bubble \( \bar{\tau} \)
Trading equilibrium and Preliminary Results

**Definition**

A trading equilibrium is a Perfect Bayesian Equilibrium such that if \( \sigma(t, t_i) > 0 \) arbitrager \( t_i \) believes \( \sigma(t, t_j) > 0 \) for all \( t_j < t_i \).

**Lemma**

1. \( \sigma(t, t_i) \in \{0, 1\} \)
2. \( \sigma(t, t_i) = 1 \Rightarrow \sigma(t, t_j) = 1 \) for all \( t_j < t_i \) and \( \sigma(t, t_i) = 0 \Rightarrow \sigma(t, t_j) = 0 \) for all \( t_j > t_i \)
3. Arbitrageurs use trigger strategies, once they go short (at \( T(t_i) \)) \( \forall t > T(t_i) \) they keep their position \( \sigma(t, t_i) = 1 \).
Sell Out Condition

- The expected payoff to arbitrageur $t_i$ from selling out at time $t$ is given by

$$\int_{t_i}^{t} e^{-rs} \left[ 1 - \beta \left( s - T^{*-1}(s) \right) \right] p(s) \pi(s|t_i) \, ds + e^{-rt} p(t) \left( 1 - \Pi(t|t_i) \right)$$

- Arbitrageurs will keep maximum short position if

$$\frac{\pi(t|t_i)}{(1 - \Pi(t|t_i))} > \frac{g - r}{\beta (t - T^{*-1}(t))}$$

- and maximum long position otherwise.
Persistence of Bubbles

- Under the common knowledge of bubble there is unique equilibrium in which bubble burst immediately.
- In this model existence of bubble is never a common knowledge:
  - arbitrageurs become sequentially informed.
  - they don’t know their position in "line".
  - this breaks common knowledge in the model.
- If $\kappa$ or $\eta$ is large enough, then we can show that bubble will always persist for some time;
- That is there is no equilibrium in which all arbitrageurs sell their stocks at the moment they receive the news.
Persistent of Bubbles (exogenous crash)

This is true if

\[ t_0 + \eta \kappa + \tau^1 > t_0 + \bar{\tau} \]
Persistent of Bubbles (endogenous crash)

- Otherwise, bubble will burst for endogenous reasons

- If $\kappa$, $\eta$ or $g - r$ are large enough then each arbitrageur waits for a strictly positive period of time

- Hence, the bubble burst at $\tau^* > 0$
Impact of news

- At the heart of the problem is the coordination problem between arbitrageurs;
- A public news may act like a coordination device;
- Hence even if the news have little informative content it may lead to a market crash.
Conclusions

- Presence of rational, fully informed arbitrageurs do not preclude existence of bubble;
- A bubble may last for a long time even if agents are aware of it;
- This is possible due to lack of common knowledge;
- News can have large impact on behavior of agents by acting like a coordination device.
Appendix (News)

- News arrive with Poisson arrival rate $\theta$;
- They are uninformative and serve only as a coordination device;
- They are observed only by traders who are aware of the bubble for time interval $\tau_e$;
- "News" leads to multiplicity of equilibria;
- There is an equilibrium such that:
  - all arbitrageurs who observe the news sell out;
  - if the bubble burst they stay out of the market;
  - if the bubble doesn’t burst they re-enter the market.

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