Can Sticky Price Models Generate
Volatile and Persistent Real Exchange Rates?

Chari, Kehoe, and McGrattan (REStud 2002)

Discussion by Fernando Leibovici
Real Exchange Rate

Definition

- $RER \equiv \frac{eP^*}{P}$
- Price of foreign output relative to domestic output in a common currency

US-Europe 1973Q1-1994Q4

<table>
<thead>
<tr>
<th></th>
<th>SD relative to GDP</th>
<th>Autocorrelation</th>
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</thead>
<tbody>
<tr>
<td>$RER$</td>
<td>4.36</td>
<td>0.83</td>
</tr>
<tr>
<td>$e$</td>
<td>4.67</td>
<td>0.86</td>
</tr>
<tr>
<td>$\frac{P^*}{P}$</td>
<td>0.71</td>
<td>0.87</td>
</tr>
</tbody>
</table>
Sticky Prices and RER Fluctuations

Mechanism

- Under sticky prices, \( \Delta \) money supply lead to
  - \( \Delta R \) such that money market clears
  - \( \Delta e \) such that all currencies offer same nominal expected rate of return
  - Stable \( \frac{P^*}{P} \)
  - Thus, \( \Delta \frac{eP^*}{P} \)
Sticky Prices and RER Fluctuations

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This paper asks:

- Can this mechanism account for the volatility and persistence of RER?
  1. Standard frictionless model + sticky prices
  2. RER volatility
  3. RER persistence
  4. Quantitative evaluation
Setup

- 2 countries → $H$ and $F$

- Agents in each country
  - Representative household
  - Representative producer of final consumption-capital good
  - Continuum $[0, 1]$ of intermediate good producers
  - Central bank

- Commodities
  - Continuum $[0, 1]$ of $H$ and $F$ intermediate goods → Traded internationally
  - $H$ and $F$ final consumption-capital goods
  - $H$ and $F$ money
  - Labor
Problem

\[ \max_{Y^H_{it}, Y^F_{it}} P_t Y_t - \int_0^1 P_{H,it} Y^H_{it} \, dt - \int_0^1 P_{F,it} Y^F_{it} \, dt \]

subject to

\[ Y_t = \left[ a_1 \left( \int_0^1 Y^\theta_{H,it} \, dt \right)^{\frac{\rho}{\sigma}} + a_2 \left( \int_0^1 Y^\theta_{F,it} \, dt \right)^{\frac{\rho}{\sigma}} \right]^{\frac{1}{\rho}} \]

Remark

- Home-bias \( \rightarrow a_1 > a_2 \)
Intermediate Good Producers in $H$

Technology

\[ Y_{H,it} + Y_{H,it}^* = K_{it}^{\alpha} L_{it}^{1-\alpha} \]

\[ K_{i,t+1} = (1 - \delta)K_{it} + X_{it} - \phi \left( \frac{X_{it}}{K_{it}} \right) K_{it} \]

Market structure

- Monopolistic competition
- International market segmentation $\rightarrow$ Pricing-to-market
Intermediate Good Producers in $H$: Pricing

**Exogenous stickiness**
- Prices are fixed for $N$ periods

**Endogenous stickiness: staggeredness**
- Interval $[0, 1]$ of producers is divided into $N$ sub-intervals
- Every period, producers from only 1 sub-interval adjust prices
- This introduces a backward-looking component to current optimal prices
Intermediate Good Producers in $H$: Pricing (cont.)

Without sticky prices

$$P_{H, it} = \frac{P_{\tau} mc_{it}}{\theta}$$

With sticky prices

$$P_{H, it} = \frac{\sum_{\tau=t}^{t+N-1} \sum_{S^\tau} Q_{\tau} \Lambda_{H, \tau} P_{\tau} mc_{i\tau}}{\theta \sum_{\tau=t}^{t+N-1} \sum_{S^\tau} Q_{\tau} \Lambda_{H, \tau}}$$

$$\Lambda_{H, \tau} = \theta_1 y_\tau P_{\tau}^{\theta_2} P_{H, \tau}^{\theta_3}$$
Representative Household

Problem

$$\max_{C_t, M_t, L_t, B_t} E_0 \sum_t \beta^t U(C_t, L_t, M_t / P_t)$$

subject to

$$P_t C_t + M_t + \sum_{S_{t+1}} Q(S_{t+1}, S^t) B(S_{t+1}) \leq P_t w_t L_t + M_{t-1} + B_t + \Pi_t + T_t$$

Remark

- Complete markets
Central Bank and Market Clearing

Central bank

\[ M_t = \mu_t M_{t-1} \]
\[ T_t = M_t - M_{t-1} \]

Market clearing conditions

\[ Y_t = C_t + \int_0^1 X_{it} \, di \]
\[ L_t = \int_0^1 L_{it} \, di \]
\[ B(S^t) + B^*(S^t) = 0 \forall S^t \]
Utility specifications

- Separable utility
  \[
  U(C, L, M/P) = \frac{1}{1-\sigma} \left[ \left( \omega C^{\frac{\eta-1}{\eta}} + (1 - \omega) \left( \frac{M}{P} \right)^{\frac{\eta-1}{\eta}} \right) \right]^{1-\sigma} + \psi \frac{(1-L)^{1-\gamma}}{1-\gamma}
  \]

- Nonseparable utility
  \[
  U(C, L, M/P) = \frac{1}{1-\sigma} \left[ \left( \omega C^{\frac{\eta-1}{\eta}} + (1 - \omega) \left( \frac{M}{P} \right)^{\frac{\eta-1}{\eta}} \right) (1 - L)^{\xi} \right]^{1-\sigma}
  \]
Calibration: Key Parameters

- Sticky prices $\rightarrow N = 4$
- Risk aversion $\rightarrow \sigma = 5$
- Money growth process
  - AR(1) with persistence 0.68
  - Cross-country correlation $= 0.5$ (to match GDP cross-country correlation)
  - SD $= 2.3\%$ to match GDP volatility (vs. 1.15\% in the data)
- Only disturbances are monetary shocks
Results: Real Exchange Rate

<table>
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<tr>
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<th>SD relative to GDP</th>
<th>Autocorrelation</th>
<th>Half-life</th>
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<tbody>
<tr>
<td>Data</td>
<td>4.36</td>
<td>0.83</td>
<td>3.72</td>
</tr>
<tr>
<td>Model (separable $U$)</td>
<td>4.27</td>
<td>0.62</td>
<td>1.45</td>
</tr>
<tr>
<td>Model (nonseparable $U$)</td>
<td>0.05</td>
<td>0.77</td>
<td>2.65</td>
</tr>
</tbody>
</table>

→ Tension between getting volatility and persistence right
Real Exchange Rate Volatility

Perfect international risk sharing

\[ q_t \equiv \frac{e_t P_t^*}{P_t} = \frac{U_{c,t}^*}{U_{c,t}} \]

Log-deviations around steady-state

- Separable utility

\[ \hat{q}_t \approx \sigma \left( \frac{c_t}{c_t^*} \right) \]

- Nonseparable utility

\[ \hat{q}_t \approx \sigma \left( \frac{c_t}{c_t^*} \right) - \Omega(\sigma) \left( \frac{L_t}{L_t^*} \right), \quad \Omega, \Omega' > 0 \]

→ Need high \( \sigma \) and separable utility
Real Exchange Rate Persistence

\[ \hat{q}_t \approx \sigma \left( \frac{c_t}{c_t^*} \right) \]

1. Persistent \( \hat{q}_t \) → Persistent \( \frac{c_t}{c_t^*} \)

2. To get persistent \( \frac{c_t}{c_t^*} \), need high price stickiness
   - Otherwise prices adjust fast, pushing the economy back to steady-state

→ So, what determines the speed of price adjustment?

• Tension between stickiness and cost movements
Consider $\uparrow C_t$ and $\uparrow q_t$ after a positive shock to the money supply

1. Labor supply with separable utility

\[ \uparrow w_t L_t^\sigma = \frac{\psi}{\downarrow U_{c,t}} \]

2. Marginal cost

\[ \uparrow mc_t = \frac{\uparrow w_t L_t^\alpha}{(1 - \alpha)K_t^\alpha} \]

3. Optimal prices

\[ \uparrow P_{H,it} = \frac{\sum_{\tau=t}^{t+N-1} \sum_{S_t} Q_{\tau} \Lambda_{H,\tau} \uparrow mc_{i\tau}}{\theta \sum_{\tau=t}^{t+N-1} \sum_{S_t} Q_{\tau} \Lambda_{H,\tau}} \]

4. $\uparrow P_t$

5. $\downarrow C_t$ and $\downarrow q_t$
Results: Volatility
Results: Persistence
Sensitivity Analysis

- Sticky wages
- Incomplete markets
- Real shocks: productivity and government consumption
- Nonseparable utility
- High exports
Conclusion

Can sticky prices generate volatile and persistent RER? “Some success”

- $\sigma = 5$ and separable utility $\rightarrow$ Volatility but can’t get enough persistence
- Nonseparable utility $\rightarrow$ Persistence but can’t get volatility

Remarks

- Staggered pricing are a weak source of endogenous stickiness (CKM 2000)
- Alternative sources of real pricing rigidities may deliver the puzzle?
- Correlation between $C$ and $RER$ is too high $\rightarrow$ Too much intl risk-sharing
Appendix: US-Europe RER
Appendix: Intermediate Good Producers in $H$

Problem

$$\max_{P_{H, it}, P_{H, it}^*, X_{it}, L_{it}} \sum_{t=0}^{\infty} \sum_{S^t} Q(S^t) \left[ P_{H, it} y_{H, it} + e_t P_{H, it}^* y_{H, it}^* - P_t w_t L_{it} - P_t X_{it} \right]$$

subject to

$y_{H, it}, y_{H, it}^*$ CES demand functions

$y_{H, it} + y_{H, it}^* = F(K_{it}, L_{it})$

$K_{it} = (1 - \delta)K_{i, t-1} + X_{it} - \phi \left( \frac{X_{it}}{K_{i, t-1}} \right) K_{i, t-1}$

$P_{H, i, \tau} = ... = P_{H, i, \tau+N-1}, \ \tau = t, t+N, ...$

$P_{H, i, \tau}^* = ... = P_{H, i, \tau+N-1}^*, \ \tau = t, t+N, ...$