"Time-Varying Risk, Interest Rates, and Exchange Rates in General Equilibrium"

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by Fernando Alvarez, Andrew Atkeson and Patrick Kehoe

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Introduction

- Consider the risk, in nominal terms faced by an investor in the US choosing between bonds denominated in either dollars or euros.
- If e_t is the exchange rate between euros and dollars then the risk premium, in logs, is equal to the expected log dollar return in euro bond minus the log dollar return on a dollar bond

$$p_t = i_t^* + \mathbb{E}_t \log e_{t+1} - \log e_t - i_t$$

  where i_t^* and i_t are the logs of the euro and dollar gross interest rates.
- Nominal exchange rates are viewed to be roughly random walks, so the expected depreciation of the currency, $\mathbb{E}_t \log e_{t+1} - \log e_t$ is roughly constant.
- The observed variation in $i_t^* - i_t$ is thus almost entirely do to variation in the risk premia.

Forward Premium Anomaly

- A more nuanced view of the of the data finds that when a currency's interest rate is high that currency is expected to appreciate.
- This is the *forward premium anomaly* and goes against intuition as one would expect investors would demand a higher interest rate on currencies which are expected to fall in value.
- Documented by the regression

$$\log e_{t+1} - \log e_t = a + b(i_t - i_t^*) + u_{t+1}$$

  such regressions usually yield estimates of b that are zero or negative
- The contribution of this paper is to generate a simple general equilibrium monetary model which creates both a variance in risk premium and the forward premium anomaly.
- This is done by allowing the asset market to be segmented, only a fraction of the model's agents choose to participate

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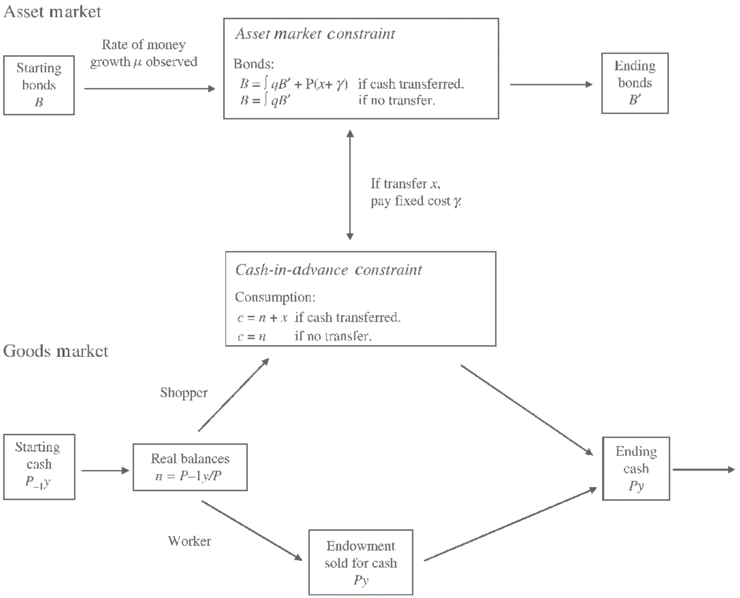
- Can transfer cash between asset market and goods market for a fixed cost γ. Distributed over households by $f(\gamma)$

Timing


Asset market
Starting bonds B
Rate of money growth μ observed
Asset market constraint
Bonds:
B = ∫ qB′ + P(x+ γ) if cash transferred.
B = ∫ qB′ if no transfer.
Ending bonds B′
If transfer x, pay fixed cost γ.
Cash-in-advance constraint
Consumption:
c = n + x if cash transferred.
c = n if no transfer.
Goods market
Shopper
Starting cash P−1y
Real balances n = P−1 y/P
Worker
Endowment sold for cash Py
Ending cash Py

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$$B(s^t, \gamma) = \int_{s_{t+1}} q(s^t, s_{t+1}) B(s^t, s_{t+1}, \gamma) ds_{t+1} + P(s^t)[x(s^t, \gamma) + \gamma] z(s^t, \gamma)$$

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- Agents have access to both asset markets but no arbitrage implies that $q(s^t, s_{t+1}) = q^*(s^t, s_{t+1}) e(s^t) / e(s^{t+1})$. WLOG home households only buy home debt.

Goods Market Details

- Households come in with real balances $n(s^t, \gamma)$.
- Consumption is then given by

$$c(s^t, \gamma) = n(s^t, \gamma) + x(s^t, \gamma) z(s^t, \gamma) \tag{1}$$

- The real balance next period is then given by $n(s^{t+1}, \gamma) = \frac{P(s^t)y}{P(s^{t+1})}$ with the caveat that in period $t = 1$ the real balance $n(s^1, \gamma)$ is given by $M_0 / P(s^1)$
- Agents have preferences

$$\sum_{t=1}^{\infty} \beta^t \int U(c(s^t, \gamma)) g(s^t) ds^t$$

over consumption of the home good.

Equilibrium

- The resource constraint for the home market is given by

$$\int \left[c(s^t, \gamma) + \gamma z(s^t, \gamma) \right] f(\gamma) d\gamma = y \tag{2}$$

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$$\int \left(n(s^t,\gamma) + [x(s^t,\gamma) + \gamma] z(s^t,\gamma)\right) f(\gamma) d\gamma = M(s^t)/P(s^t) \tag{3}$$

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- An *Equilibrium* is then a collection of bond and good prices (q,q^*) and (P,P^*) together with bond holdings (B,B^*) and allocations (c,x,z,n) (similarly for foreign) such that the households are maximizing, the government's budget constraint holds, and the resource and money market clearing constraints hold.

The Planner's Problem

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- In this economy inflation is only distorting because it induces households to use real resources to pay the fixed cost γ. This and the fact that households have a complete set of nominal claims means that an equilibrium will solve the planners problem

$$\max_{c_A, z} \sum_{t=1}^{\infty} \beta^t \int_{s^t} \int_{\gamma} U(c(s^t, \gamma)) f(\gamma) g(s^t) d\gamma ds^t$$

subject to the resource constraint and

$$c(s^t, \gamma) = z(s^t, \gamma) c_A(s^t, \gamma) + [1 - z(s^t, \gamma)] y/\mu_t$$

Solving the planners problem

- The planners problem above is derived under the assumption that $\bar{B}(\gamma)$ and $\bar{B}^*(\gamma)$ are chosen such that Pareto weights are equal.
- It is clear that that the planners problem is time independent and thus it reduces to finding $c_A(\mu)$ and $\bar{\gamma}(\mu)$ to solve

$$\max U(c_A(\mu))F(\bar{\gamma}(\mu)) + U(y/\mu)\left[1 - F(\bar{\gamma}(\mu))\right]$$

subject to

$$c_A(\mu)F(\bar{\gamma}(\mu)) + \int_0^{\bar{\gamma}(\mu)} \gamma f(\gamma) d\gamma + (y/\mu)[1 - F(\bar{\gamma}(\mu))] = y$$

- If $\gamma < \bar{\gamma}(\mu_t)$ then $c(s^t, \gamma) = c_A(\mu_t)$ and otherwise $c(s^t, \gamma) = y/\mu_t$.

Pricing

- The active consumers are the ones who price the assets and thus the pricing kernels become (similarly for foreign kernel)

$$m(s^t, s_{t+1}) = \beta \frac{U'(c_A(\mu_{t+1}))}{U'(c_A(\mu_t))} \frac{1}{\mu_{t+1}}$$

- The risk free dollar return then satisfies $R_{t+1}^{-1} = \mathbb{E}_t m_{t+1}$(also ${R_{t+1}^*}^{-1} = \mathbb{E}_t m_{t+1}^*$)
- No arbitrage $R_{t+1} = R_{t+1}^* e_{t+1}/e_t$ then gives us that $m_{t+1} e_{t+1}/e_t$ is a pricing kernel for the foreign good, which combined with complete markets gives us

$$\log e_{t+1} - \log e_t = \log m_{t+1}^* - \log m_{t+1}$$

- Taking logs of our equations for the risk free rate we get $i_t = -\log \mathbb{E}_t m_{t+1}$ and thus our risk premium can be expressed as

$$p_t = (\mathbb{E}_t \log m_{t+1}^* - \mathbb{E}_t \log m_{t+1}) - (\log \mathbb{E}_t m_{t+1}^* - \log \mathbb{E}_t m_{t+1})$$

Quadratic Approximation

- Take the second order log approximation of $U'(c_A(\mu_t))$ around $\bar{\mu}$ to get

$$\log U'(c_A(\mu_t)) = \log U'(c_A(\bar{\mu})) - \phi\hat{\mu} + \frac{1}{2}\eta\hat{\mu}^2$$

where $\hat{\mu}_t = \log \mu_t - \log \bar{\mu}$. Under CRRA $\phi = \sigma \frac{d \log c_A(\mu)}{d \log u}|_{\mu=\bar{\mu}}$

- With this quadratic approximation we have that the pricing kernel is given by

$$\log m_{t+1} = \log \frac{\beta}{\bar{\mu}} - (\phi + 1)\hat{\mu}_{t+1} + \frac{1}{2}\eta\hat{\mu}_{t+1}^2 + \phi\hat{\mu} - \frac{1}{2}\eta\hat{\mu}^2$$

- We will assume that the log of home money growth has normal innovations so

$$\hat{\mu}_{t+1} = \mathbb{E}_t\hat{\mu}_{t+1} + \epsilon_{t+1}$$

ϵ_{t+1} has mean zero and variance σ_ϵ^2

Forward Premium Anomaly

- The interest rate differential can be written as $i_t - i_t^* = \mathbb{E}_t \log e_{t+1} - \log e_t - p_t$. Letting ν_t be the real exchange rate $e_t P^* t / P_t$ one obtains

$$\mathbb{E}_t \log e_{t+1} - \log e_t = (\mathbb{E}_t \log \nu_{t+1} - \log \nu_t) + \mathbb{E}_t \left[\log \mu_{t+1} - \log \mu_{t+1}^*\right]$$

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- As the real exchange rate is the ratio of the marginal utilities of the active agents one can show that

$$\frac{d}{d\hat{\mu}_t}(\mathbb{E}_t \log \nu_{t+1} - \log \nu_t) = \phi \left[\frac{d(\mathbb{E}_t \hat{\mu}_{t+1})}{d\hat{\mu}_t} - 1\right]$$

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  an increase in money growth will lead to an expected appreciation of the exchange rate
- We also know the following formula, which gives us

$$\frac{dp_t}{d\hat{\mu}_t} = -\frac{\eta(\phi+1)\sigma_\epsilon^2}{1 - \eta\sigma\epsilon^2} \frac{d\mathbb{E}_t \hat{\mu}_{t+1}}{d\hat{\mu}_t}$$

Forward Premium Proposition

Proposition

If these inequalities are satisfied

$$\frac{\phi(1-\eta\sigma_\epsilon^2)}{1+\phi} < \frac{d\mathbb{E}_t\hat{\mu}_{t+1}}{d\hat{\mu}_t} \leq \frac{\phi}{1+\phi}$$

then for μ_t *close to* $\bar{\mu}$*, a change in money growth leads the interest rate differential and the expected exchange rate depreciation to move in opposite directions.*

Numerics

- Note that planners problem can be solved without knowing the process for μ.
- One can then specify

$$\hat{\mu}_{t+1} = g(\hat{\mu}_t) + \epsilon_{t+1}$$

to obtain the behavior one requires for the relationship between exchange rate and interest rate. (for instance their baseline is chosen such that the exchange rate is Martingale)

- These numerical results match the data qualitatively with exchange rates being more variable than interest rate differentials
- Endogenous segmentation is critical, as exogenous segmentation does not give necessary variation in interest rate differentials.

Advantages

- Market segmentation is a compelling mechanism for explaining the variation in the market price for risk.
- The change in consumption of active agents provides an intuitive mechanisms for the variation in discount factor and explaining the puzzles presented in the paper
- More over the paper is able to account for these puzzles with a simple general equilibrium model.

Limitations

- Unfortunately the results are very fragile and rely on knife edge situations.
- Model relies on the cost of transferring wealth from asset to goods market being real.
- It also relies upon the initial endowment of bonds being such that the planer's problem with equal weights constitutes a competitive equilibrium.
- It would be interesting to see if these results hold in a more generic economy, especially allowing the cost of transferring wealth to be in nominal terms.