Search and Rest Unemployment

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Introduction

- Search unemployment: costly reallocation in which a worker leaves current industry for another.
- Rest unemployment: costless activity in which a worker remains in industry but does not work, waits for industry conditions to improve.
- Equilibrium search and rest unemployment can be expressed in closed-form as a function of model primitives.
- Industry wage dynamics are determined by the same parameters.
- Use industry wage data to determine the relative contributions of search unemployment and rest unemployment to total unemployment.

Intermediate good industries

- $t \in [0, \infty)$.
- Continuum of industries indexed by $j \in [0, 1]$
- Each industry produces distinct intermediate good using CRS technology and labor input.
- Labor productivity in industry j subject to an idiosyncratic shock:

$$d \log x(j,t) = \mu_x dt + \sigma_x dz(j,t)$$

- Price of intermediate good, $p(j,t)$, and wage, $w(j,t)$, are determined competitively in each j at every instant, expressed in units of the final good.
- Industry j shuts according to Poisson process with arrival rate δ.

Final goods sector

- Final good produced with CRS technology:

$$Y(t) = \left(\int_0^1 y(j,t)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \ \theta > 0, \ \theta \neq 1$$

- Final goods sector takes price of intermediate goods $\{p(j,t)\}$ as given and determines $y(j,t)$ to maximize profits.

$$y(j,t) = \frac{Y(t)}{p(j,t)^{\theta}}$$

Households

- Representative household, unit measure of members.
- Full risk sharing.
- Household activities:
  - $L(t)$ household members are located in an industry.
    - $E(t)$ of these workers are employed at the industry wage and receive leisure 0.
    - $U_r(t)$ of these workers are rest employed and receive leisure b_r.
  - $U_s(t)$ workers are search unemployed and searching for a new industry, receiving leisure b_s.
  - The remaining workers are inactive, receiving leisure b_i, $b_i > b_s$.
- Switching between employment and rest unemployment is costless, as is switching between searching and inactivity.
- Worker cannot leave industry without spell of search unemployment.

Households, continued

- Workers can exit their industry for search unemployment or inactivity for three reasons:
  - They do so endogenously.
  - Their industry is dissolved (occurring at rate δ).
  - They exogenously quit (occurring at rate q).
- Workers in search unemployment find their industry of choice at rate α (directed search).
- Household preferences:

$$\int_0^\infty e^{-\rho t}\left[u\left(C(t)\right) + b_i\left(1 - E(t) - U_r(t) - U_s(t)\right) + b_r U_r(t) + b_s U_s(t)\right] dt$$

Household problem

We consider a stationary competitive equilibrium.

- Households must allocate members to employment across industries, rest unemployment across industries, search unemployment, and inactivity.
- Households value workers by the expected present value of marginal utility that they generate from leisure or income, $\upsilon \in [\underline{\upsilon}, \overline{\upsilon}]$.
- An individual who is permanently inactive contributes b_i/ρ to the household: $\underline{\upsilon} = b_i/\rho$.
- The expected present value of a searcher is $\underline{\upsilon}$; the searcher earns flow utility b_s and finds the best industry at rate α with capital gain $\overline{\upsilon} - \underline{\upsilon}$. Hence, $\rho\underline{\upsilon} = b_s + \alpha(\overline{\upsilon} - \underline{\upsilon})$, so

$$\overline{\upsilon} = b_i \left(\frac{1}{\rho + \kappa} \right), \quad \kappa \equiv \frac{1}{\alpha} \frac{b_i - b_s}{b_i}.$$

Wages

- Let $\omega(j,t)$ denote the log wage under full employment measured in units of marginal utility.

$$\omega(t) = \frac{\log Y(t) + (\theta - 1)\log x(j,t) - \log l(j,t)}{\theta} + \log u'(C(t))$$

  - If goods are substitutes and $\theta > 1$, more productive industries hire more workers.
  - If goods are poor substitutes and $\theta < 1$, an increase in productivity decreases employment.
- If $\omega(j,t) < \log b_r$, the household gains utility from allocating workers to rest unemployment.
- Hence, employment falls in industry j until the wage is equal to b_r.

Wages, cont.

Claim: The expected present value of a household member in an industry j depends only on $\omega(j,t)$,

$$v(\omega) = \mathbb{E}\left(\int_0^\infty e^{-(p+q+\delta)t}\left(\max\left\{b_r, e^{\omega(j,t)}\right\} + (q+\delta)\underline{v}\right) dt \middle| \omega(j,0)=\omega\right),$$

where expectations are taken with respect to the stochastic process for $\omega(j,t)$.

- We can now establish that $v(w)$ lies on a closed and bounded interval by the exit and entry behavior of workers.
- Hence, we have two thresholds for the log full employment wage, $\overline{\omega} > \underline{\omega}$, where

$$v(\omega) \in [\underline{v}, \overline{v}]\ \forall\omega,\ v(\overline{\omega}),\ \text{and } v(\underline{\omega}) = \underline{v} \text{ if } \underline{\omega} > -\infty. \tag{1}$$

Wage dynamics

Let $\hat{\omega} \equiv \min\{\underline{\omega}, \log b_r\}$.

- Consider the case in which $\hat{\omega} > \underline{\omega}$.
  - When $\omega(j,t) \in [\hat{\omega}, \overline{\omega})$, log employment falls at rate q, there is no rest unemployment, and wages change randomly.
  - When $\omega(j,t) \in (\underline{\omega}, \hat{\omega}]$, there is rest unemployment, log wages are constant at $\hat{\omega}$, and employment and rest unemployment change randomly with productivity.
- This implies $d\omega(j,t) = \mu dt + \sigma dz(j,t)$, where

$$\mu \equiv \frac{\omega - 1}{\omega}\mu_x + \frac{q}{\theta}, \ \ \sigma_\omega \equiv \frac{\omega - 1}{\omega}\sigma_x, \ \text{and}\ \sigma \equiv |\sigma| \tag{2}$$

- If finite, the thresholds $\overline{\omega}$, $\underline{\omega}$ act as reflecting barriers, as ω outside the barriers would be offset by exit/entry of workers.

Proposition 1

Equations (1) and (2) uniquely define $\underline{\omega}$ and $\overline{\omega}$ as functions of model parameters. A proportional increase in b_i, b_r, and b_s raise $e^{\underline{\omega}}$ and $e^{\overline{\omega}}$ by the same proportion. Moreover, $\underline{\omega} < \log b_i < \overline{\omega} < \infty$, with $\underline{\omega} > -\infty$ if and only if $b_r < b_i$.

Proposition 2

There exists a $\overline{b}_r$ such that in equilibrium, $b_r \gtreqless e^{\underline{\omega}}$ if and only if $b_r \gtreqless \overline{b}_r$, with $\overline{b}_r = B(\kappa, \rho + q + \delta, \mu, \sigma) b_i$ for some function B, positive-valued and decreasing in κ with $B(0, p + q + \delta, \mu, \sigma) = 1$.

Aggregation

- Steady state density of workers across log full employment wages can be expressed in closed form.
- Closed form expressions for rest unemployment and search unemployment can be derived as a function of $\overline{\omega}$, $\underline{\omega}$, $\hat{\omega}$, and model primitives.
- Unemployment rate and composition of unemployment is determined by relative advantage of leisure of each activity
- Output (and hence consumption and employment) depends on the absolute comparison of leisure versus market production.
- Effect of productivity on equilibrium employment depends on substitution and wealth effects.

Limiting Economy

- With the restriction $\mu_x + \frac{1}{2}(\theta - 1)\sigma_x^2 = 0$, invariant distribution still exists when $\delta \to 0$.
- With q=0,

$$\frac{U_r}{L} = \frac{\theta(\hat{\omega} - \underline{\omega}) + e^{-\theta(\hat{\omega} - \underline{\omega})} - 1}{\theta(\overline{\omega} - \underline{\omega})} \text{ and } \frac{U_s}{L} = \frac{\theta\sigma^2}{2\alpha(\overline{\omega} - \underline{\omega})}.$$

- Under parameter restrictions, $\mu = -\frac{1}{2}\sigma^2$.
- Then $-(\overline{\omega} - \underline{\omega})/\mu$ is the average duration of an industry spell.
- Hence, U_s/L equals the average duration of an industry spell over the average duration of a search unemployment spell.

Auxiliary Statistical Model

- Statistical model for industry wages:

$$\log w_{j,t+1} = \beta_w \log w_{j,t} + \log w_j + \epsilon_{j,t+1},\ \ \epsilon_{j,t+1} \sim N(0, \sigma_w) \quad (3)$$

- Proposition: If $\mu_x = -\frac{1}{2}(\theta - 1)\sigma_x^2$ and $\delta \to 0$, $\hat{\beta}_w$ and $\hat{\sigma}_w/\sigma$ depend only on q, $\theta\sigma$, $\alpha U_s/L$, and U_r/L.
- Estimate (3) using US 5-digit NAICS industry data at different levels of aggregation. (1990 to 2008, $J = 297$ Industries)
- Simulate the model for various values of the reduced form parameters that determine wage persistence.
- Estimate (3) from simulated data.

Results

- Five-digit aggregation of NAICS data: $\hat{\beta}_w = 0.874$.
- Simulation, no rest unemployment: $U_s/(U_s + L) = 0.055$, $\alpha = 3.2$, $b_r = 0$.
  - $q = 0 \Rightarrow$ largest possible $\hat{\beta}_w$ is 0.58
  - $q = 0.12$ (two-thirds of quits are exogenous) $\Rightarrow$ largest possible $\hat{\beta}_w$ is 0.71
- Simulation, three-fourths of total unemployment is rest unemployment: $U_s/(U_s + L) = 0.013$, $U_r/(U_r + L) = 0.042$, $\alpha = 3.2$
  - $q = 0 \Rightarrow$ largest possible $\hat{\beta}_w$ is 0.822, $\kappa = 2.7$
  - $q = 0.04 \Rightarrow$ largest possible $\hat{\beta}_w$ is 0.886, $\kappa = 10.1$
- High search employment implies that industry wages are frequently reflecting off of their lower bounds as industry workers search for better industries: decreases persistence of industry wages.
- Industries with rest unemployment pay a constant wage: increases persistence of industry wages.