Uncertainty, Productivity, and Unemployment in the Great Recession

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Facts about Unemployment and Productivity

- Decrease in output from 2008–2009 followed by recovery
- Persistent increase in unemployment from 2008–2010
- Rise in productivity from 2009–2010 after initial decrease
Standard Models Fail

Standard models of labor market frictions cannot produce the observed productivity and unemployment trends.
Goal of Paper

This paper aims to construct a dynamically tractable model that can produce these features, while being consistent with certain, non-targeted, cross-sectional facts.

- Motivated by cross-sectional firm growth patterns, the key modification is to introduce a shock to the variance of the innovation in firm level productivity.
Relation to the Literature


- The key theoretical contribution is to extend the existence and uniqueness proof of a Block Recursive equilibrium with multi-worker firms facing DRS production technology.
- Builds on recent “uncertainty” shock literature, but focuses on role of labor market imperfections.
Population and Technology

- A continuum of equally productive workers of mass 1
- An unrestricted mass of firms could enter the economy
- Workers and firms are risk neutral with same discount rate
- Time varying (Markov) aggregate and idiosyncratic productivity \((y_t, z_t)\)
- Production for firm with \(n\) workers: \(e^{(y+z)} F(n)\)
- \(F(n)\) increasing and concave
- Sunk entry cost \(k_e\) and period operating cost \(k_f\)
Labor Market

- There is a continuum of submarkets indexed by utility $x \in [\underline{x}, \bar{x}]$ promised to workers
- Firms post dynamic contracts (guaranteeing utility $x$) at cost $c$ per vacancy
- Workers direct their search to a particular submarket $x$ and accept offer with certainty
- Submarkets are characterized by tightness $\theta(x, y) = \text{vacancies}(x, y)/\text{searchers}(x, y)$
- Workers find job with prob $p(\theta)$
- Firms find workers with prob $q(\theta) := (p(\theta)/\theta)$
- Employed workers find job with reduced efficiency $\lambda p(\theta)$
Worker Problem: Unemployed

\[ U(y) = \max_{\hat{x}_u(\hat{y})} \quad b + \beta \mathbb{E}_{\hat{y}} [(1 - p(\theta(\hat{x}_u, \hat{y}))U(\hat{y})) + p(\theta(\hat{x}_u, \hat{y}))\hat{x}_u] \]
Worker Problem: Employed

Contract specifies:
\[ \{ w(W), \hat{\tau}(\hat{y}, \hat{z}, W), \hat{x}(\hat{y}, \hat{z}, W), \hat{d}(\hat{y}, \hat{z}, W), \hat{W}(\hat{y}, \hat{z}, W) \}, \]
where \( \hat{W} \) is promised utility guaranteed next period to worker.

\[
W(y, z; \{ w, \hat{\tau}, \hat{x}, \hat{d}, \hat{W} \}) = \\
w + \beta \mathbb{E}_{\hat{y}, \hat{z}} \left[ \hat{d} \hat{U}(\hat{y}) + (1 - \hat{d}) \left[ \hat{\tau} \hat{U}(\hat{y}) + (1 - \hat{\tau}) \lambda p(\hat{x}) \hat{x} + (1 - \hat{\tau})(1 - \lambda p(\hat{x})) \hat{W} \right] \right]
\]
Firm Problem

Firms hire a continuum of workers with potentially different contracts. $\phi(W)$ denotes the CDF of promised utilities within a firm: $n = \int d\phi$.

$$J(y, z, n, \phi) = \max_{w, \hat{\tau}, \hat{x}, \hat{d}, \hat{W}, \hat{\nu}, \hat{X}} e^{(y+z)} F(n) - k_f - \int wd\phi + \beta E\hat{y},\hat{z} [(1 - \hat{d})(-c\hat{\nu} + J(\hat{y}, \hat{z}, \hat{n}, \hat{\phi}))]$$

s.t.

$$\forall W, \ W(y, z; \{w, \hat{\tau}, \hat{x}, \hat{d}, \hat{W}\}(W)) \geq W$$

Law of motion for $n$ and $\phi(W)$
Simplifying Firm Problem

- Due to the contract structure, the optimal firm policies also maximize the joint surplus of a firm and its workers, $V$
- This greatly simplifies the solution as wages and the promise-keeping constraint drop out
- As the promise-keeping constraint drops out, the problem can be further simplified to remove $\phi$ as a state
- This, with free entry, allows for a block-recursive equilibrium
Free Entry

After realization of $y$, new firms may enter submarket $X$ by paying $k_e$ to draw $z$ and then post $v(y, z)$ vacancies leading to the free entry condition:

$$k_e \geq \max_{v(z)} \mathbb{E}_g \left[ J(y, z, vq(\theta(X, y)), \phi) - cv \right]^+$$

Rearranging yields the complementary slackness condition:

$$\forall X, \theta(X) \left[ \max_{v(z)} \mathbb{E}_g \left[ V(y, z, n) - \left( \frac{c}{q(\theta(X, y))} + X \right)n \right]^+ - k_e \right] = 0$$

Free entry makes $\theta$ a function of joint surplus $V$, independent of distribution $g$, allowing for block recursivity.
Properties of Block Recursive Equilibrium

- If there is positive free-entry, then a Block Recursive competitive equilibrium exist (under some technical assumptions)
- There exists a unique solution to the social planner’s problem
- If a Block Recursive competitive equilibrium exists, it coincides with the unique efficient allocations
Optimal Firm Policies

Optimal hirings, firings, quits, layoffs, and exit as a function of $(z, n)$ for fixed $y$: 
Reaching the Goal

Calibration follows standard practice in the literature. Productivities follow AR(1) processes. After calibration, run the following experiments:

▸ Negative permanent aggregate TFP shock
▸ Positive permanent shock to the variance of the innovation to firm productivity
▸ Both shocks at same time
▸ Shocks that replicate the 2009-2010 scenario
Permanent Decrease in Aggregate Productivity

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Permanent Increase in Firm Productivity Variance

(a) Output
(b) Unemployment
(c) Productivity
(d) Sales IQR

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Combining Permanent Shocks

(a) Output

(b) Unemployment

(c) Productivity

(d) Sales IQR

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Calibrated Productivity Shocks

Graphs showing:

(a) Output
(b) Unemployment
(c) Output per person
(d) Output
(e) Unemployment
(f) Output per person

Data and Model comparisons over the years 2008 to 2010.
Timeline
Uncertainty Shock
A Block Recursive competitive equilibrium is:

- A set of value functions:
  \[ V(y, z, n), U(y), J(y, z, n, \phi), W(y, z; \{w, \hat{\tau}, \hat{x}, \hat{d}, \hat{W}\} \]

- Optimal policy functions: \( \{w^*, \hat{\tau}^*, \hat{x}^*, \hat{d}^*, \hat{W}^*, \hat{\nu}^*, \hat{X}^*\} \)

- Where \( w^* \) depends only on \((y, z, W)\) and the rest on \((y, z, \hat{y}, \hat{z}, W)\)

- Labor market tightness: \( \theta^*(x, y) \)

Such that:

- Policies are optimal and satisfy all constraints
- There exists positive entrants every period
Firm Response to Negative Productivity Shock

![Graph showing firm response to negative productivity shock with axes labeled size n and idiosyncratic productivity z, with curves for separation, inactivity, exit, and hiring.](image-url)
Firm Response to Uncertainty Shock
Labor Market Flows
Appendix

Entry and Exit

(a) Entry

(b) Exit

(c) Entry

(d) Exit

Data entry

Model entry

Model

Entry and Exit

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